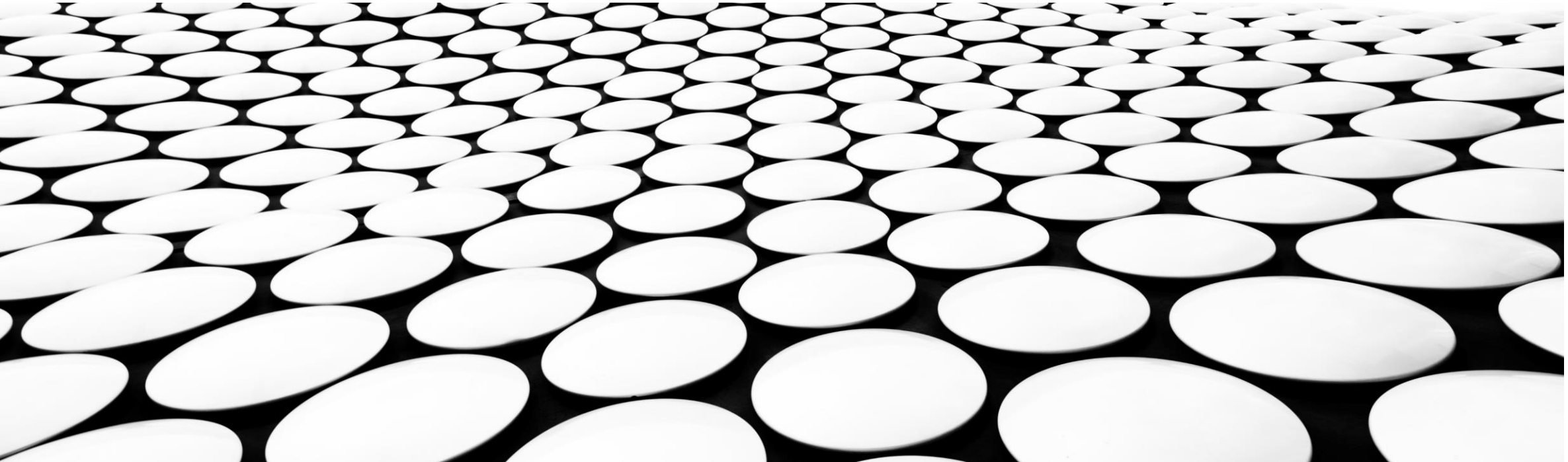

MATH CIRCLE AT FAU

10/5/2024



THE ISLAND OF KNIGHTS AND KNAVES (AND WEREWOLVES)



The island of knights and knaves, where knights **ALWAYS** tell the truth, and knaves **ALWAYS** lie has had an outbreak of lycanthropy; werewolves, that is. Some knights and some knaves become werewolves on nights of a full moon.

You visit the island and interview three people, A, B, and C; it is known that exactly one of them is a werewolf.

They tell you:

A: "C is a werewolf."

B: "I am not a werewolf."

C: "At least two of us are knaves."

Our problem here has two parts.

- a. Is the werewolf a knight or a knave?
- b. C. If you want to pick one of them as a traveling companion, and want to be sure its not the werewolf, which one would you pick?



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a. Is the werewolf a knight or a knave?

b. C. If you want to pick one of them as a traveling companion, and want to be sure its not the werewolf, which one would you pick?

SOLUTION: Suppose A is a knight. Then C is a werewolf. In addition, B is telling the truth, so B is a knight. Then C is lying. Conclusion: C is the werewolf and a knave.

Assume next that A is a knave. Then C is not a werewolf. With C saying at least two of us are knaves, C has to be a knight As a knave, C and A would be knaves, and C would be telling the truth; impossible! So C is a knight, C is telling the truth, so B must be a knave for two knaves to be present. Then B lies, and is the werewolf: Conclusion: Bis the werewolf and is a knave.

