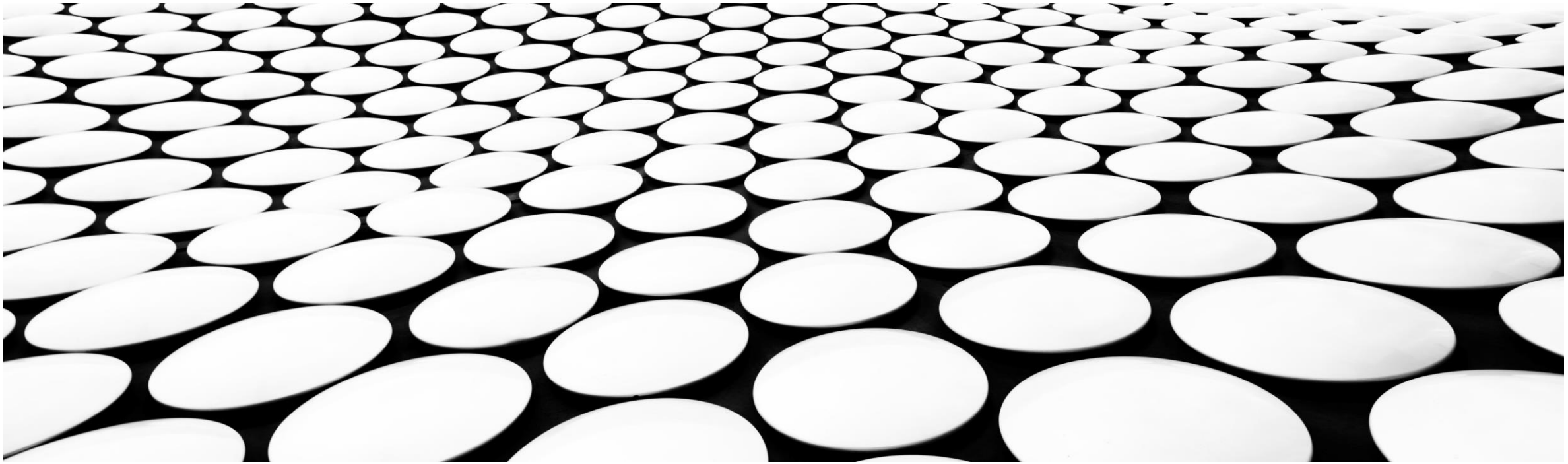
MATH CIRCLE AT FAU

11/02/2024



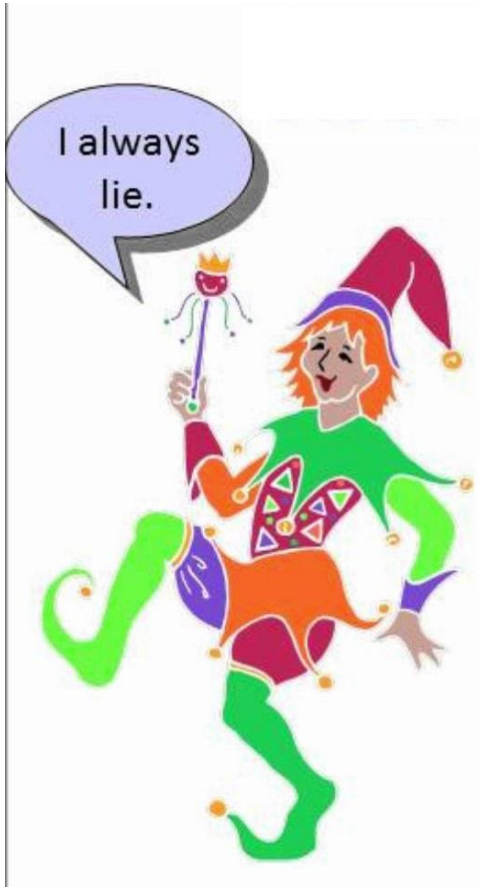
THE ISLAND OF KNIGHTS AND KNAVES

Here we are on the island of knights and knaves; The knights who can only tell the truth, the knaves who always lie.

You visit the island and meet two people, Alice and Bob.

Alice tells you: “We are both knaves.”

What are Alice and Bob?



