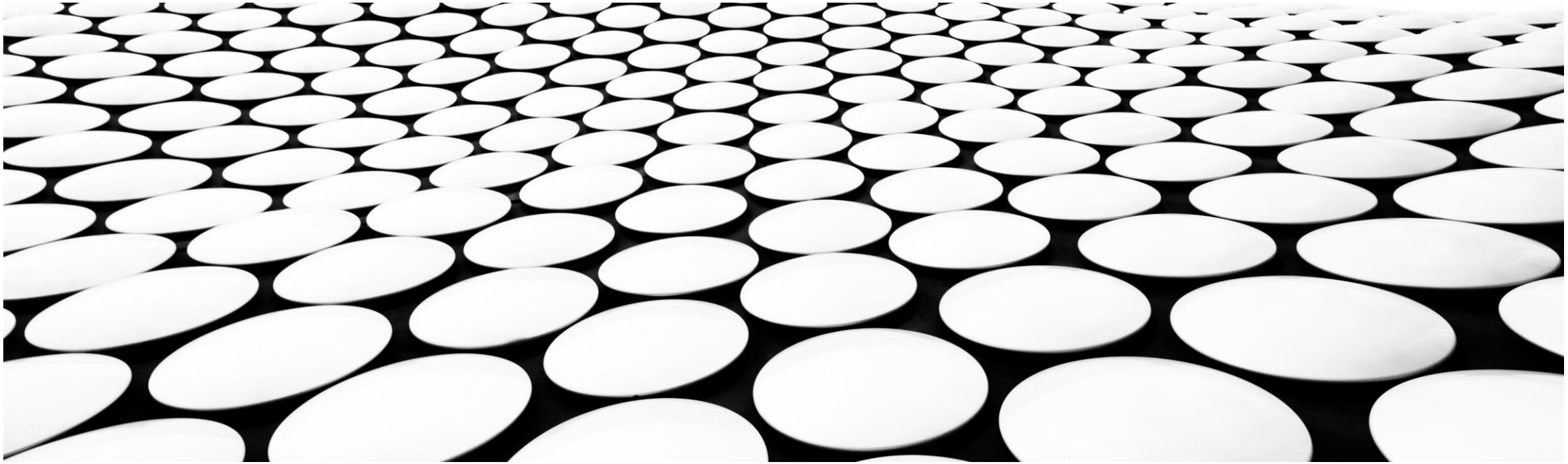
MATH CIRCLE AT FAU

11/16/2024



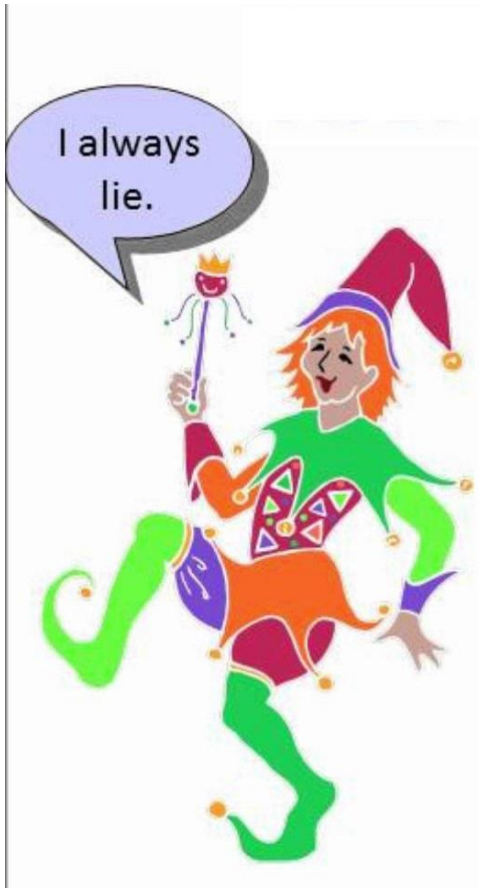
THE ISLAND OF KNIGHTS AND KNAVES

Here we are on the island of knights and knaves; The knights who can only tell the truth, the knaves who always lie.

You visit the island and meet one of the locals, Al.