Answer: **The werewolf is a knave and is either B or C. Pick A as your traveling companion.**

THE ISLAND OF KNIGHTS AND KNAVES (AND WEREWOLVES)



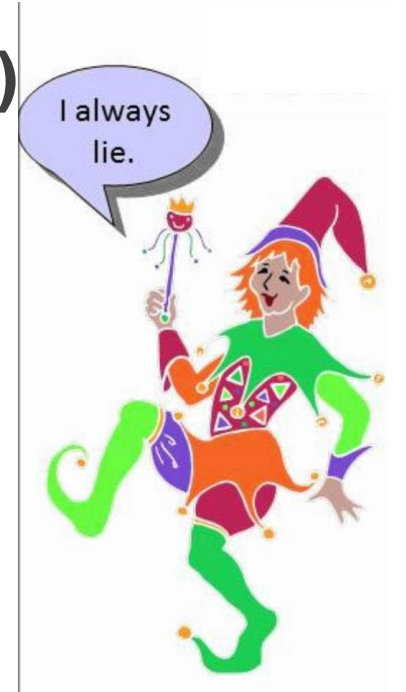
Again, A, B, C are knights or knaves and exactly one is a werewolf. They tell you:

A: "I am a werewolf."

B: "I am a werewolf."

C: "At least one of us is a knight."

Give a complete classification of A, B, C.



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Give a complete classification of A, B, C.

SOLUTION: To come.

THE ISLAND OF KNIGHTS AND KNAVES (AND WEREWOLVES)

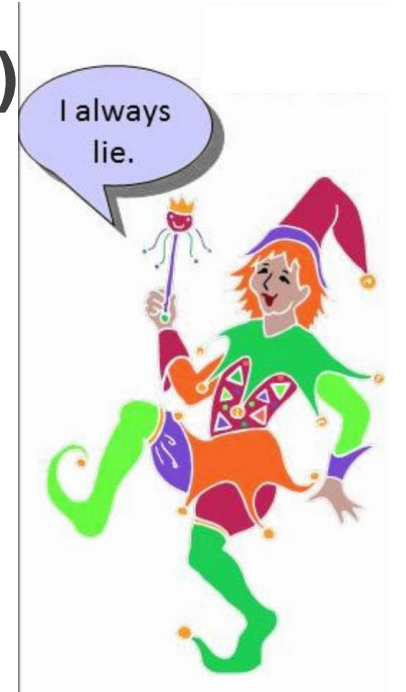


Again, A, B, C are knights or knaves. At least one of them is a werewolf and none of them is both a knight and a werewolf. Only two speak this time.

A: "At least one of the three of us is a knight"

B: "At least one of the three of us is a knave"

Which one is. or which ones are the werewolves?



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A: "At least one of the three of us is a knight"

B: "At least one of the three of us is a knave"

Which one is. or which ones are the werewolves?

SOLUTION: B must be a knight; otherwise, there is at least one knave (namely B) and B would be telling the truth. With B a knight, A is also telling the truth, so A is a knight. Since there is a werewolf and its not a knight, C must be the knavish werewolf.

A, B are knights, C is a knave and the werewolf.

THE ISLAND OF KNIGHTS AND KNAVES (AND WEREWOLVES)

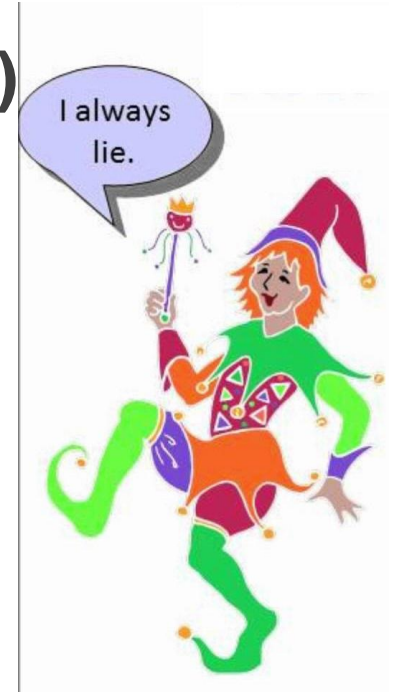


As in the previous problem A, B, C are knights or knaves and exactly one of them is a werewolf. It is also known that the werewolf is a knight. Once again, only two speak.

A: "At least one of the three of us is a knave"

B: "C is a knight"

Who is the werewolf?



As in the previous problem A, B, C are knights or knaves and exactly one of them is a werewolf. It is also known that the werewolf is a knight. Once again, only two speak.

A: "At least one of the three of us is a knave"

B: "C is a knight"

Who is the werewolf?

SOLUTION: A has to be a knight. If B were a knight, all three are knights, so B is a knave, and then so is C.

A is a knight and the werewolf; B, C are knaves.