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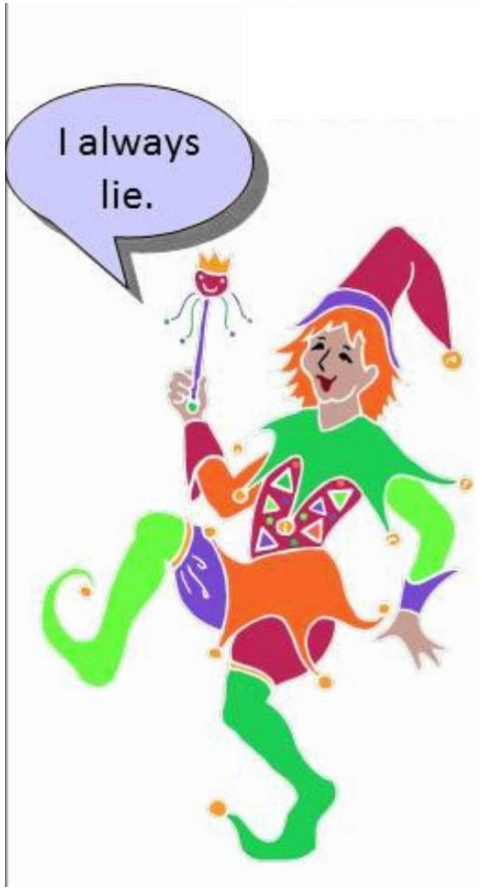
The solution: Alice is a knave and Bob is a knight.



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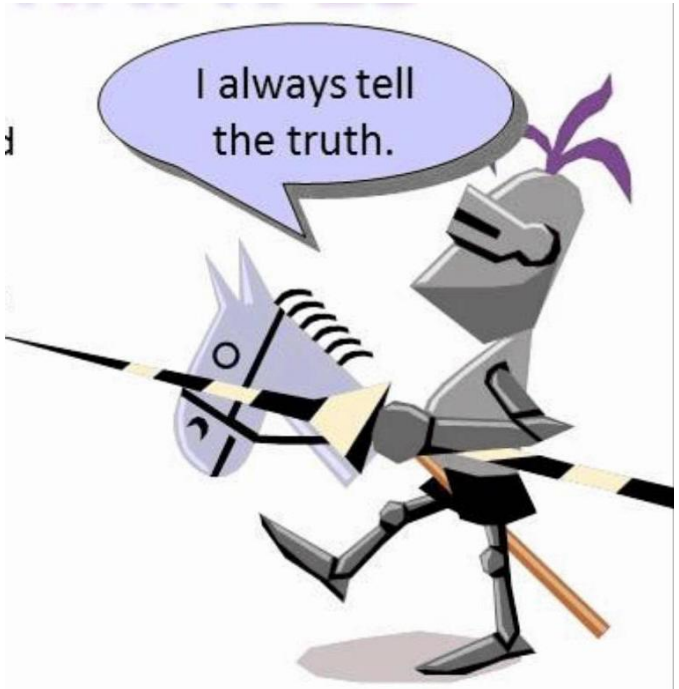

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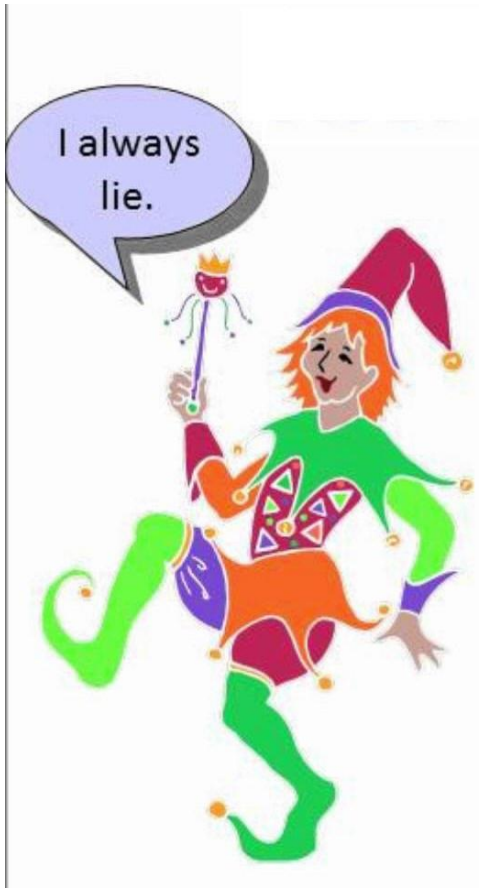
1. Did you get it?
2. Can you explain the logic for us?