Al tells you: “I love dogs.”
He then goes on to tell you
“If I love dogs then I love cats

Is Al a knight or a knave?



THE ISLAND OF KNIGHTS AND KNAVES

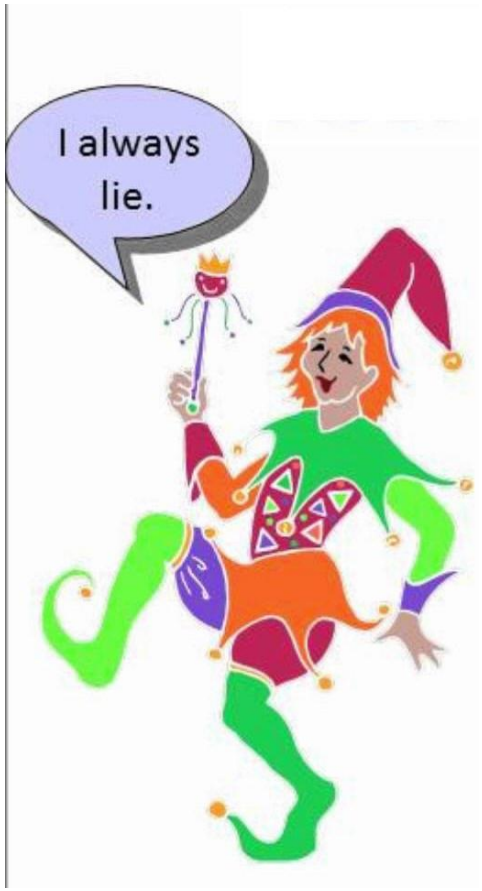
We are still on the island of knights and knaves; The knights who can only tell the truth, the knaves who always lie.

You visit the island and meet three locals, Ali, Baba, and Chippy..

Ali tells you: “Baba is a knight.”

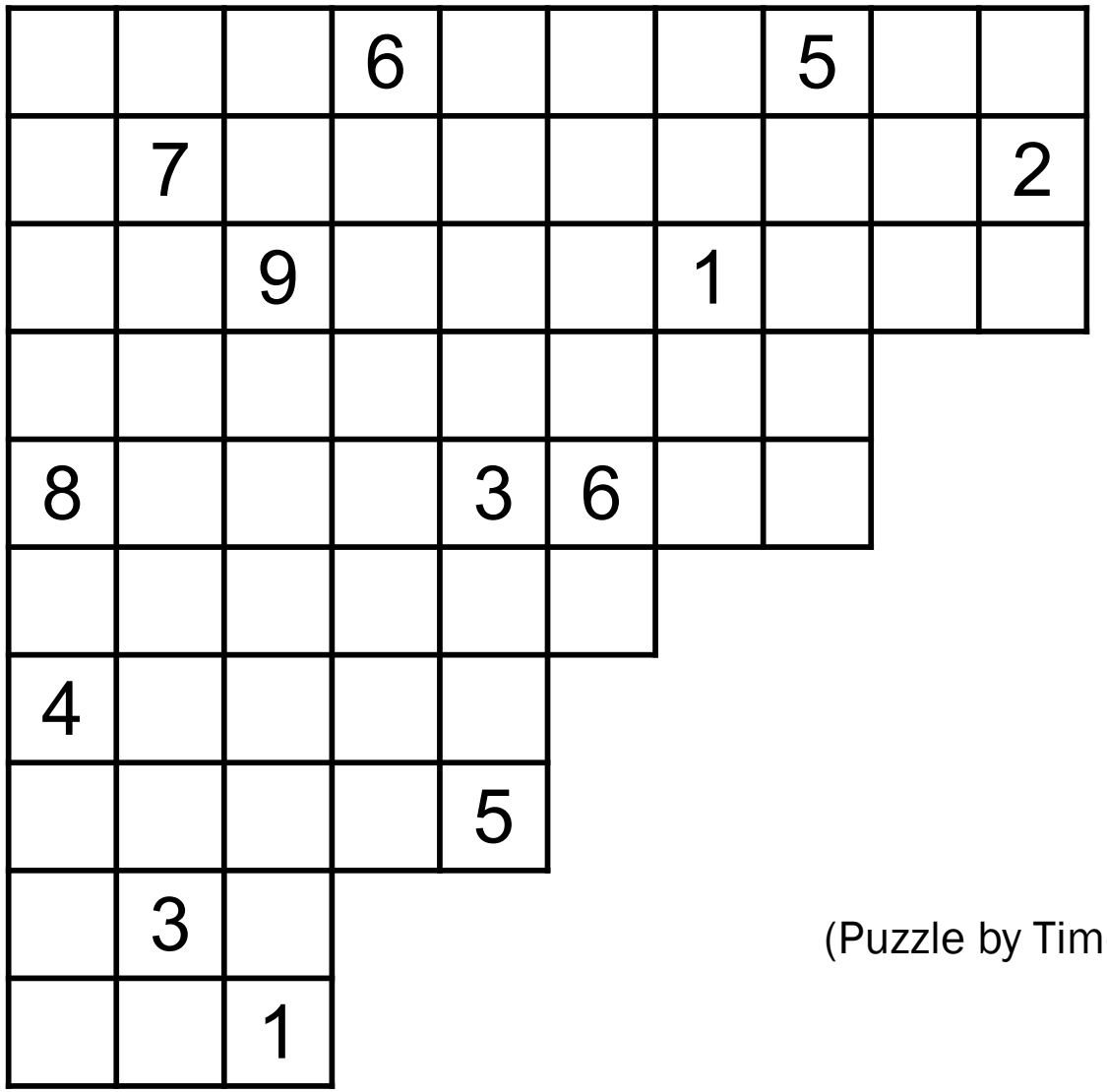
Baba tells you: “If Ali is a knight, so is Chippy.”

