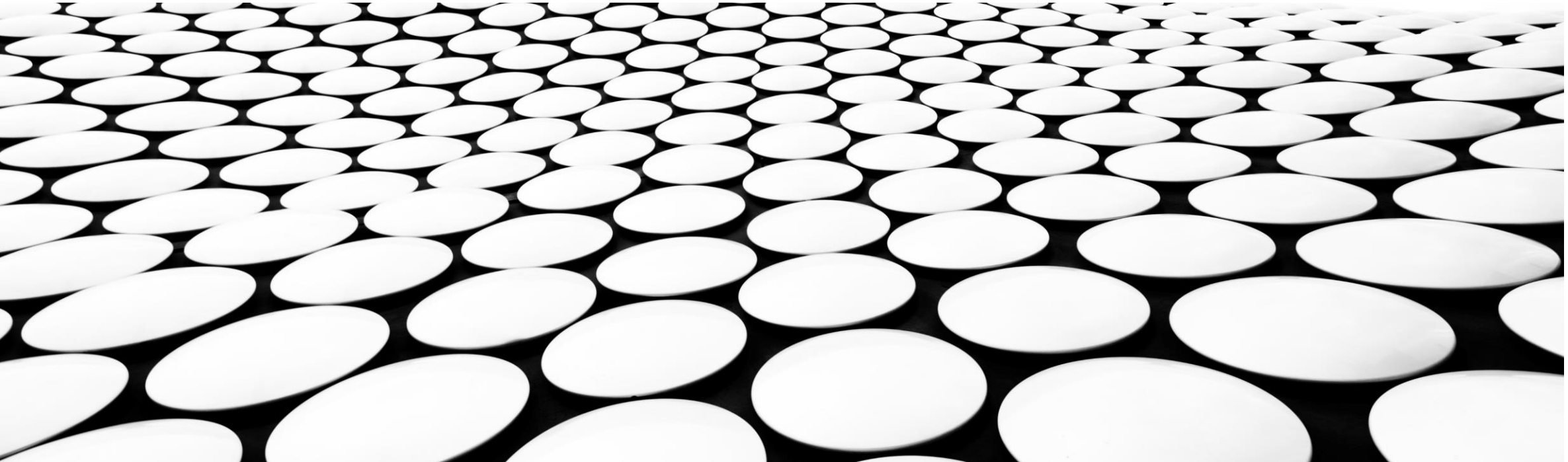

MATH CIRCLE AT FAU

9/7/2024



THE ISLAND OF KNIGHTS AND KNAVES



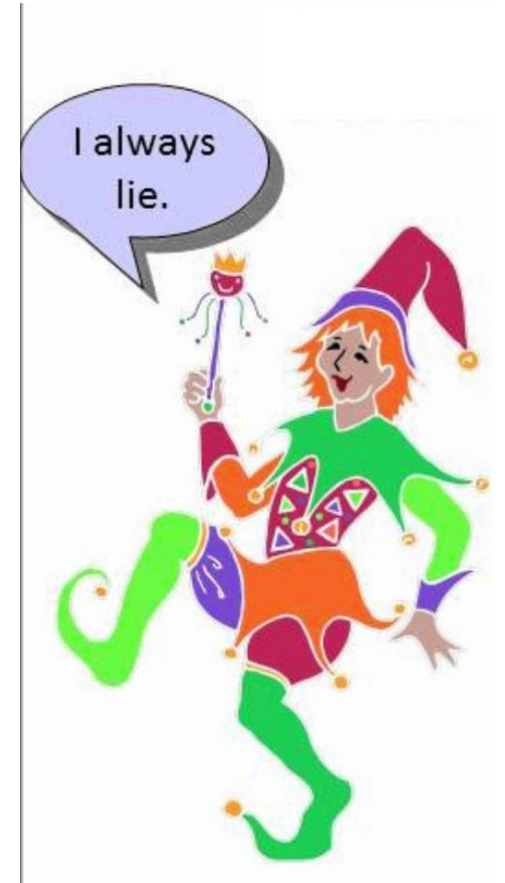
Here we are on the island of knights and knaves; The knights who can only tell the truth, the knaves who always lie.

You visit the island and meet three people, Alph, Beth and Gam.

Alph tells you: "All of us are knaves."

Beth says: "Exactly one of us is a knight."

What are Alph, Beth, and Gam?