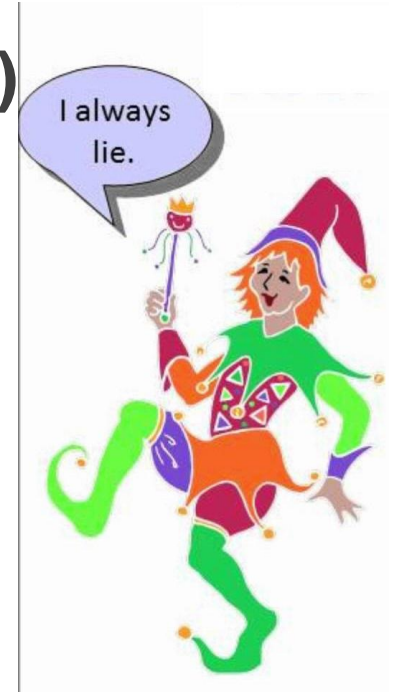
THE ISLAND OF KNIGHTS AND KNAVES (AND WEREWOLVES)



Now we have only two islanders, A and B; just one of them is a werewolf. They tell you.

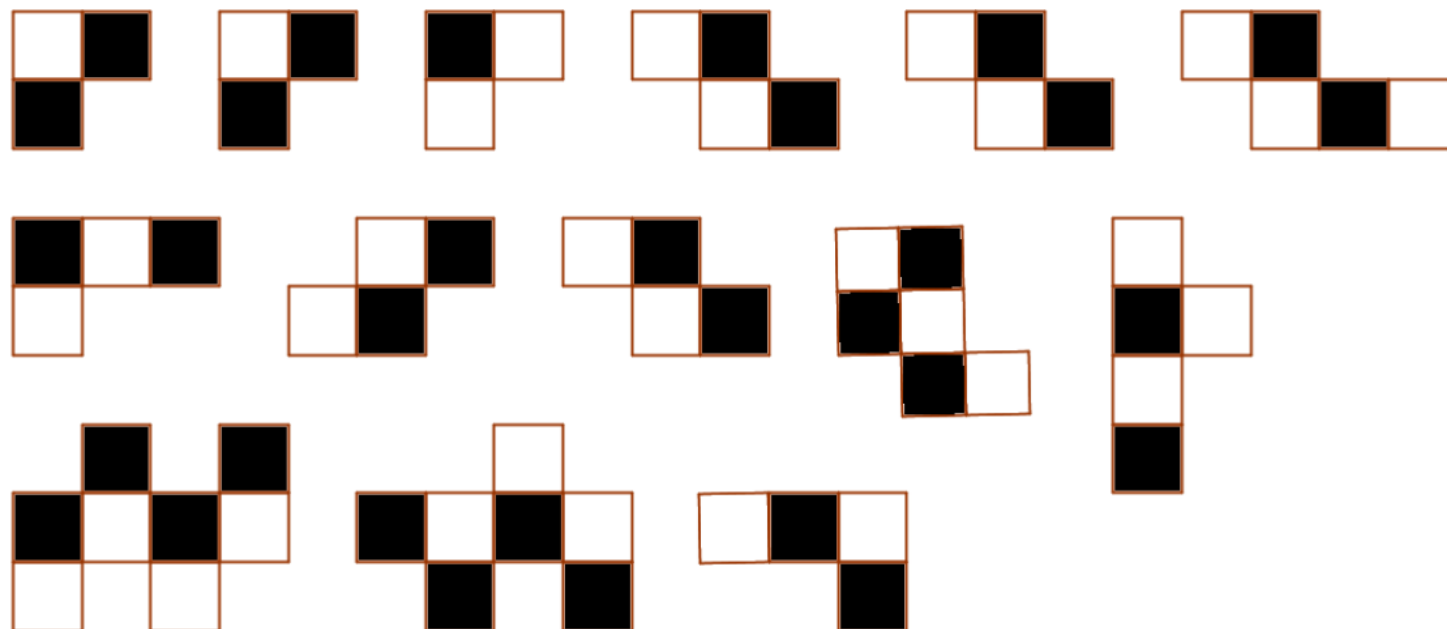
- A: "The werewolf is a knight"
- B: "The werewolf is a knave"

Which one would you select for your traveling companion?



ACTIVITY

A chess board has been cut into 14 pieces as shown.
See if you can put it together again.



Now we have only two islanders, A and B; just one of them is a werewolf. They tell you.

