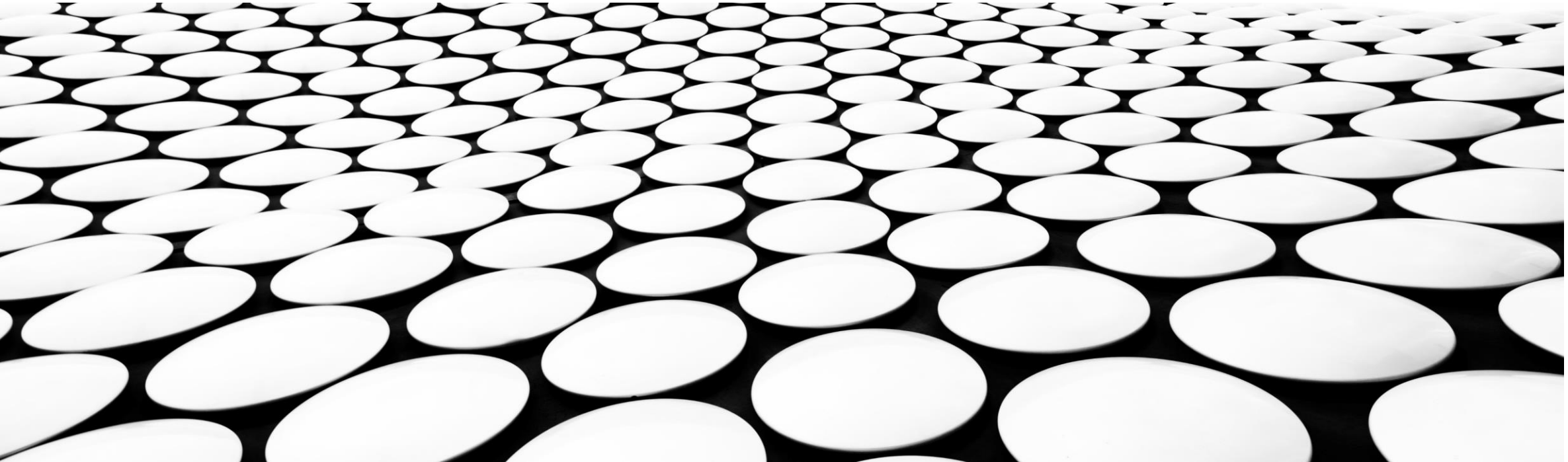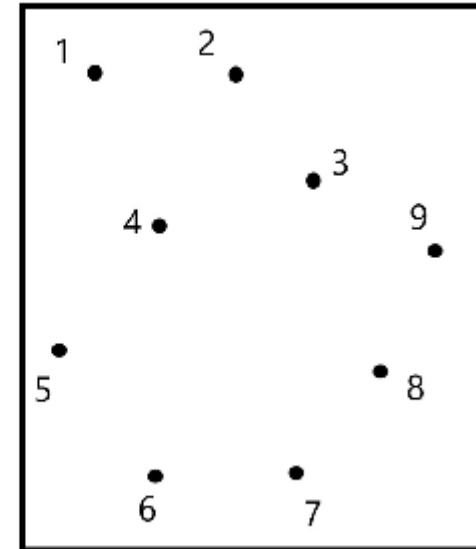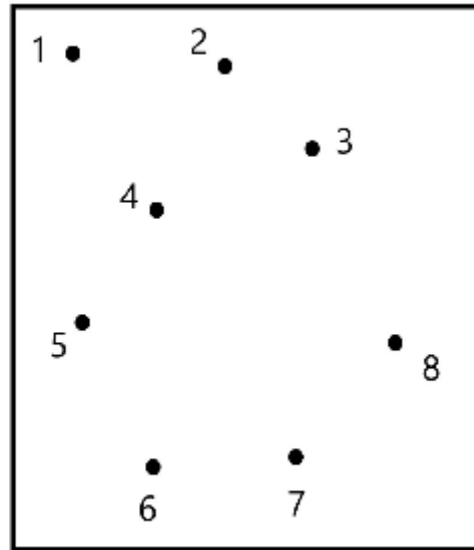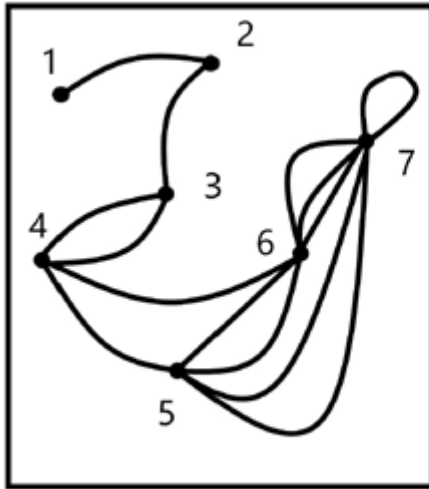# MATH CIRCLE AT FAU