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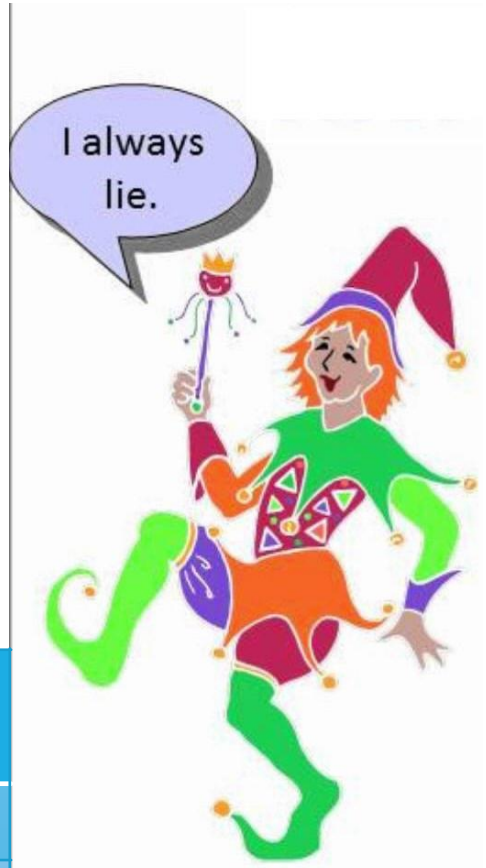
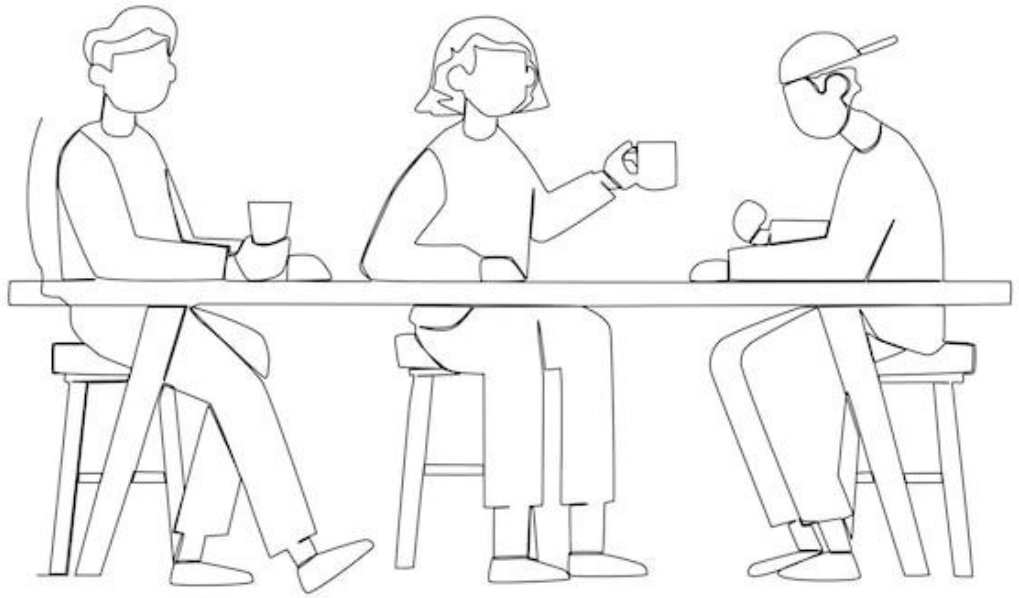
Truth table

Alice	Bob	Alice’s Statement
True	True	False
True	False	True
False	True	True
False	False	True



ACADEMICIANS AT LUNCH

Two men and one woman were having lunch together in the university cafeteria; they were Professor Merle White of the mathematics department, Professor Leslie Black of philosophy, and Jean Brown a secretary from the registrar’s office.
"Isn't it remarkable," observed the female members of the group, "that our last names are Black, Brown and White and that one of us has black hair, one has brown hair, and one white."
"It is indeed," replied the person with black hair, "and have you noticed that not one of us has hair that matches his or her name?"
"Wow! you're right!" exclaimed Professor White.
If the female member does not have brown hair, what is the color of Professor Black's hair?



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"Wow! you're right!" exclaimed Professor White.

If the female member does not have brown hair, what is the color of Professor Black's hair?

Solution. Professor White must have brown hair; it can't be white (that would match his or her name), nor black, since he/she responds to a comment by the black-haired person. Since the woman can't have brown hair, that means that Professor White is one of the men. The woman cannot have black hair, since the black haired person responded to her comment, so the woman has white hair. We next notice that Jean Brown has to be one of the men. In fact, suppose JB is the woman. If JB is the woman her hair would be white, but with Professor White having brown hair, that would force Professor Black to have black hair; a no-no. Thus, JB is a man, his hair is black, and Professor Black is the woman, and has white hair.