A: "The werewolf is a knight"

B: "The werewolf is a knave"

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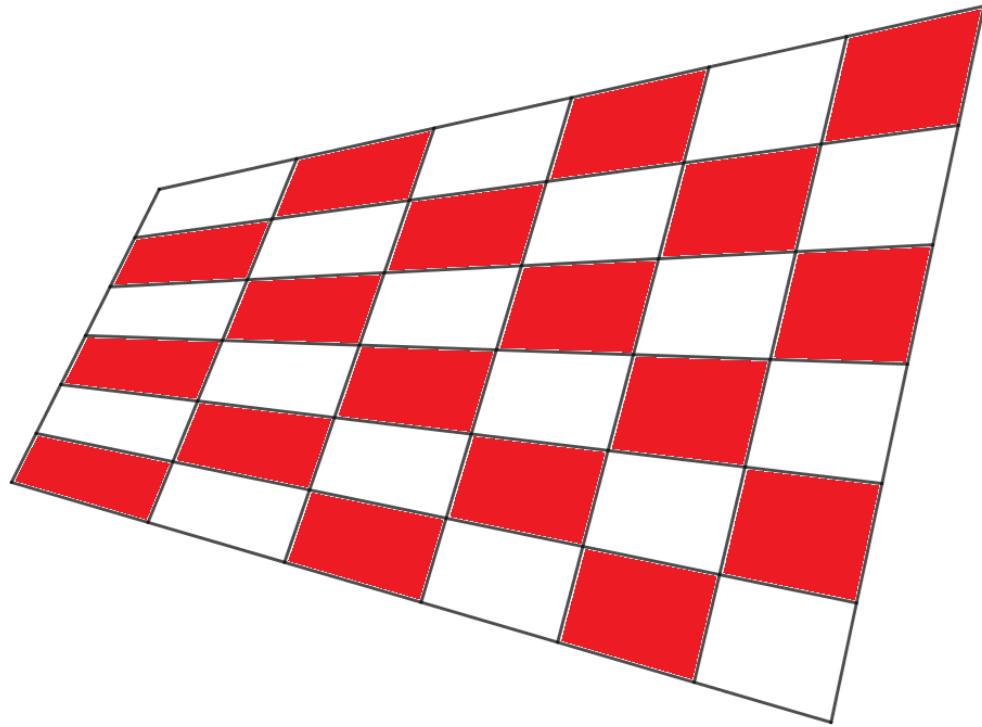
Suppose A is a knight. Then A is the werewolf. Suppose now A is a knave. Then the werewolf is a knave. This makes what B says true, so B is a knight; again A is the werewolf.

Select B.



It is Geometry Today

SKEWED PIZZA

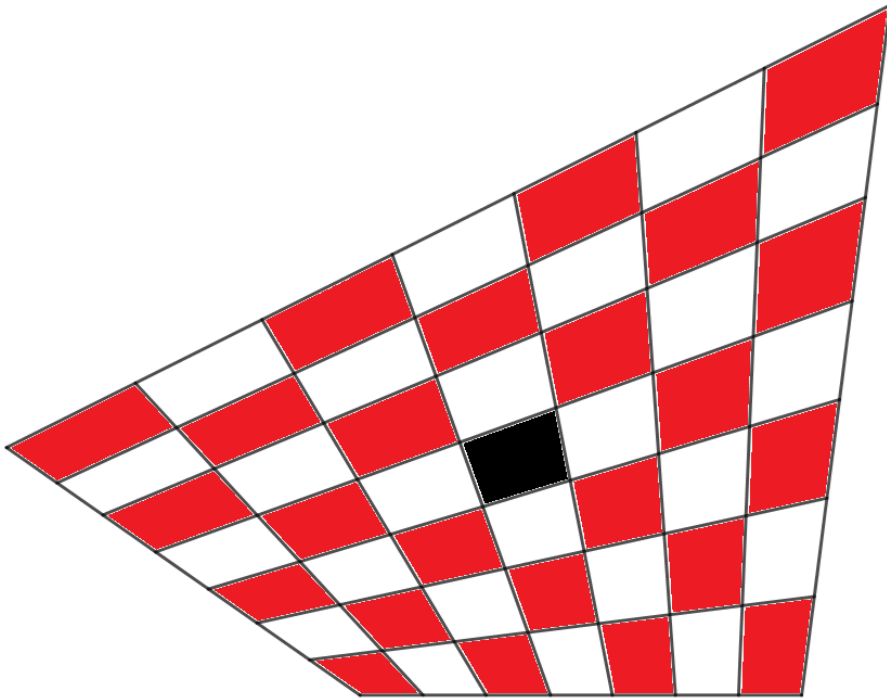


A pizza is made in the shape of a convex quadrilateral. Each of the four sides of the pizza is divided into n equal segments and straight-line cuts are made connecting corresponding division points on opposite sides. The resulting pieces are colored red and white, in a checkerboard pattern, as in the figure.