9/21/2024

JOINING DOTS

We discussed at length problems related to these graphs. You had to be there!

The Joys of Counting

STRINGING UP DIGITS, PART DEUX

The counting numbers from 1 to a positive integer n are written as a long string of digits:

$$123456789101112131415161718192021 \ldots n$$

This results in a string of 6993 digits. What is n?

STRINGING UP DIGITS, PART DEUX

The counting numbers from 1 to a positive integer n are written as a long string of digits:

$$123456789101112131415161718192021 \ldots n$$

This results in a string of 6993 digits. What is n?

SOLUTION

There are 9 one digit numbers, occupying 9 places.
There are 90 two digit numbers, occupying 180 places.
There are 900 three digit numbers, occupying 2700 places.
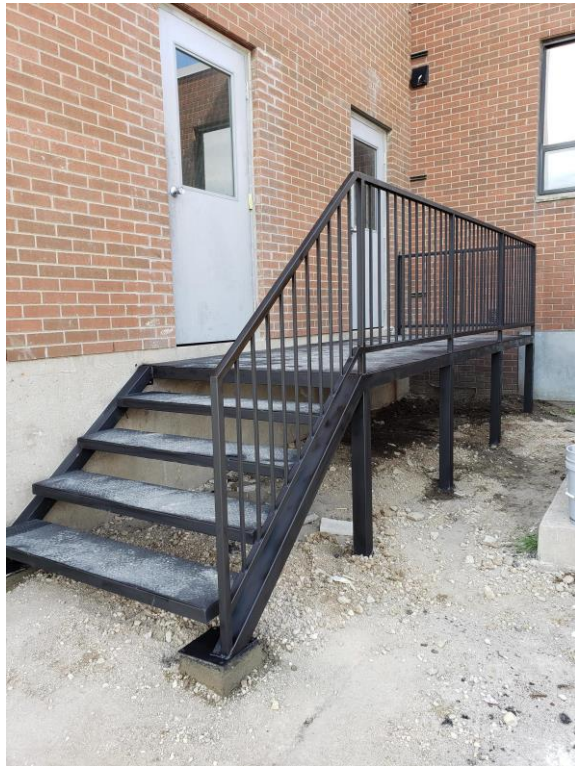There are 9000 four digit numbers, occupying 36,000 places.
Since

$$9 + 180 + 2700 = 2889 < 7093 < 9 + 180 + 2700 + 36,000 = 38889,$$

n is a four digit number. The numbers from 1000 to n (included) contribute $4(n - 1000 + 1)$ digits. Thus

$$6993 = 2889 + 4(n - 1000 + 1).$$

From this one can solve for n. One gets $\boxed{n = 2025}$

STAIRS TO GOOD FORTUNE

- A staircase has 5 steps. You can walk up the staircase by taking one or two steps at a time. In how many ways can you walk up the staircase?