THE ISLAND OF KNIGHTS AND KNAVES

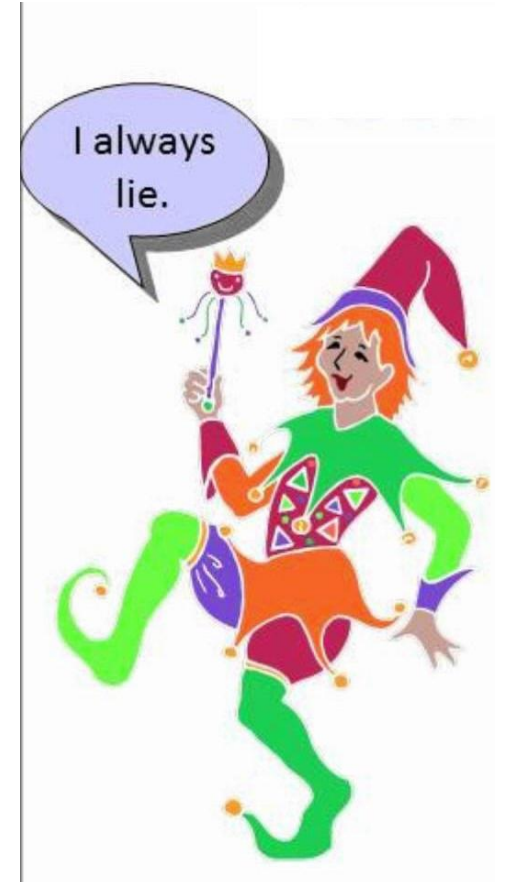


Still in the island of knights and knaves; The knights who can only tell the truth, the knaves who always lie.

On the island you meet Alfie and Balfie.

Alfie tells you: "Either I am a knave or Balfie is a knight."

What are Alfie and Balfie?



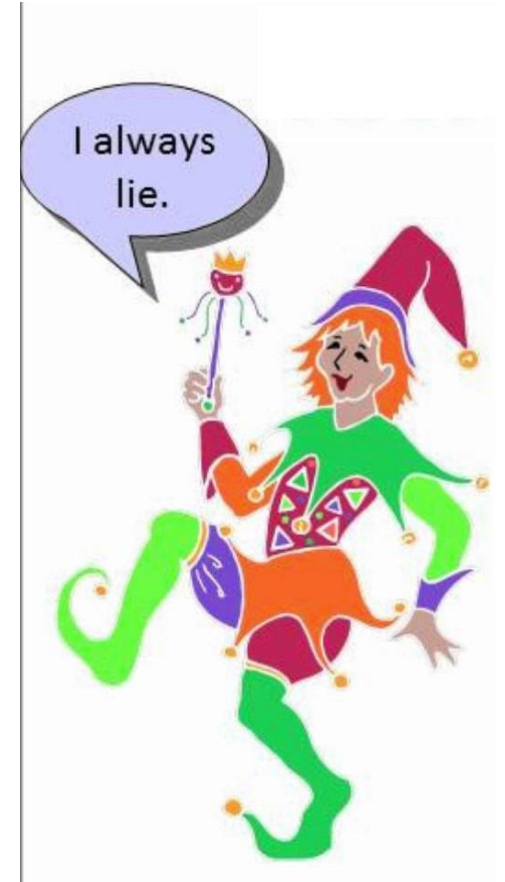
THE ISLAND OF KNIGHTS AND KNAVES



Still in the island of knights and knaves; The knights who can only tell the truth, the knaves who always lie.

You meet Awkwy, a somewhat confused individual who tells you: "Either I am a knave or $2 + 2 = 5$."

What can you conclude?



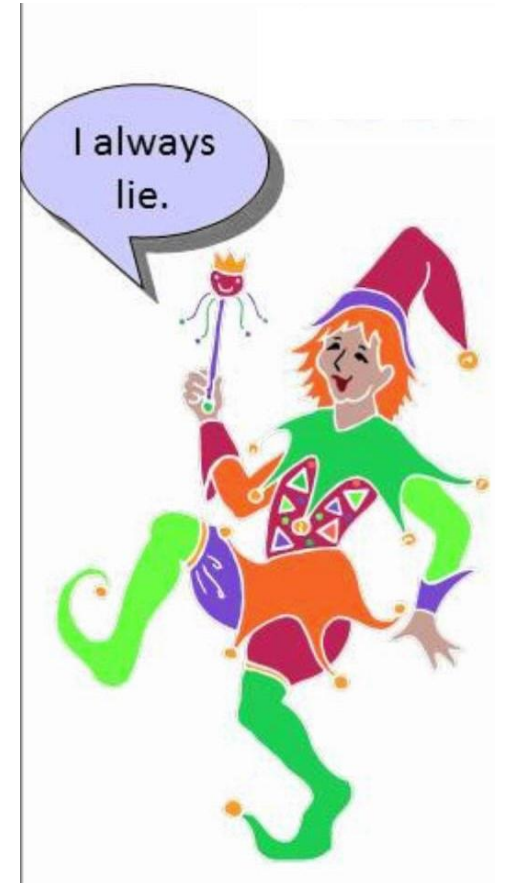
THE ISLAND OF KNIGHTS AND KNAVES



One more in the island of knights and knaves; The knights who can only tell the truth, the knaves who always lie.