What are Ali, Baba, and Chippy?



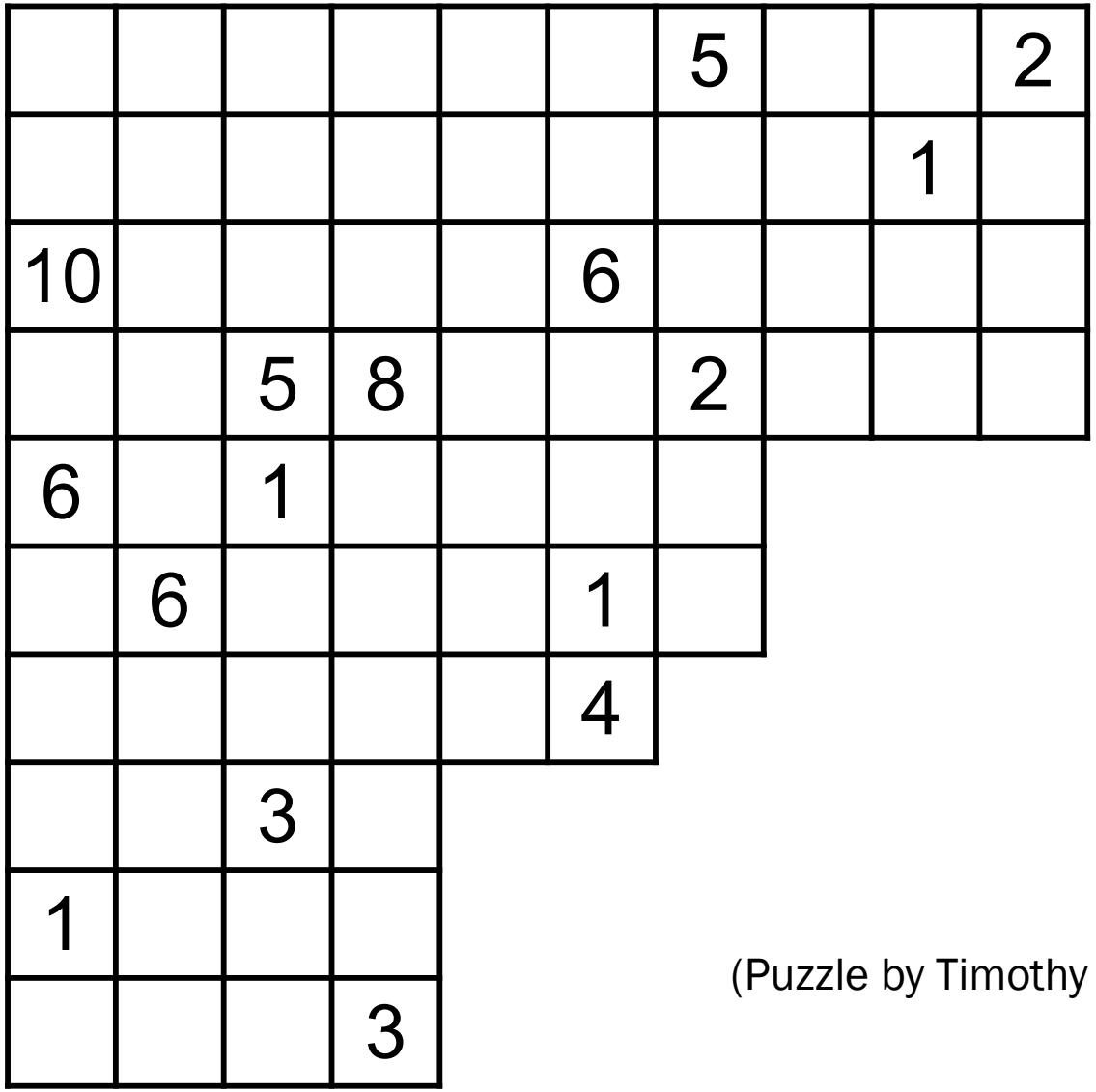
LATIN TABLEAU

In a Latin tableau, each row must contain some permutation of he numbers from 1 to r, where r is the length of that particular row. Each column must contain some permutation of the numbers from 1 to c, c being the height of that particular column. On the right there is a Latin tableau, except that some entries have been erased. Your job is to restore the missing entries.



(Puzzle by Timothy Y. Chow)

HERE IS ANOTHER ONE, TO DO AT HOME.