STAIRS TO GOOD FORTUNE

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It is not hard to list all possibilities:
1, 1, 1, 1, 1
1, 1, 1, 2
1, 1, 2, 1
1, 2, 1, 1
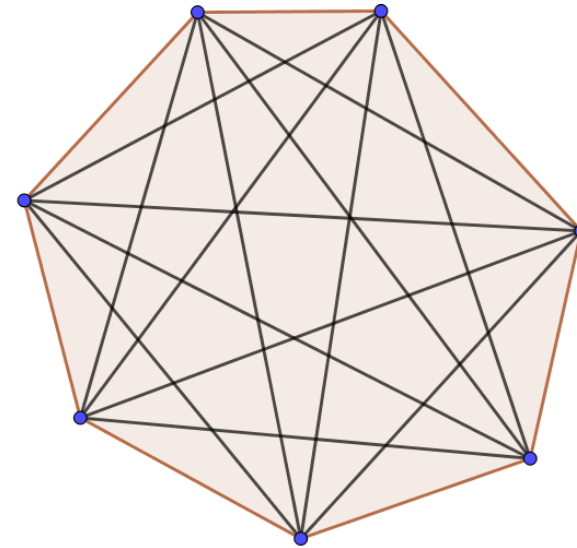1, 2, 2
2, 1, 1, 1
2, 1, 2
2, 2, 1

8 in all.

POLYGONS AND THEIR DIAGONALS.

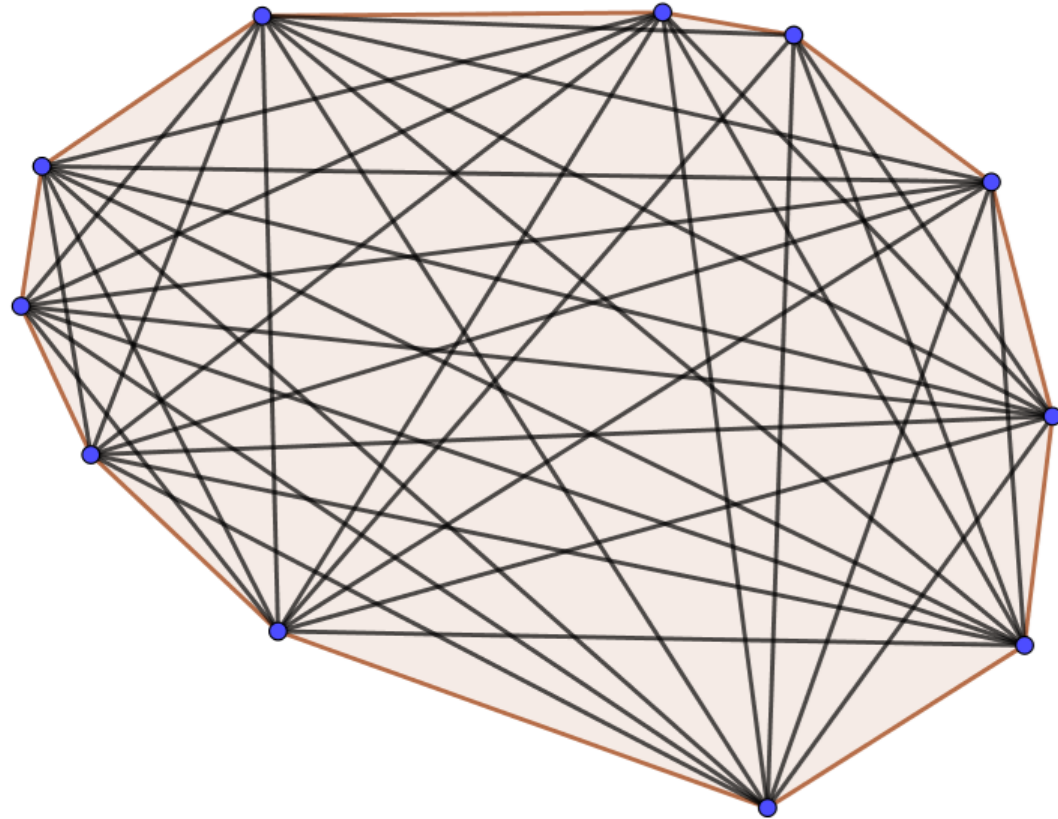- A convex polygon has 7 sides. How many diagonals does it have?