(*Martin Gardner's Best Logical and Mathematical Puzzles*.)

2016 AMC 8 PROBLEM 1

P1: The longest professional tennis match ever played lasted a total of 11 hours and 5 minutes. How many minutes was this?

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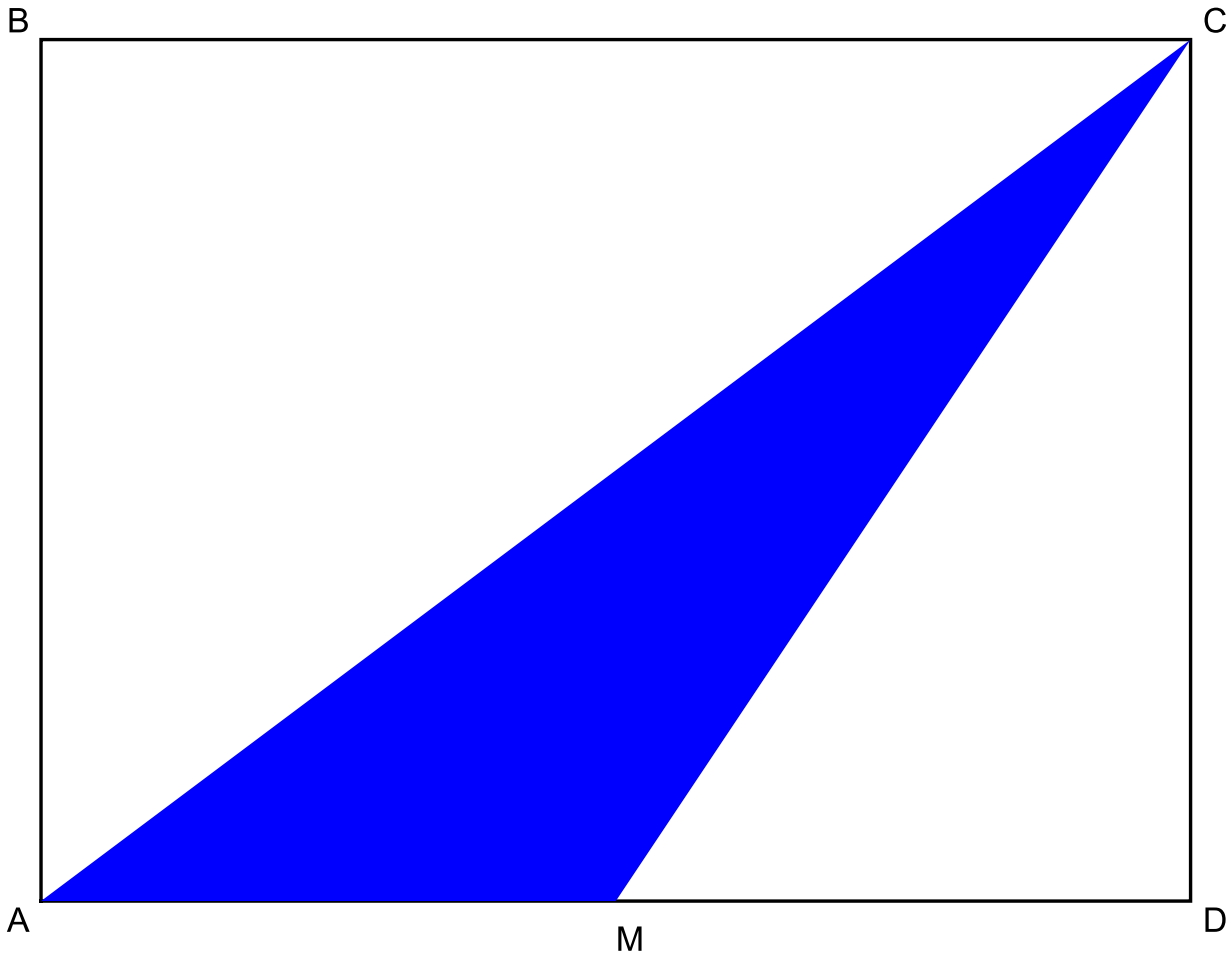
Solution: 665

Explanation: 11*60+5 = 665

2016 AMC 8 PROBLEM 2

P2: In rectangle ABCD, AB = 6 and AD= 8. Point M is the midpoint of $\overline{AD}$. What is the area of ΔACM?