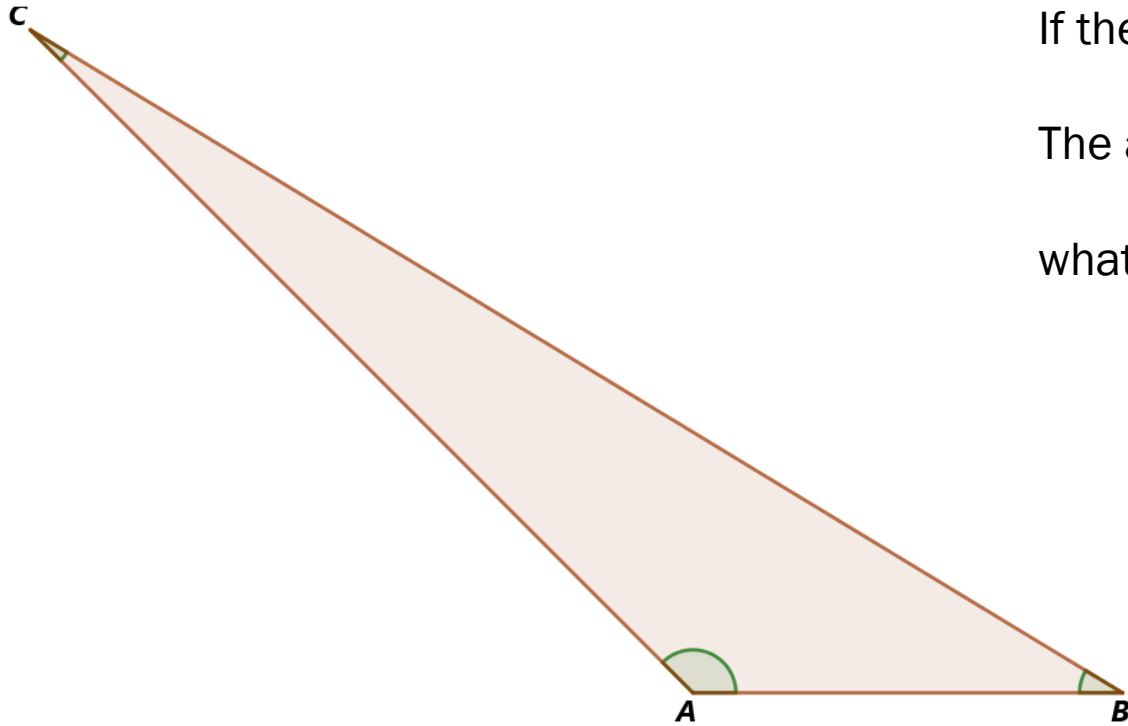
On the island, Fred the explorer (who is visiting with his team) meets two islanders, Arbie and Barbie, resting under a tree. Fred asked them "is either of you a knight?" Arbie responded and Fred knew the answer to his question.

What are Arbie and Barbie?



Geometry Basics, and More

SUM OF ANGLES, FIRST TRY.

