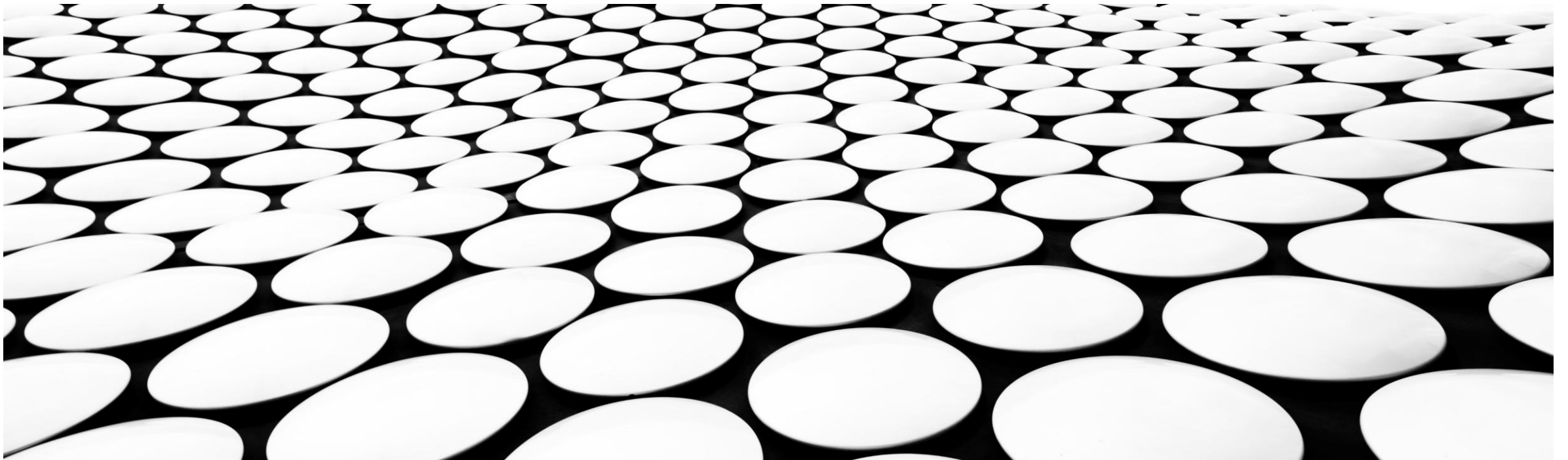

MATH CIRCLE AT FAU

12/14/2024



THE ISLAND OF KNIGHTS AND KNAVES



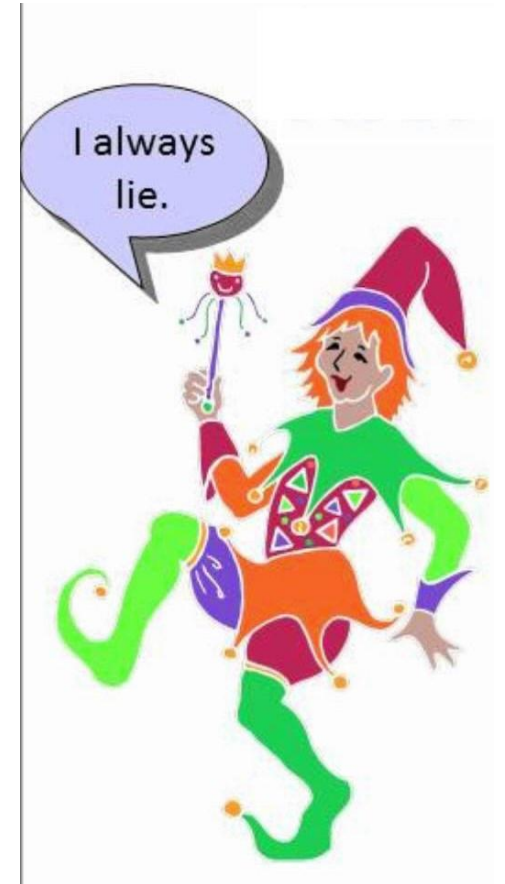
Here we are on the island of knights and knaves; The knights who can only tell the truth, the knaves who always lie.