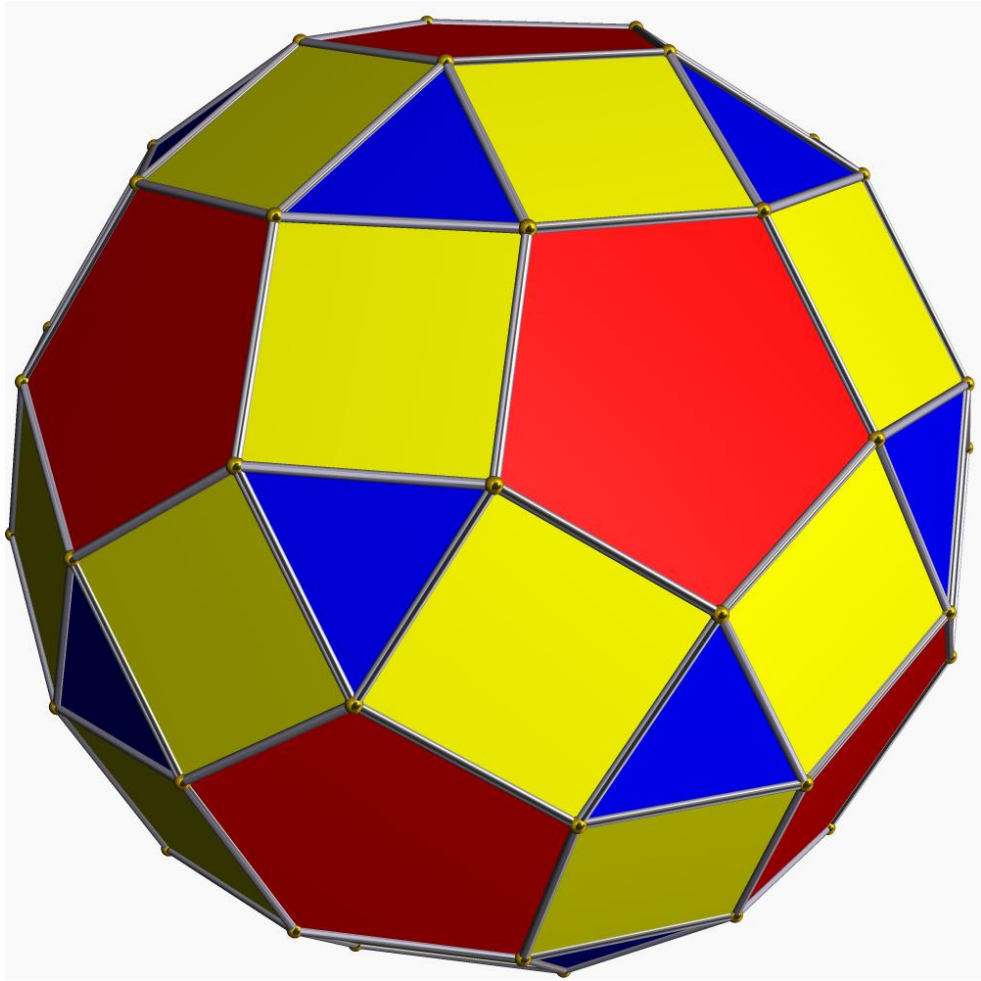


(Puzzle by Timothy Y. Chow)

YOU EITHER KNOW THIS, OR YOU'LL LEARN SOMETHING!

The pictured polyhedron has 