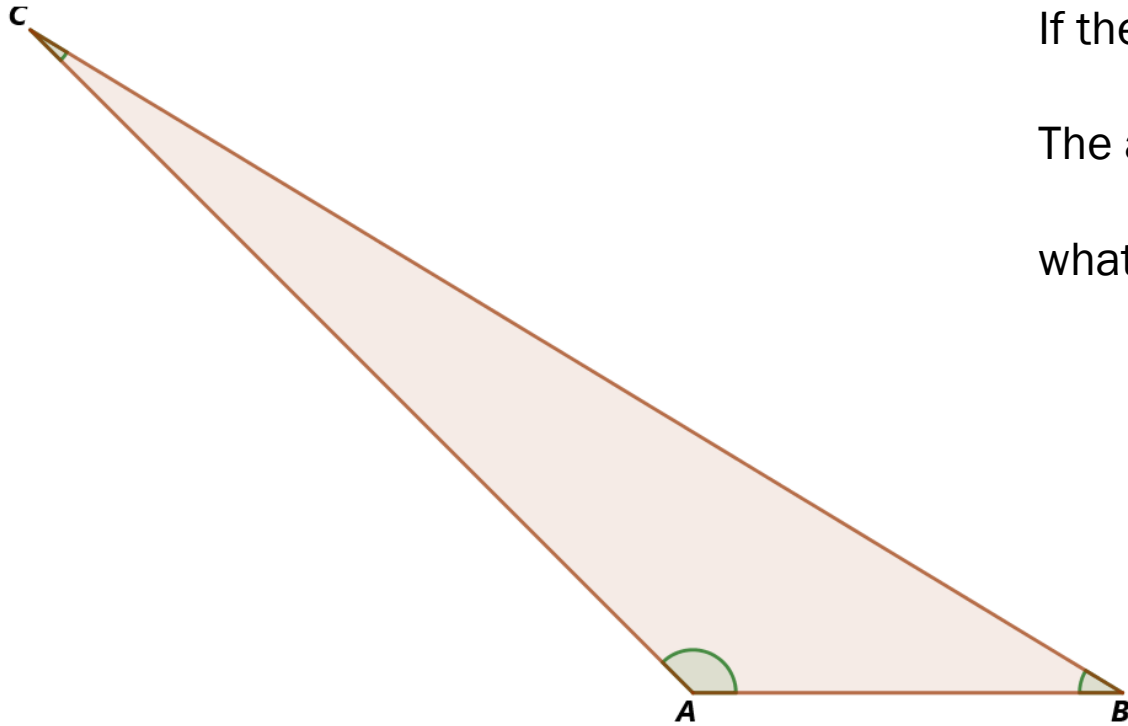


If the angle at B measures 32° ,

The angle at C measures 13° ,

what is the measure of the angle at A ?

SUM OF ANGLES, FIRST TRY.



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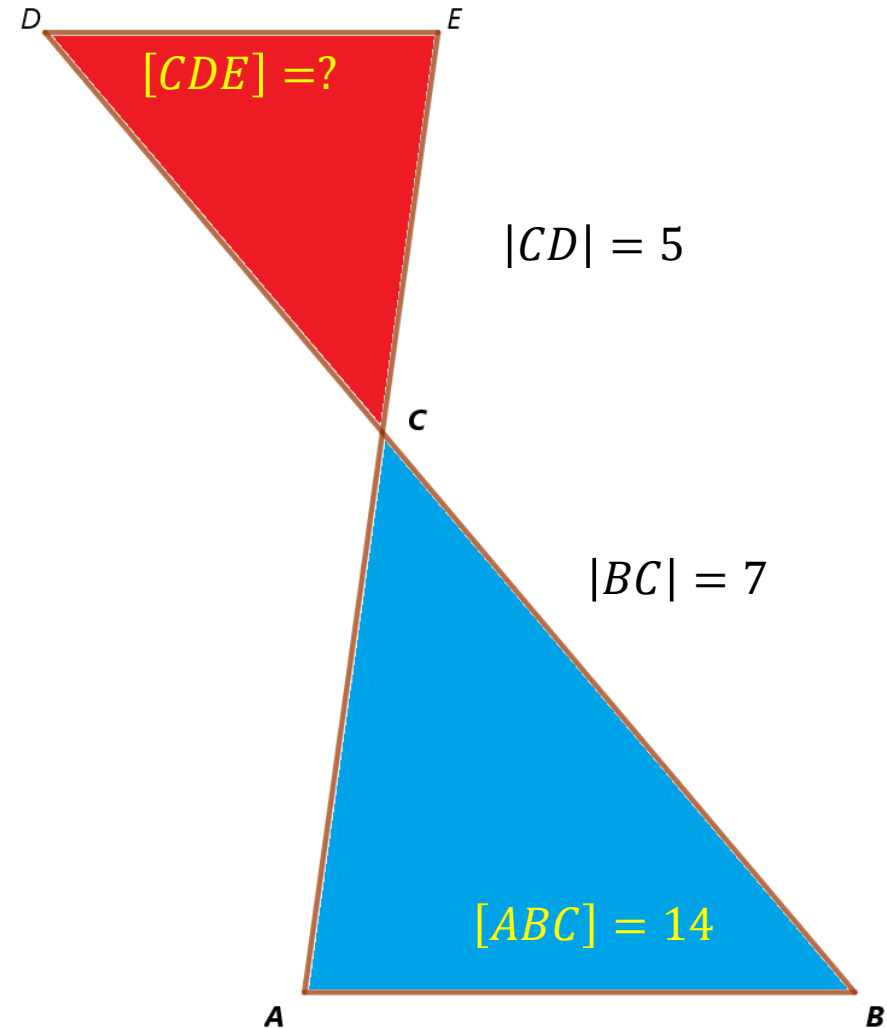
The angle at C measures 13° ,

what is the measure of the angle at A ?