You visit the island and meet Slippy, a **knave**.

Slippy tells you: *All my hats are green.*

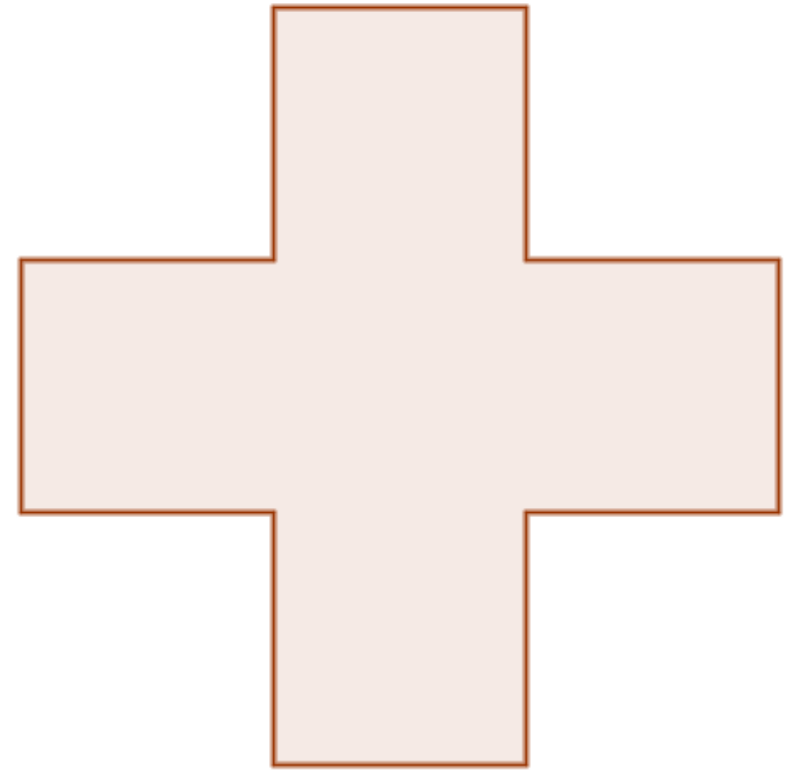
From what he said, knowing he is a lying knave, what can we conclude **for sure** from Slippy's statement?

- A. Slippy has at least one hat.
- B. Slippy has only one green hat.
- C. Slippy has no hats.
- D. Slippy has at least one green hat
- E. Slippy has no green hat.

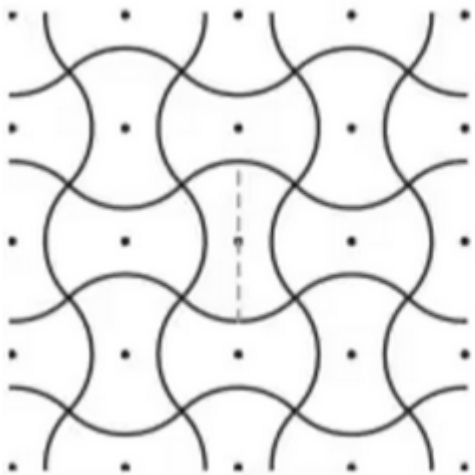


THE CRUX OF THE MATTER

The cross pictured on the right, having all arms of the same length, can be divided by two straight cuts into four (or five?) pieces that can be assembled to form a square. Try to find the cuts.



TWISTED TILES



The edges of these identical tiles are quarter circles, and their centres are the points marked. Determine the area of a tile, measured in cm^2 , given that the height of a standing tile is 12cm.