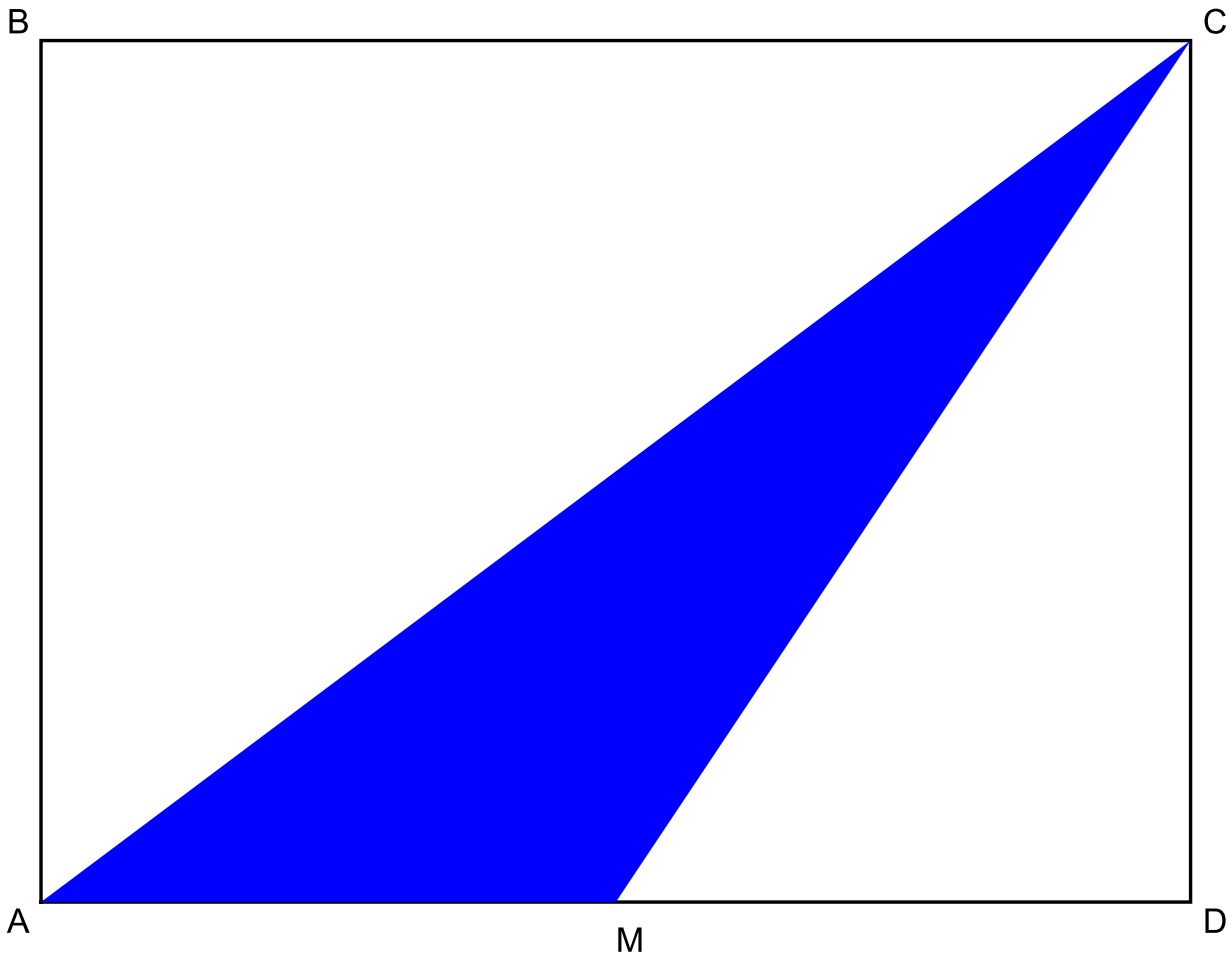


2016 AMC 8 PROBLEM 2

P2: In rectangle ABCD, AB = 6 and AD= 8. Point M is the midpoint of $\overline{AD}$. What is the area of ΔACM?

Solution: 12

Explanation: A=½*4*6=12



2016 AMC 8 PROBLEM 3

P3: Four students take an exam. Three of their scores are 70, 80, and 90. If the average of their four scores is 70, then what is the remaining score?

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Solution: 40

Explanation: Solve the linear equation (70+80+90+x)/4 = 70 for x=40.

2016 AMC 8 PROBLEM 4

P4: When Cheenu was a boy, he could run 15 miles in 3 hours and 30 minutes. As an old man, he can now walk 10 miles in 4 hours. How many minutes longer does it take for him to walk a mile now compared to when he was a boy?