$$\angle A = 135^\circ$$

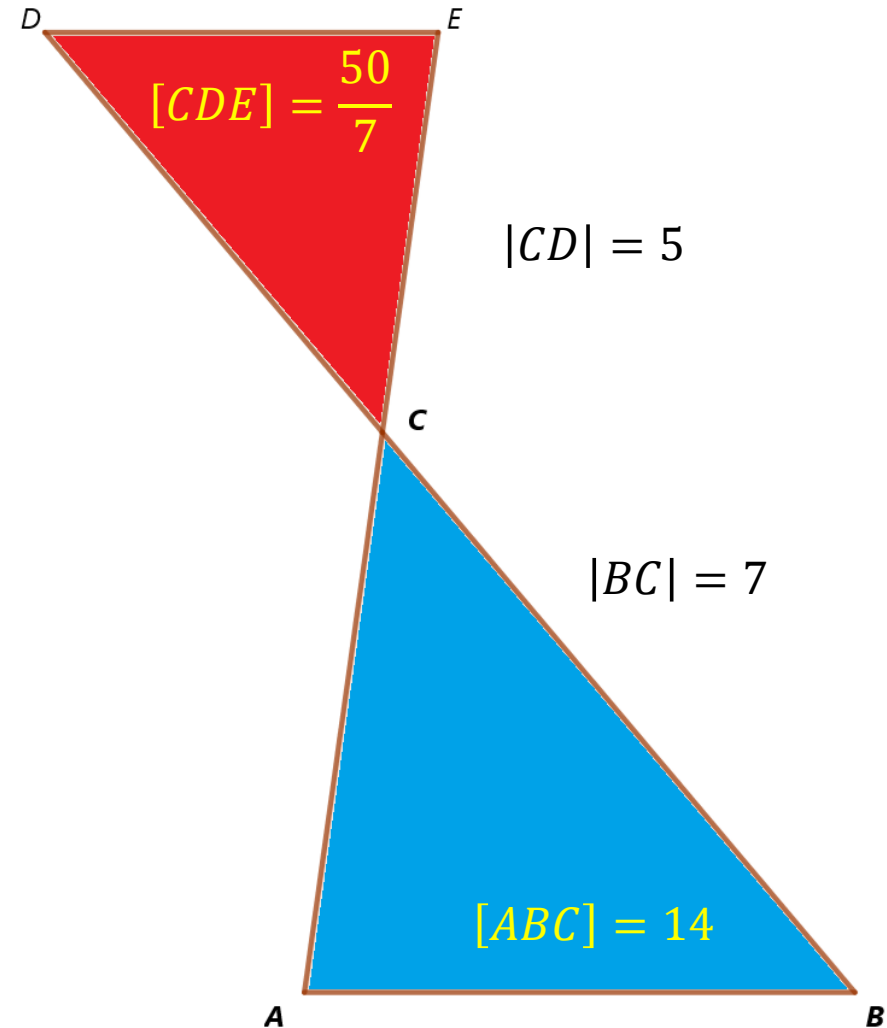
TWO TRIANGLES

- The area of triangle ABC is 14 square feet: Side BC is 7 feet long. Side CD of triangle CED is 5 feet long.
- What is the area of triangle CDE ?