Alex Belos, in The Guardian

BERTIE WOOSTER'S WANDERINGS

Every weekday Mr. Wooster returns from his club in the city (where he spends the day drinking tea and gossiping) by train, to the train station of the town where he lives. His butler Jeeves is supposed to pick him up by car and drive him home. No time is to be wasted; Jeeves is to leave the car, so he exactly arrives at the train station the moment the train pulls in. Mr. Wooster jumps into the car, and they drive immediately home. One day the train arrives early, Jeeves isn't there yet, and Mr. Wooster decides to walk home. After walking for half an hour he meets Jeeves on the way to pick him up. He gets into the car, and they arrive at the home 20 minutes earlier than usual. How many minutes early was the train?



THE DAY OF THE WALKERS

- Suppose there are exactly 9 towns in a very small country and **all distances between the towns are different**. So, for example, if two towns are at a distance of 1 mile from each other, all other towns are at a distance other than 1 mile from these two, and no other pair of towns are at a distance of 1 mile from each other.
- One morning, a person starts in each town and walks towards the **nearest** town.
- Prove:
 - (a) There are two towns A and B such that a person from A walks to B, and a person from B walks to A.
 - (b) There is a town that nobody walks to.

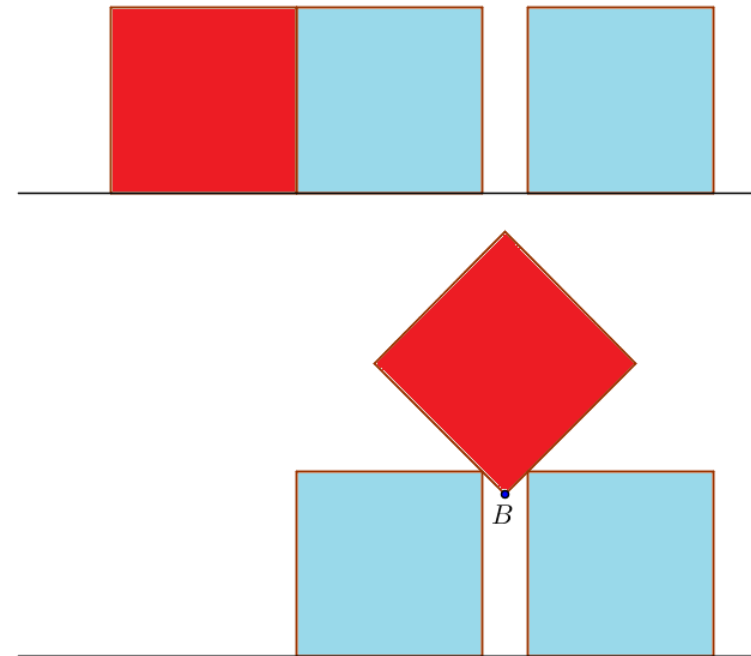
HINT: Try it first with 3 towns, 5 towns, ...

QUIRKY SQUARES

Three 4-inch squares are placed with their bases on a line; the first square and the second square are side by side, the third one is one inch to the right of the second one, as in the top picture..

The leftmost square is then lifted out and rotated 45° , then it is centered and lowered into the space between the second and third square until it touches both of these squares, as in the bottom picture.

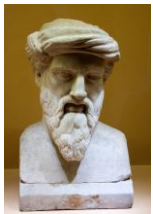
How many inches is the point B from the line on which the bases of the squares were originally placed?