60 faces and 62 vertices.

How many edges does it have?

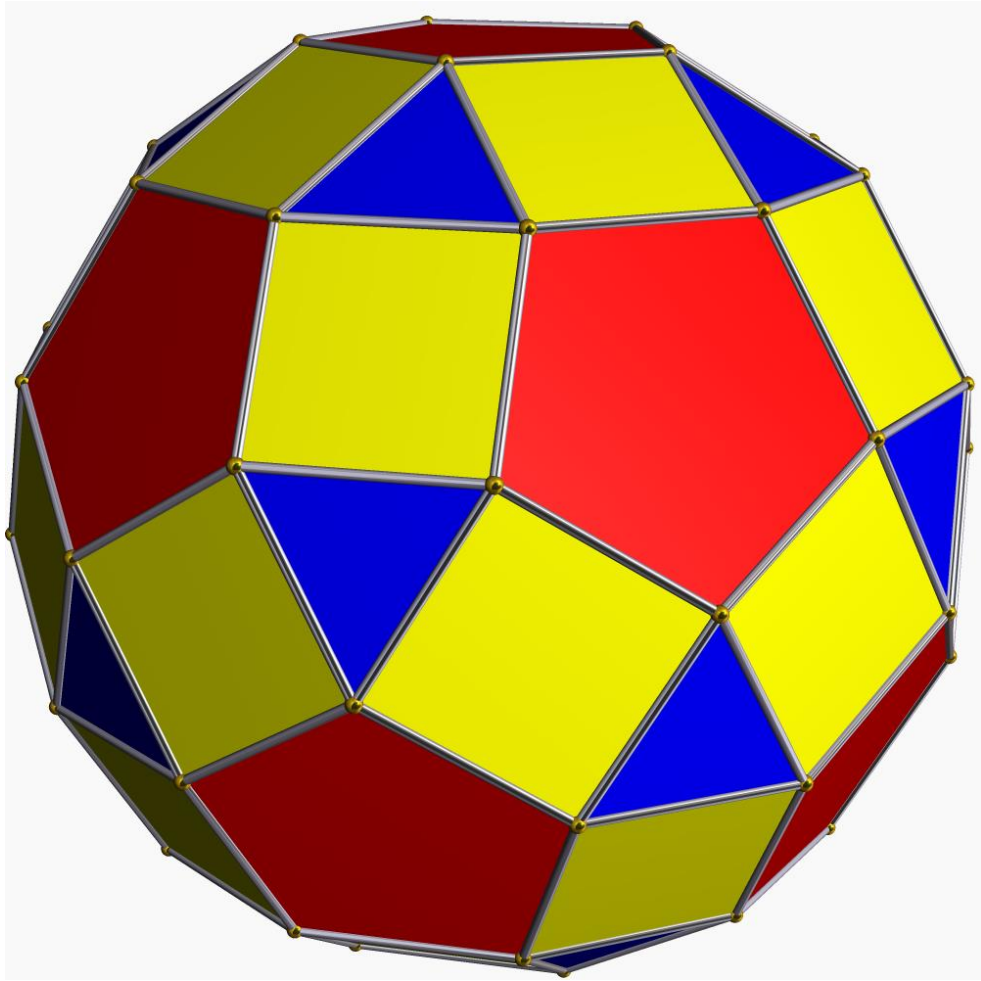


YOU EITHER KNOW THIS, OR YOU'LL LEARN SOMETHING!