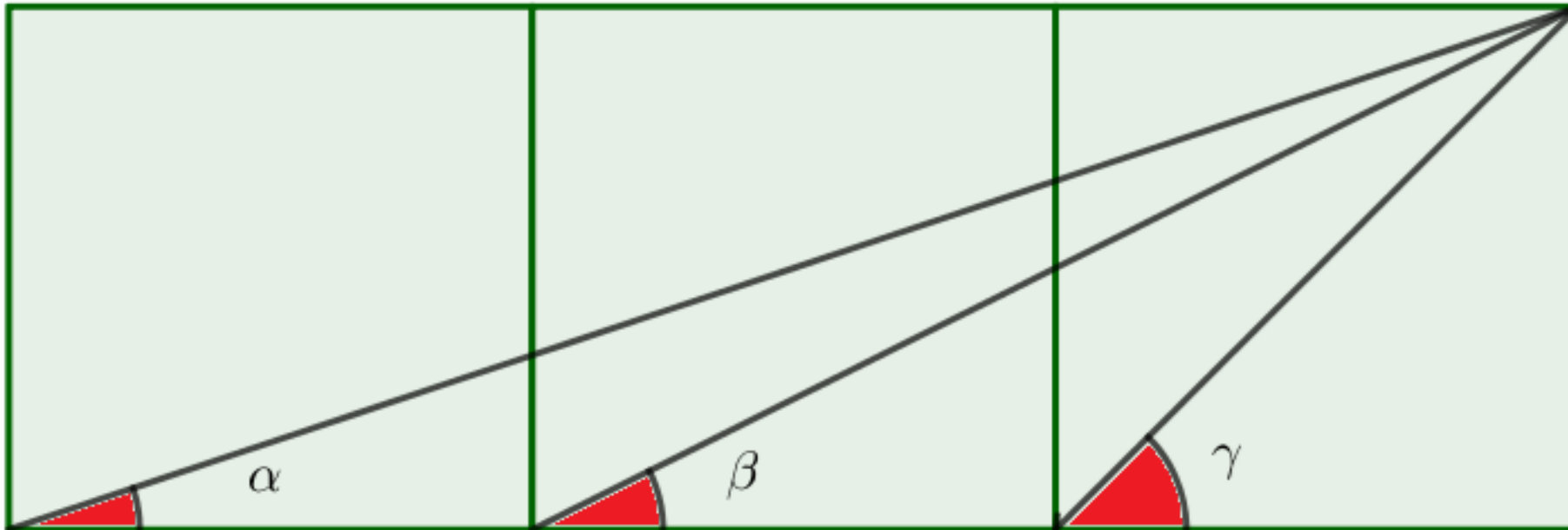


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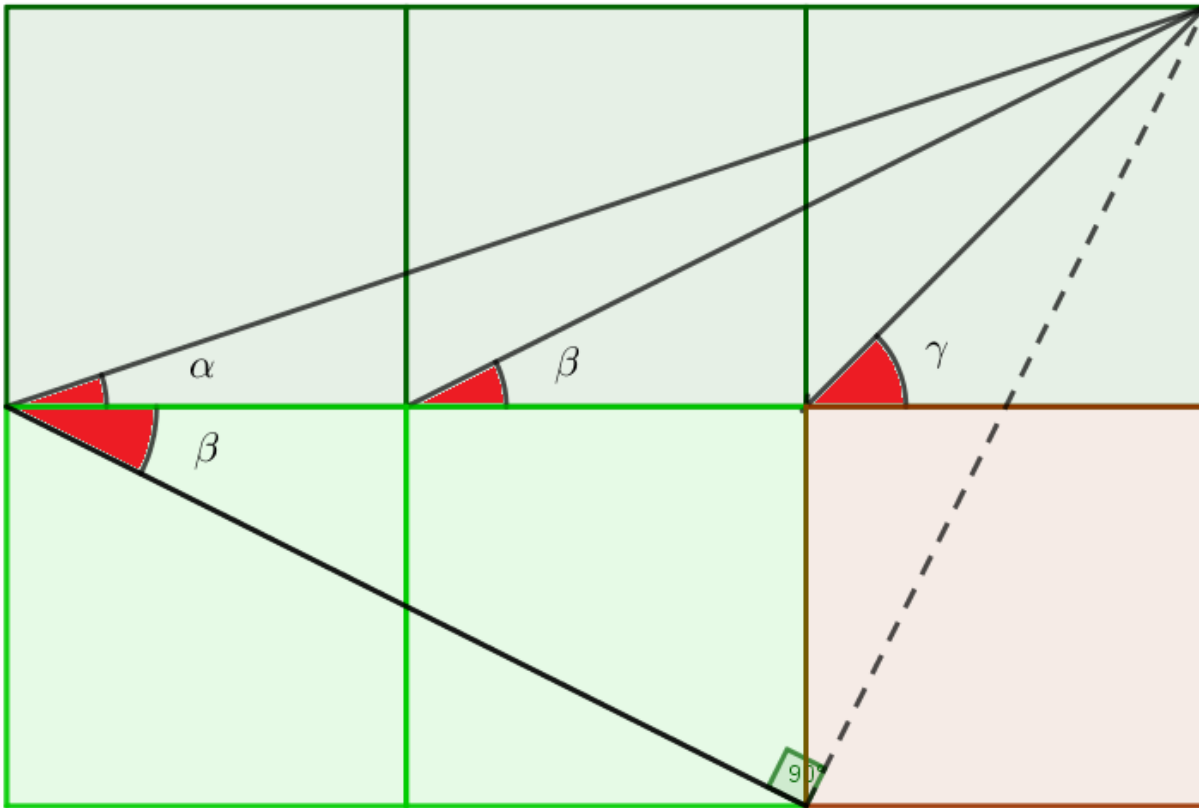
ANGULAR MYSTERY



Three equal squares are side-by-side. Lines are drawn from the top right vertex of the rightmost square to the bottom left vertices of the three squares, forming angles α , β , γ as shown in the picture. What is

$$\alpha + \beta + \gamma?$$

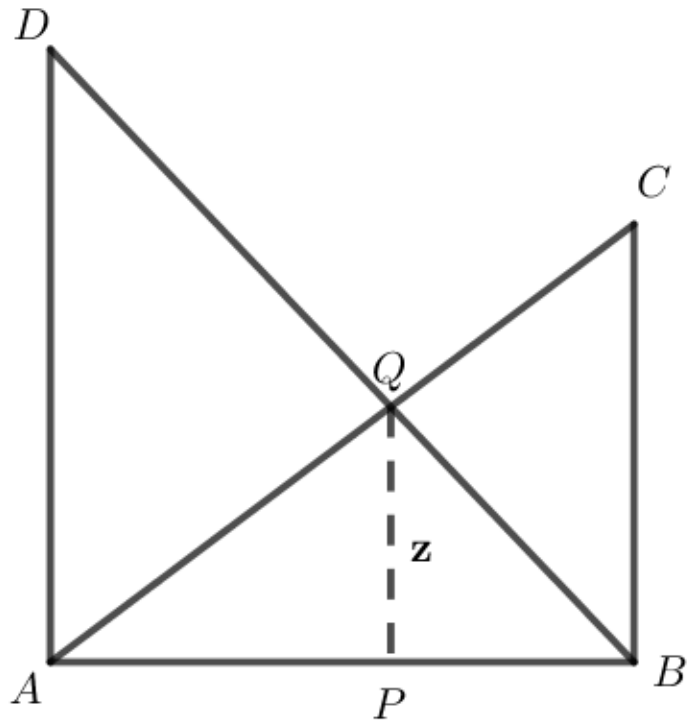
ANGULAR MYSTERY SOLVED



The picture shows that $\alpha + \beta = 45^\circ$.
Since $\gamma = 45^\circ$, the answer is

$$\alpha + \beta + \gamma = 90^\circ$$

WHERE IS PYTHAGORAS, WHEN WE NEED HIM?