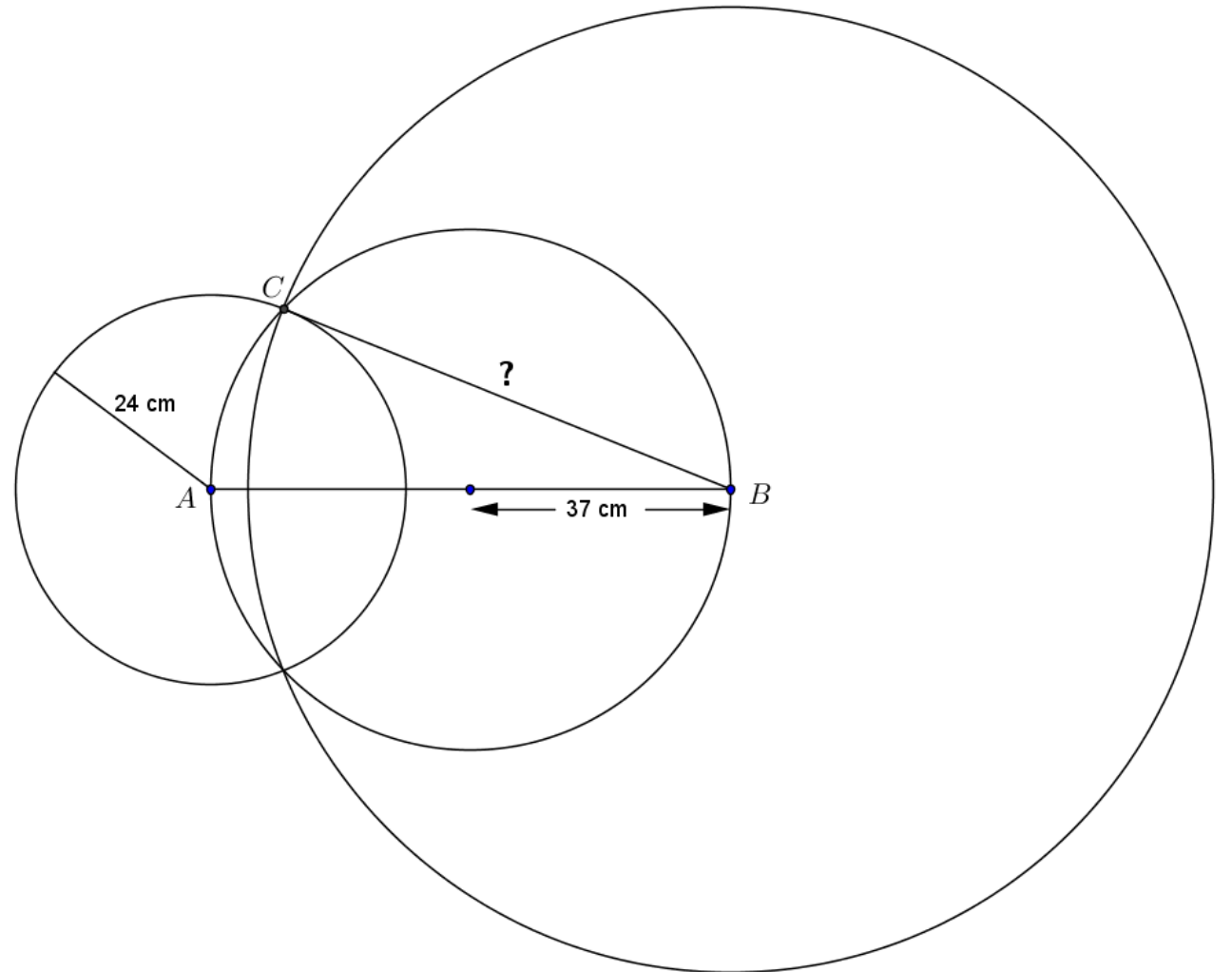
LOOKS HARD, BUT IS IT?

The circle of diameter AB has a radius 37 cm long. A circle of radius 24 cm. is drawn centered at A and it intersects at C a circle centered at B .

What is the radius of the circle of center B ?



A bust supposed to be (but almost certainly isn't) of Pythagoras. Here for inspiration.



ROOTING IT OUT

A red circle is inscribed in a square of side length 6. The green circle is tangent to the red one and to two sides of the square.

What is the radius of the green circle?

Possible answers are:

- A. $\frac{1}{2}$
- B. $9 - 6\sqrt{2}$
- C. $18\sqrt{2} - 25$
- D. $6 - 4\sqrt{2}$
- E. $6\sqrt{2} - 8$

You must justify your answer!