The pictured polyhedron has 60 faces and 62 vertices.

How many edges does it have?

Euler's formula: $V - E + F = 2$.



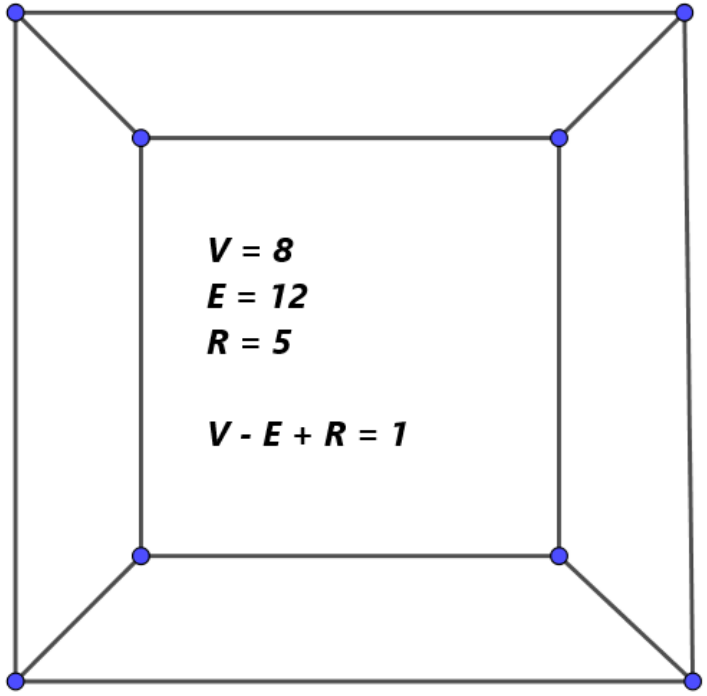
EULER'S FORMULA FOR GRAPHS

If we remove one side of a convex polyhedron, we can flatten it an open it up to a connected graph.

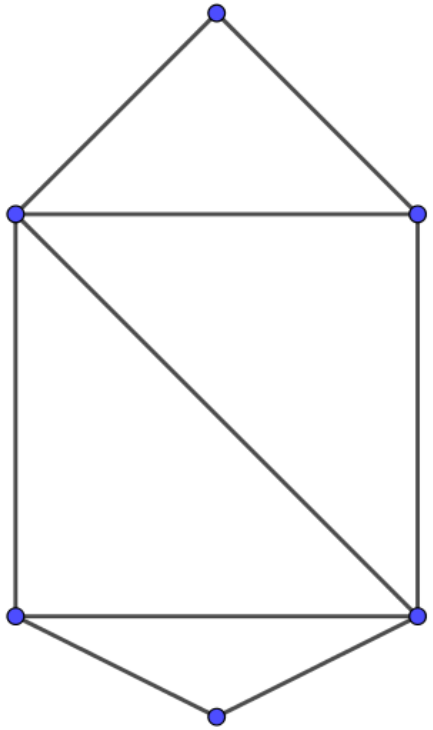
The faces become bounded regions, but there is one face missing. So, with V, E as before, R the number of regions, Euler's formula for a connected graph is