2016 AMC 8 PROBLEM 4

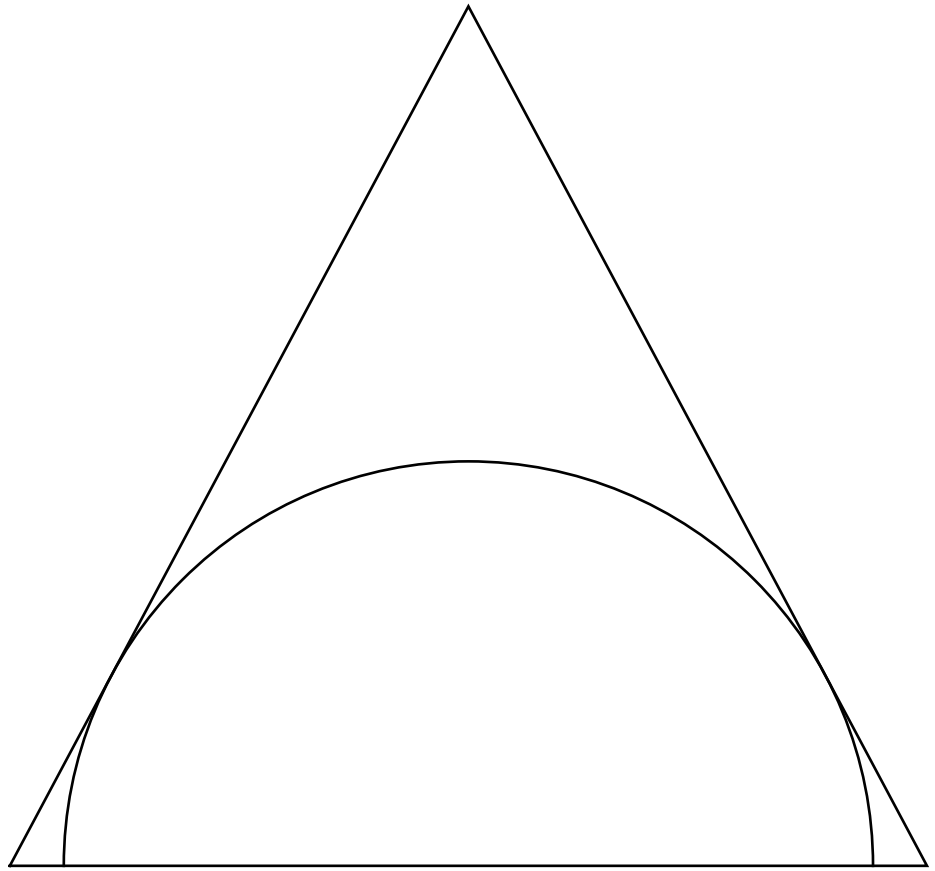
P4: When Cheenu was a boy, he could run 15 miles in 3 hours and 30 minutes. As an old man, he can now walk 10 miles in 4 hours. How many minutes longer does it take for him to walk a mile now compared to when he was a boy?

Solution: 10 minutes

Explanation: Get the rages first: t1=(3*60+30)/15=14
t2=(4*60)/15=24
t2-t1 = 24-14 = 10

FIND THE RADIUS OF THE SEMICIRCLE

- An isosceles triangle with base 16 and height 15
- The diameter of the semicircle is contained in the base
- Please find the radius of the semicircle.
- [2016 AMC 8 Problem 25]