Suppose n is even. Then there is an equal number of red and white pieces. Suppose Alice and Bob share the pizza, with Alice eating all the red pieces and Bob all the white pieces,

Do they get the same amount of pizza?

SKEWED PIZZA

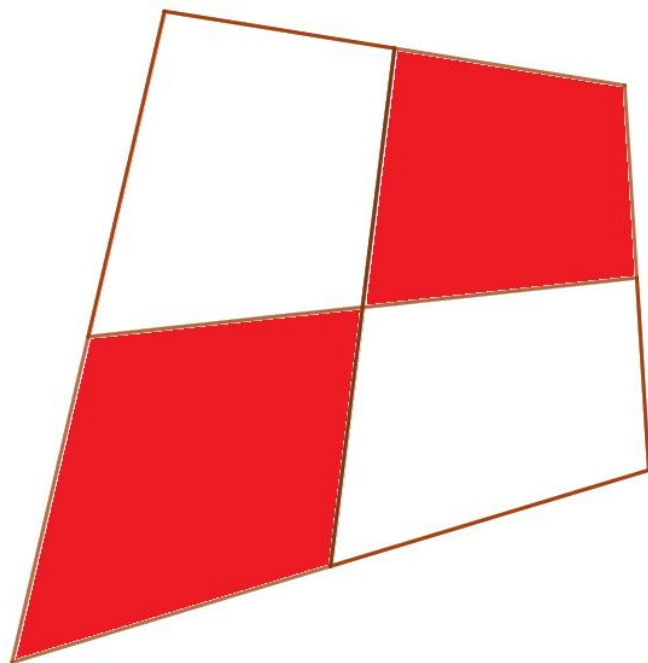


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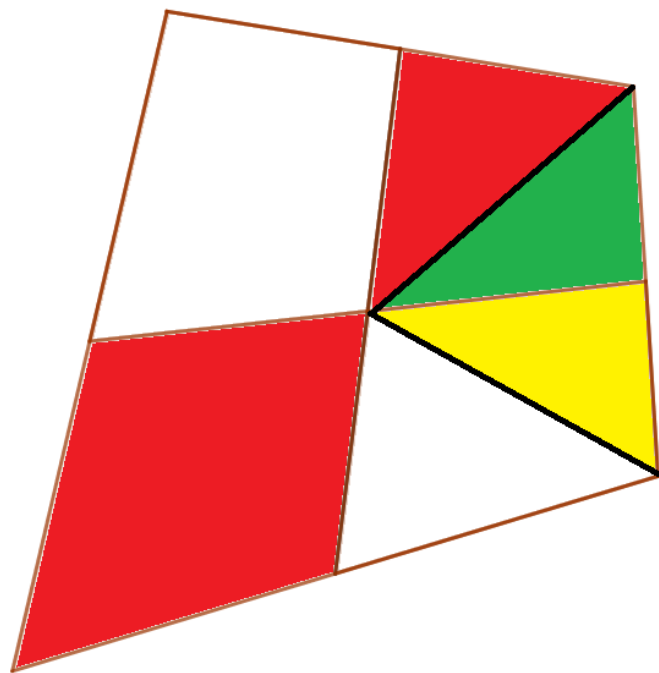
Suppose now n is odd; then there is one piece . Then there is one more piece of one color than the other. To make the division fair, Alice and Bob give the central piece to the dog before dividing the rest of the pizza as before.

Do they get the same amount of pizza?