$$V - E + R = 1.$$

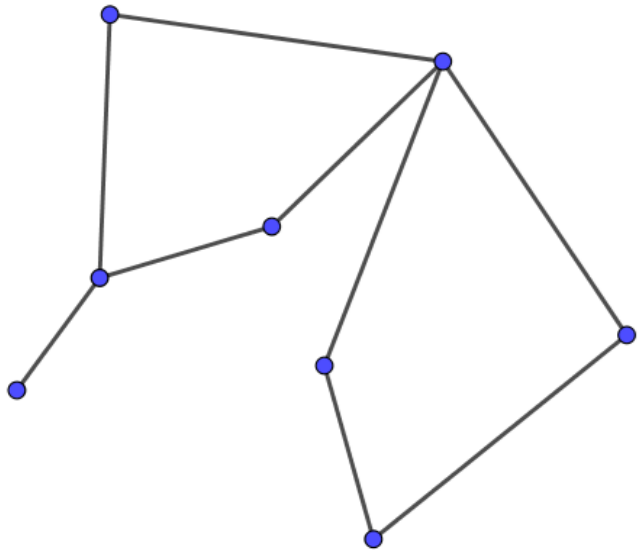
A squashed cube



GLAMOROUS GRAPHS



6 vertices, 9 edges, 4 regions



8 vertices, 9 edges, 2 regions

If a connected graph has 120 vertices and the *order* of each vertex Is 3.

How many regions does it have?

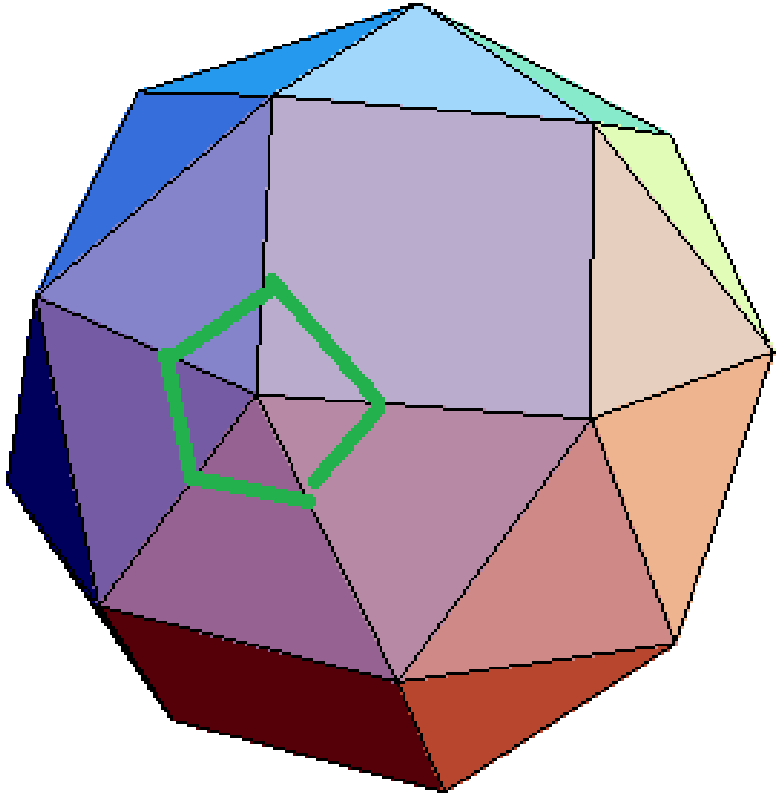
A TOUGH TOUGHIE ?

Suppose P is a convex polyhedron. Get a new polyhedron Q by cutting off a tip from each vertex. Suppose Q has V vertices, E edges, and F faces, and one of V, E, F equals 1001. How many edges does P have?

(From *Bicycle or Unicycle*,

by

D. Velleman and S. Wagon)



EULER AND SOCCER

- Soccer balls are made by stitching together pentagonal and hexagonal pieces, with three pieces meeting at each vertex. If such a ball is made using p pentagonal pieces and h hexagonal pieces, what is the answer to the following questions.

A. True or False? There could be any number of pentagonal pieces.

B. The number of pentagonal pieces must always be the same and it equals _____?

C. True or False? There could be any number of hexagonal pieces.

D. The number of hexagonal pieces must always be the same and it equals _____ ?

FROM LAST TIME: TRIANGULAR TRIANGULATIONS

- Segments are drawn in triangle ABC in such a way that D is the midpoint of BF, E is the midpoint of AD, and F is the midpoint of CE.

- If the area of triangle ABC is 1, what is the area of triangle DEF?