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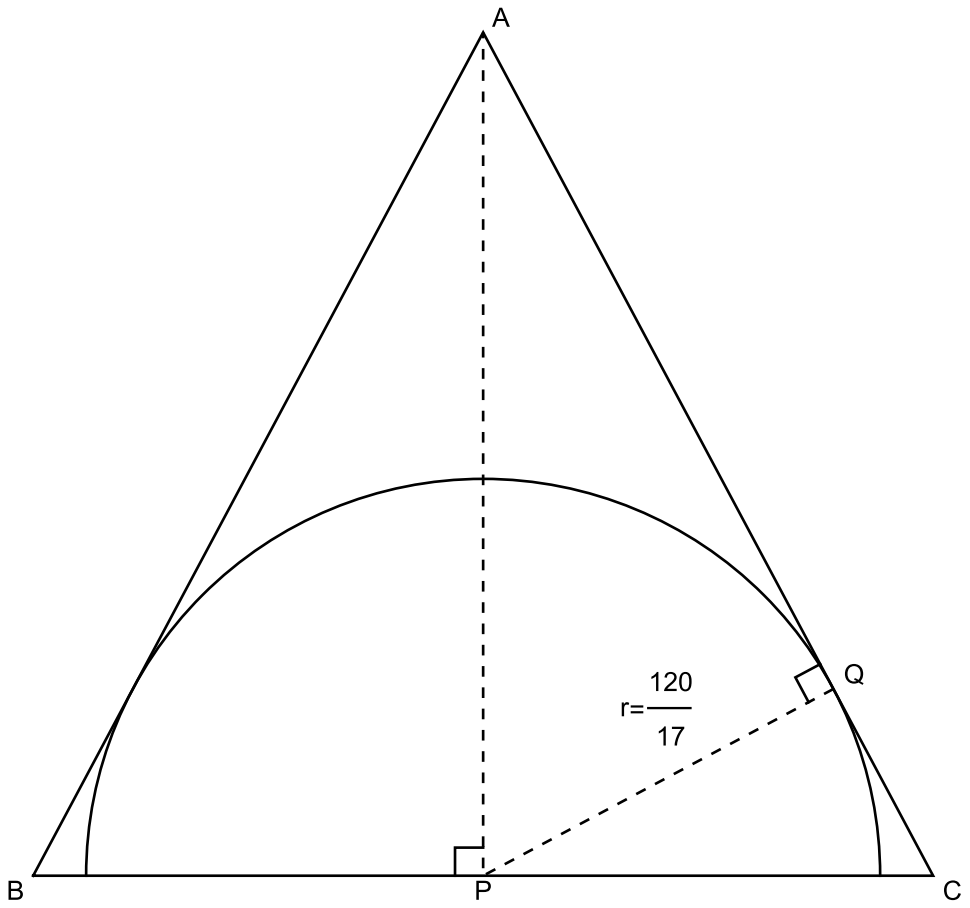
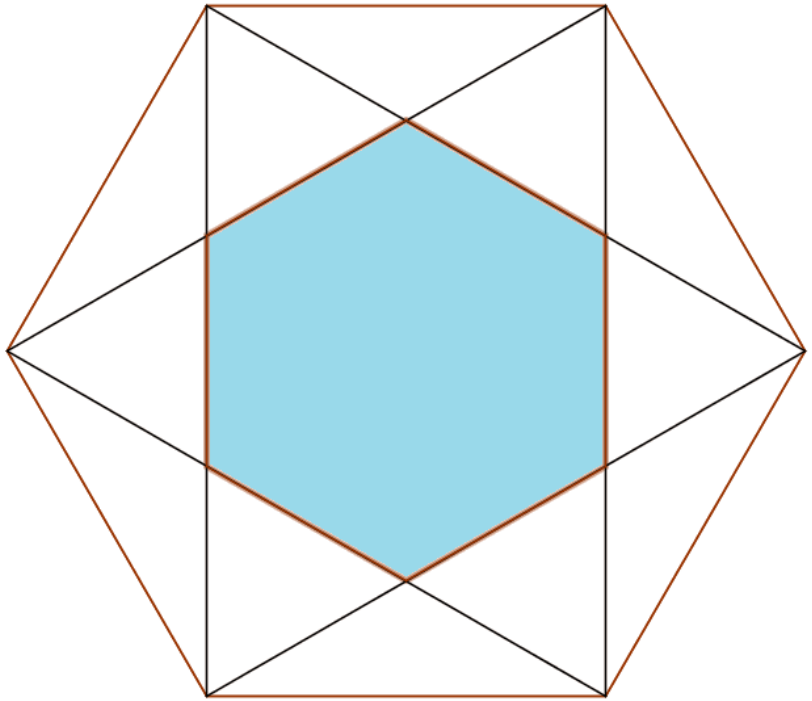
- [2016 AMC 8 Problem 25]
- Solution: r = 120/17