SKEWED PIZZA, THE SIMPLEST CASE



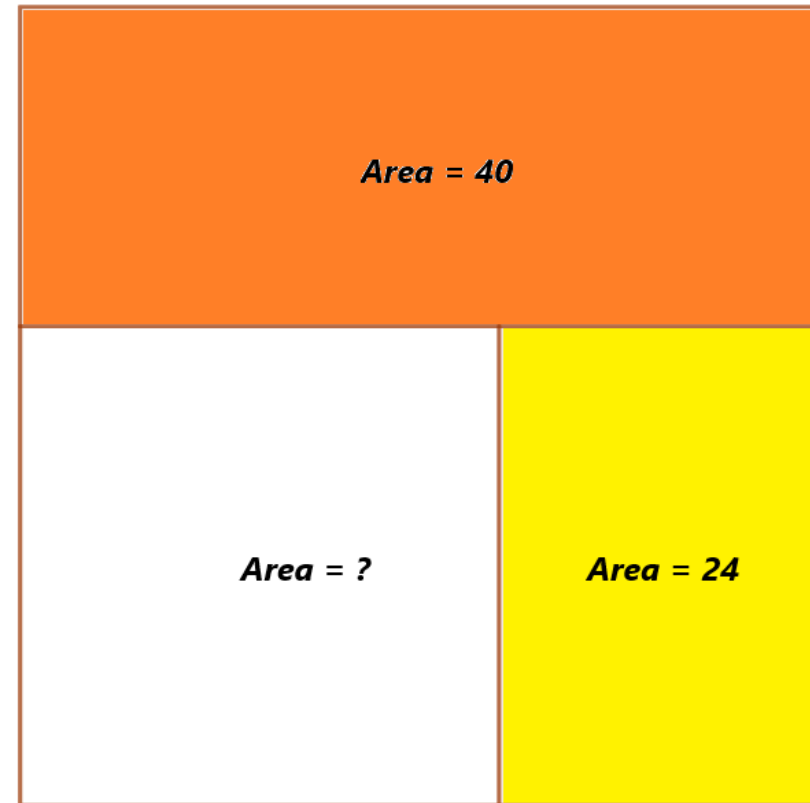
SKEWED PIZZA, THE SIMPLEST CASE, MORE DETAILS



PARTITIONING A SQUARE

A square has been partitioned into two rectangles and a smaller square, as in the picture.

If the areas of the rectangles are 40 and 24 (as shown), find the area of the smaller square.



PARTITIONING A SQUARE

A square has been partitioned into two rectangles and a smaller square, as in the picture.

If the areas of the rectangles are 40 and 24 (as shown), find the area of the smaller square.

Solution: Let x be the length of the side of the smaller square, which is also the length of one side of the smaller rectangle. Let y be the length of the other side of the smaller rectangle. Then:

