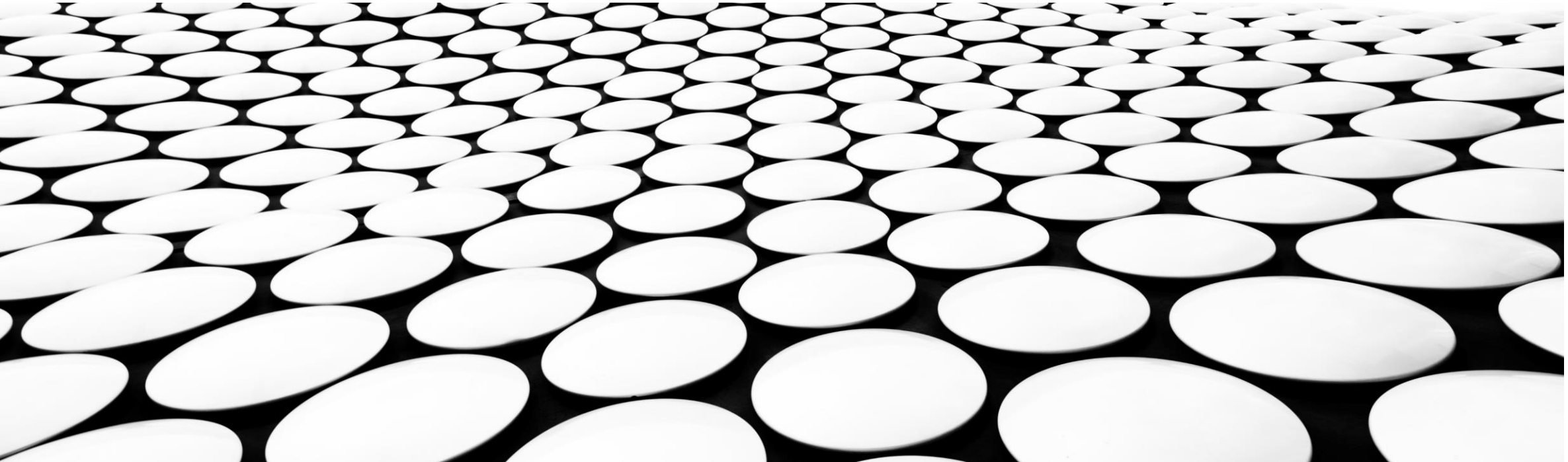


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# MATH CIRCLE AT FAU

9/7/2024



# THE ISLAND OF KNIGHTS AND KNAVES



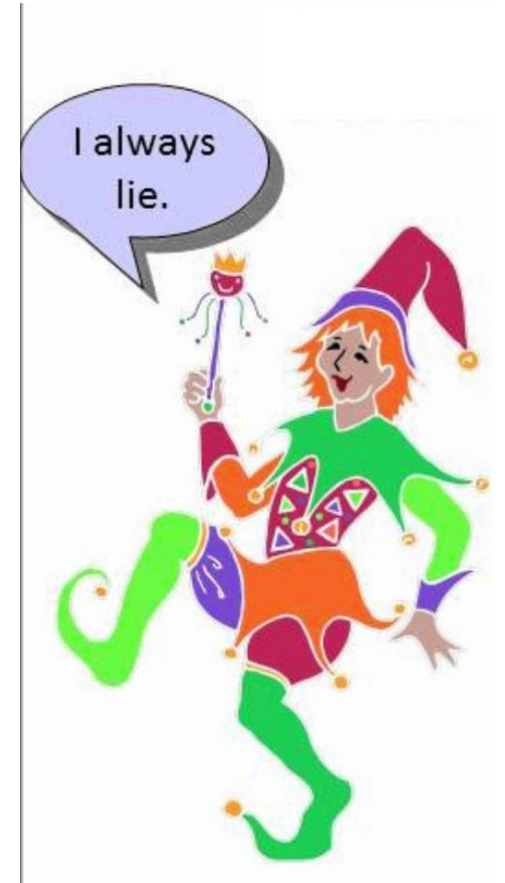
Here we are on the island of knights and knaves; The knights who can only tell the truth, the knaves who always lie.

You visit the island and meet three people, Alph, Beth and Gam.

Alph tells you: "All of us are knaves."

Beth says: "Exactly one of us is a knight."

What are Alph, Beth, and Gam?