POLYGONS AND THEIR DIAGONALS.

- A convex polygon has 7 sides. How many diagonals does it have?

- $\binom{7}{2} - 7 = 14$
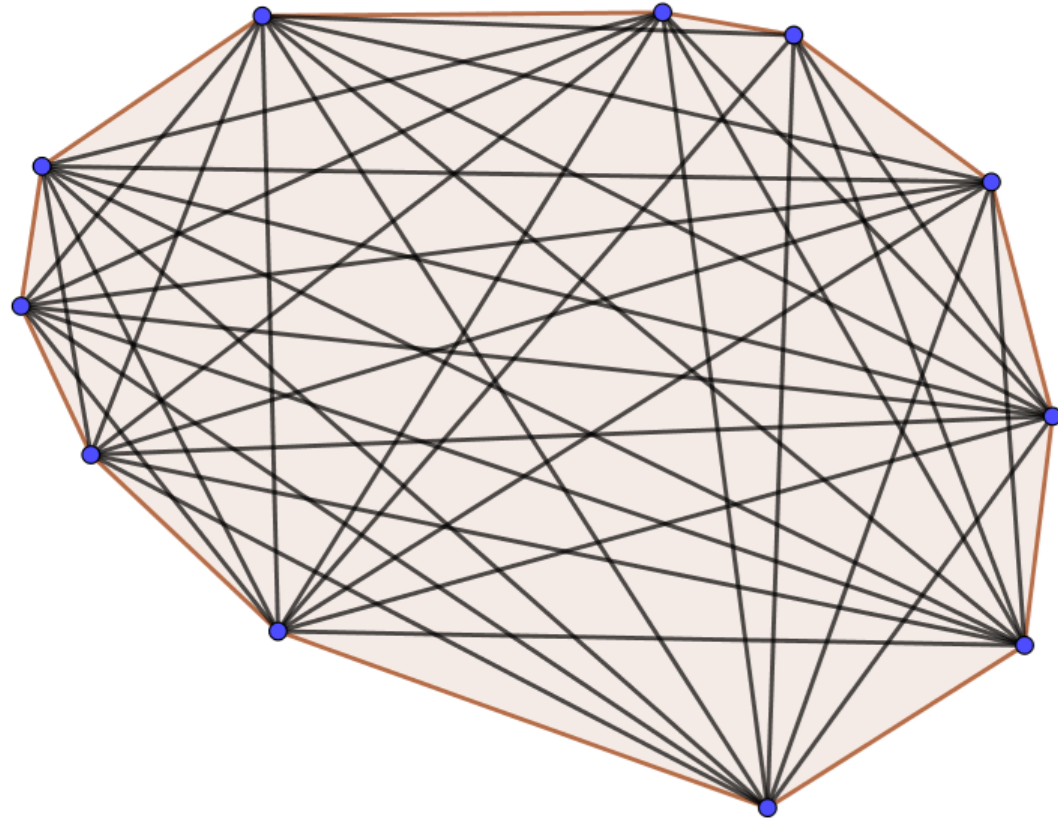
POLYGONALLY CONFUSED

A convex polygon has 54 diagonals. How many sides does it have?

POLYGONALLY CONFUSED

A convex polygon has 54 diagonals. How many sides does it have?

Same question, if the polygon has 1325 diagonals.

POLYGONALLY CONFUSED

A convex polygon has 54 diagonals. How many sides does it have?

Answer: 12 sides