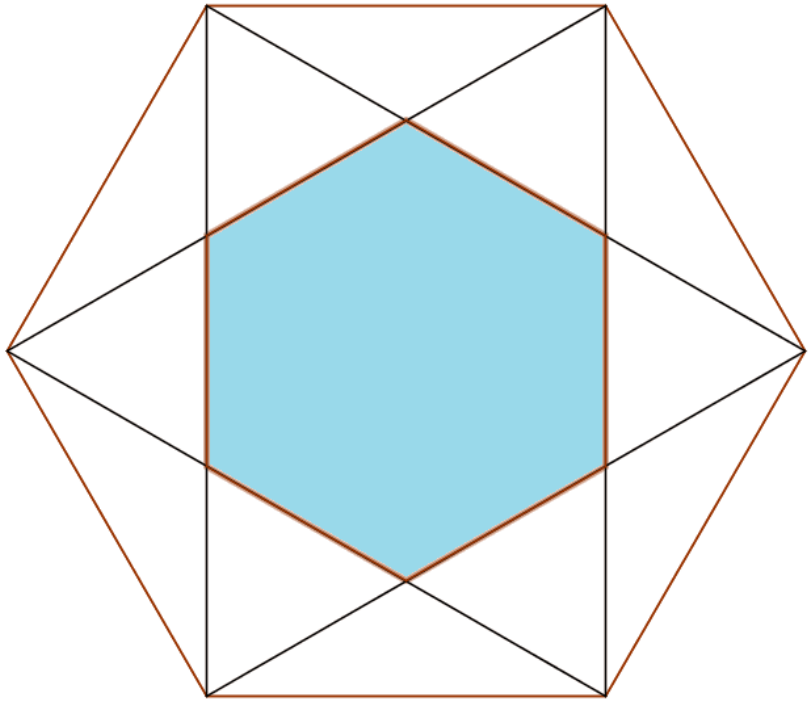
HEXAGONAL QUESTIONS



In a regular hexagon of area $120cm^2$, six diagonals are drawn to form a smaller, shaded hexagon as shown.

What is the area of the smaller hexagon, in square centimeters?

HEXAGONAL QUESTIONS – SOLUTION TO COME



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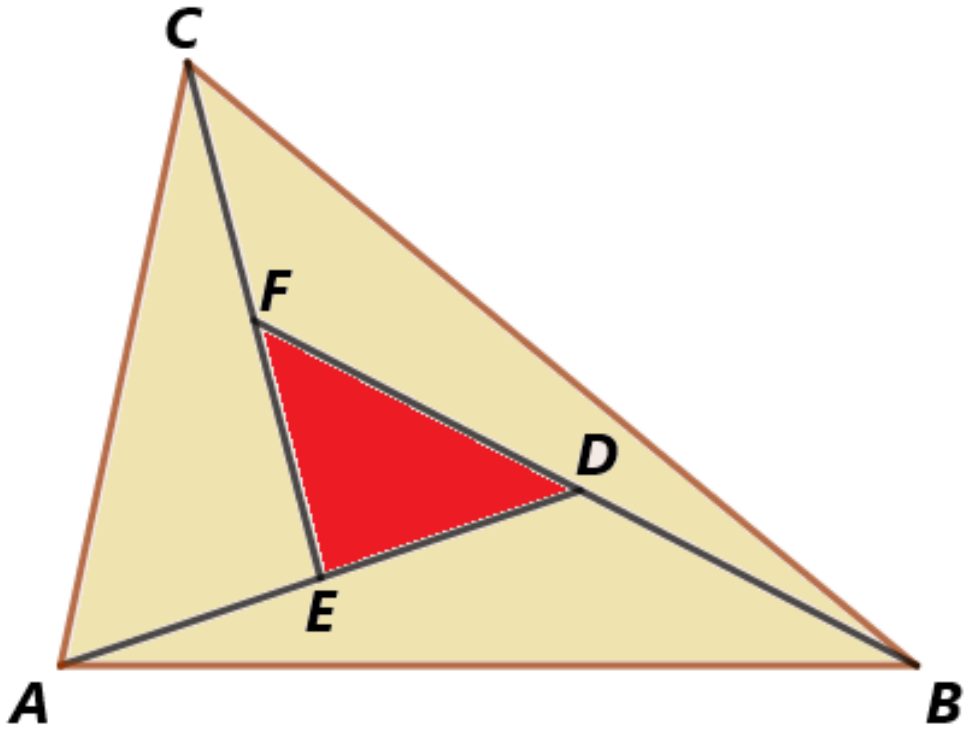
TRIANGULAR TRIANGULATIONS

- Segments are drawn in triangle ABC in such a way that D is the midpoint of BF, E is the midpoint of AD, and F is the midpoint of CE.

- If the area of triangle ABC is 1, what is the area of triangle DEF?



TRIANGULAR TRIANGULATIONS – A HINT

- Segments are drawn in triangle ABC in such a way that D is the midpoint of BF, E is the midpoint of AD, and F is the midpoint of CE.

- If the area of triangle ABC is 1, what is the area of triangle DEF?

  Hint: Relate the area of triangle ABC to the areas of the four surrounding triangles.
  Complete solution: Perhaps next time.