$$|AB| = 40$$

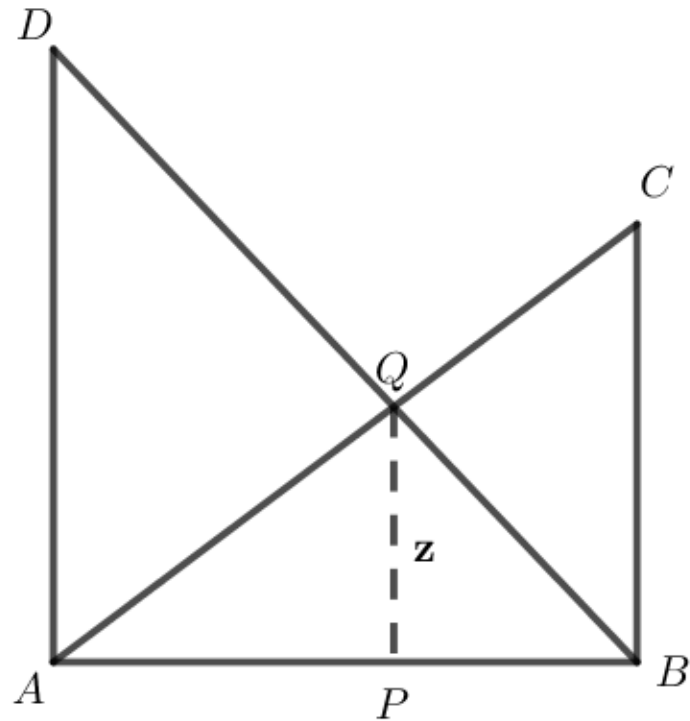
$$|AC| = 50$$

$$|BD| = 58$$

Angles $\angle DAB$ and $\angle ABC$ are right angles.

Determine $|PQ|$

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Determine $|PQ|$

Solution. Let $z = |PQ|$. By the theorem of Pythagoras, $|BC| = \sqrt{50^2 - 40^2} = 30$. $|AD| = \sqrt{58^2 - 40^2} = 42$. By similarity of triangles $\frac{z}{|BC|} = \frac{|AP|}{|AB|}$, thus $z = \frac{3}{4}|AP|$. Similarly, $\frac{z}{|AD|} = \frac{|PB|}{|AB|}$, hence $z = \frac{42}{40}|PB|$. It follows that $\frac{4}{3}z + \frac{21}{20}z = |AP| + |PB| = 40$

Solving for z :

$$z = \frac{35}{2} = 17.5$$

ANGLES GALORE

In a triangle ABC , the altitude and the median trisect the angle at A .

What is the measure of the angle at A ?

Justify your answer!!

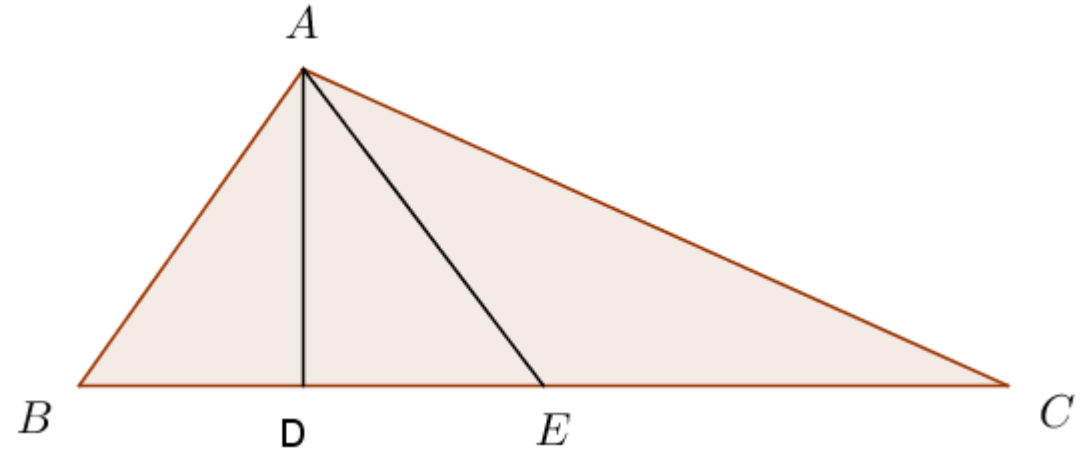
ANGLES GALORE

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What is the measure of the angle at A ?