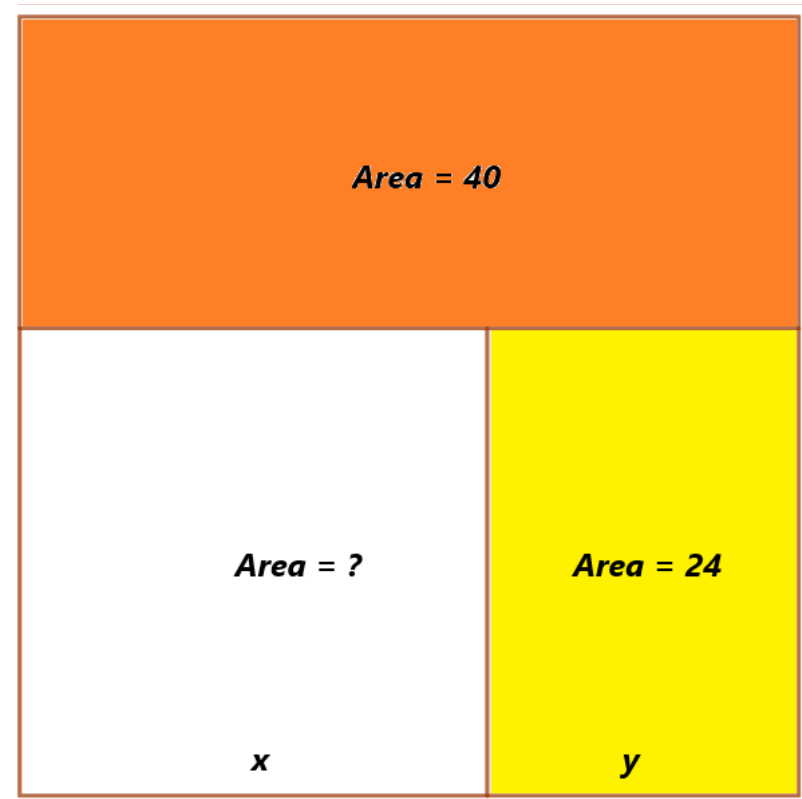
$$xy = 24$$

$$(x + y)y = 40$$

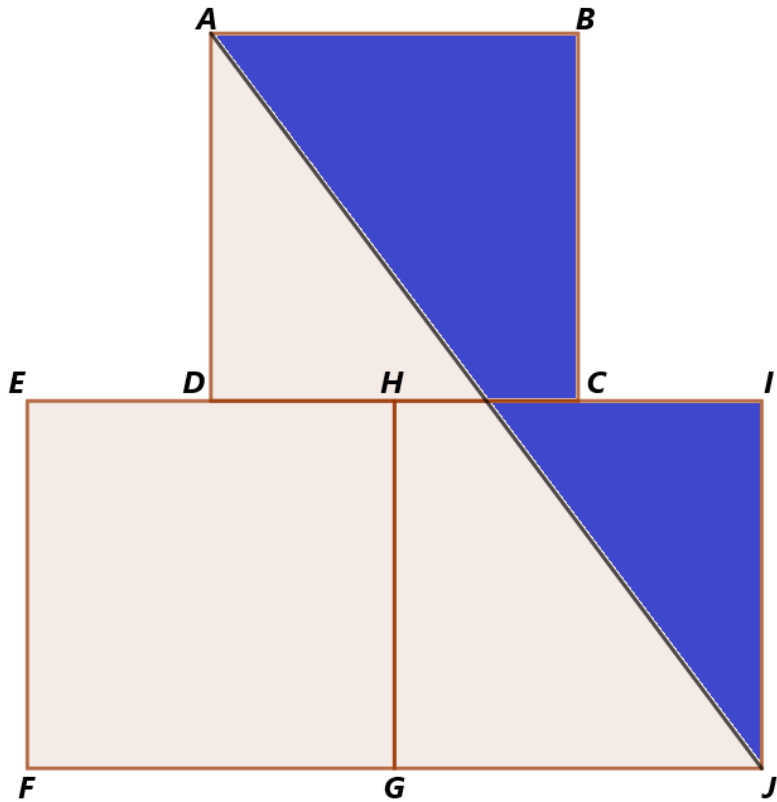
Expanding the second equation and using the first:

$xy + y^2 = 40$, $24 + y^2 = 40$, $y^2 = 16$, $y = 4$,
from the first equation $x = 6$.

$$A = 36$$

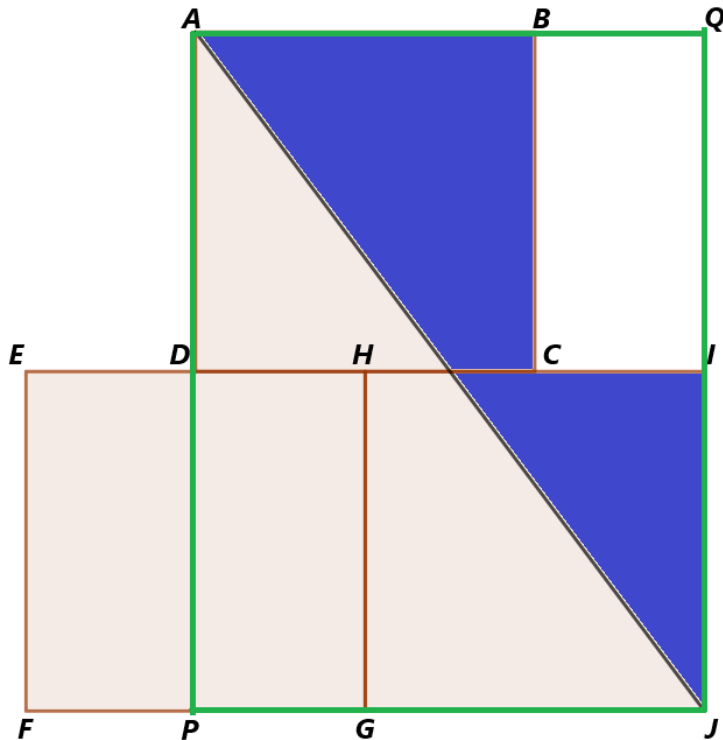


A TRIPLE OF SQUARES



Squares $ABCD$, $EFGH$, and $GHIJ$ are equal in area. Points C and D are the midpoints of sides IH and HE , respectively. What is the ratio of the area of the shaded (blue) pentagon $AJICB$ to the sum of the areas of the three squares?
(AMC 8, 2013, #24)

A TRIPLE OF SQUARES



Ratio is $\frac{1}{3}$

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(AMC 8, 2013, #24)

Solution: Let x be the length of the side of one of the squares, so the sum of the areas of the squares is $3x^2$.

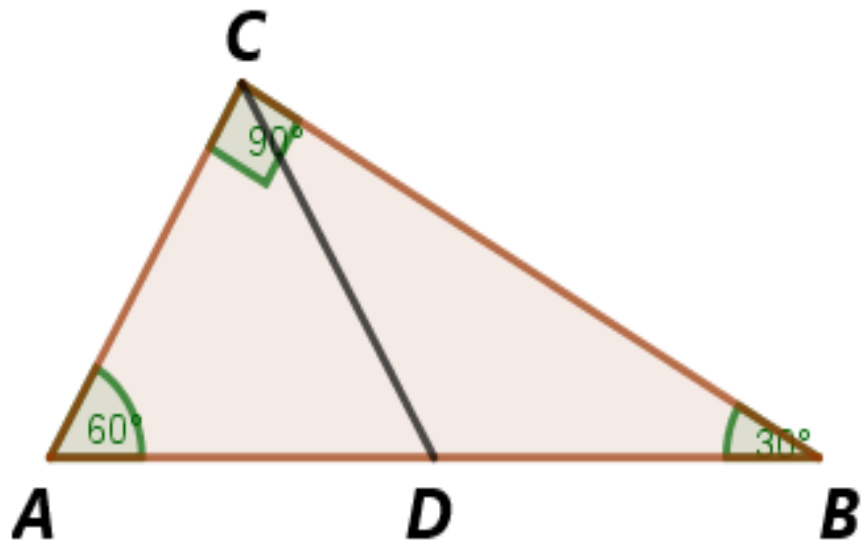
The rectangle $APJQ$ has area $\left(\frac{1}{2}x + x\right)(2x) = 3x^2$.

The blue shaded area is obtained from half of this rectangle by removing the rectangle $ECIQ$, which has area $\frac{1}{2}x^2$. Thus the area of the blue pentagon is

$$\left(\frac{1}{2}\right)(3x^2) - \frac{1}{2}x^2 = x^2.$$

The desired ratio is $\frac{x^2}{3x^2} = \frac{1}{3}$.

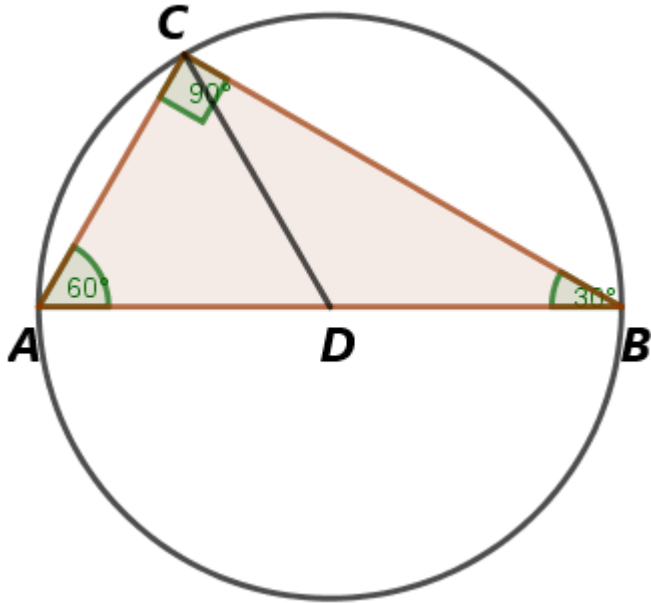
A RIGHT QUESTION



Triangle ABC is a right triangle, with the right angle at C . Segment CD is the median from C .

Show that ADC is an equilateral triangle.

A RIGHT QUESTION



Triangle ABC is a right triangle, with the right angle at C . Segment CD is the median from C .

Show that ADC is an equilateral triangle.

One possible solution: Use the fact that if we inscribe a right triangle in a circle, then the hypotenuse is the diameter. The picture shows that AD, CD are radii so that ADC is isosceles. An isosceles triangle with a base angle of 60° is equilateral.