Justify your answer!!

$$\angle A = 90^\circ$$

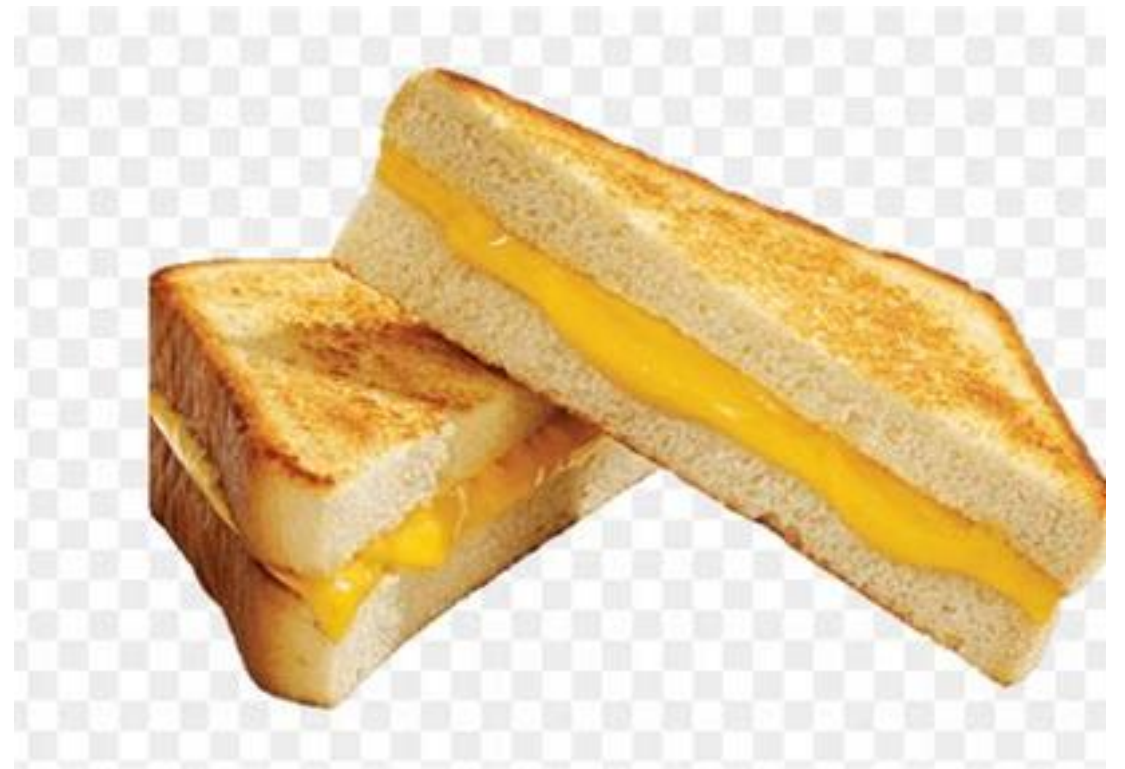


Let D and E be points on BC such that AD is an altitude and AE a median of triangle ABC , with D nearer B than C . Triangles ABD and AED are congruent, so that $DE = DB$. Since E is the midpoint of BC ,
 $DE : CE = 1 : 2$. Since AE bisects angle DAC , $AD : AC = DE : CE = 1 : 2$. Since DAC is a right triangle, we conclude that angle $DAC = 60$ degrees. It follows that $DAE = 30$ degrees, and BAC is a right angle.

The Joys of Counting

AT THE SANDWICH SHOP

- Jim is considering what sandwich to buy for lunch. He has a choice of 2 different types of bread, 3 different types of cheese, and 4 different types of meat. For his sandwich he must choose either one type of bread and one, type of cheese or one type of meat, or one type of bread, one type of cheese and one type of meat. How many different sandwiches can he choose?



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- Let's call the bread types A and B. With bread A, if he chooses the cheese or meat option, $3 + 4 = 7$ choices. Kif the cheese + meat option, he has $3 \times 4 = 12$ choices; all in all $12 + 7 = 19$ choices for bread A. The number of choices for bread B being the same, the answer is $19 + 19 = 38$ choices

STRINGING UP DIGITS

The counting numbers are written as a long string of digits:

123456789101112131415161718192021 . . .

What is the 2024th digit of this string?

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SOLUTION Suppose we move along the string, each step taking us past one digit. After 9 steps we are past the single digit numbers. There are 90 two digit numbers, so after $2 \times 90 = 180$ additional steps we reach the first three digit number, namely 100. We have now walked $9 + 180 = 189$ steps. We still have $2024 - 189 = 1835$ steps to go. There are 900 three digit numbers, so to pass them all would take $900 \times 3 = 2700$ steps, which is more than we have to take. The end of our quest will be a three digit number. We now advance in steps of three, every three steps is a new three digit number. Now $1835 = 3 \times 611 + 2$. It follows that 1835 steps place us at the second digit of the number that is the 611th three digit number. That number is $100 + 611 = 711$; the second digit of 711 is 1. The answer is 1.