# THE ISLAND OF KNIGHTS AND KNAVES

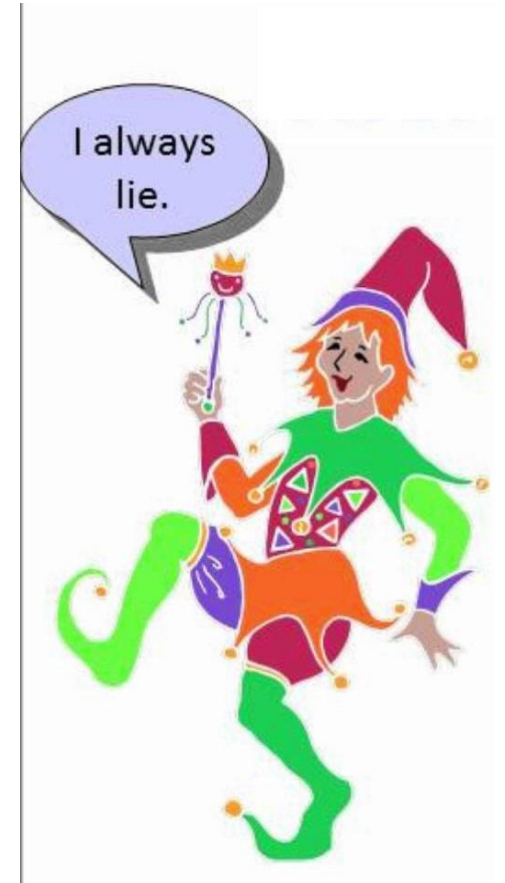


Still in the island of knights and knaves; The knights who can only tell the truth, the knaves who always lie.

On the island you meet Alfie and Balfie.

Alfie tells you: "Either I am a knave or Balfie is a knight."

What are Alfie and Balfie?



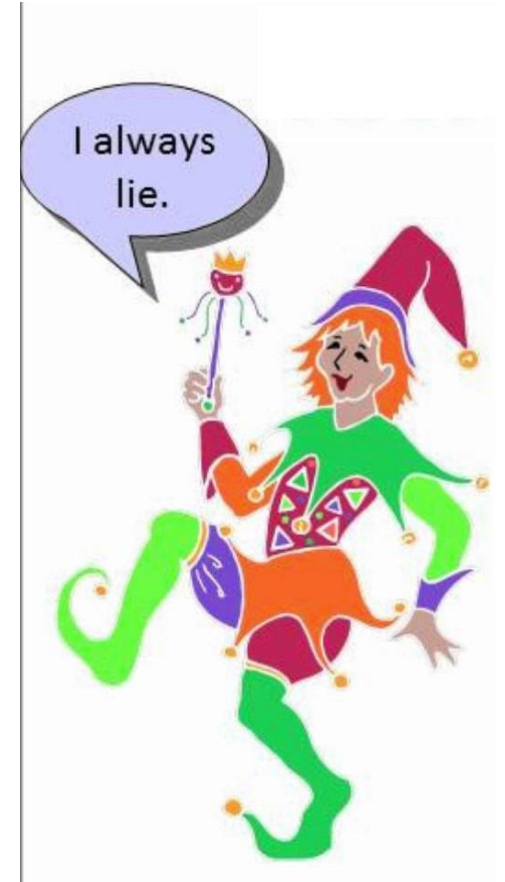
# THE ISLAND OF KNIGHTS AND KNAVES



Still in the island of knights and knaves; The knights who can only tell the truth, the knaves who always lie.

You meet Awkwy, a somewhat confused individual who tells you: "Either I am a knave or  $2 + 2 = 5$ ."

What can you conclude?



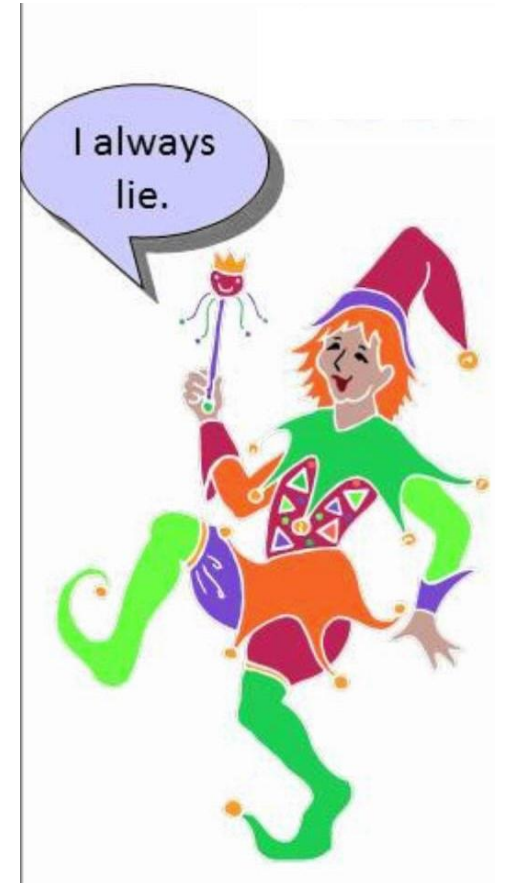
# THE ISLAND OF KNIGHTS AND KNAVES



One more in the island of knights and knaves; The knights who can only tell the truth, the knaves who always lie.

On the island, Fred the explorer (who is visiting with his team) meets two islanders, Arbie and Barbie, resting under a tree. Fred asked them "is either of you a knight?" Arbie responded and Fred knew the answer to his question.

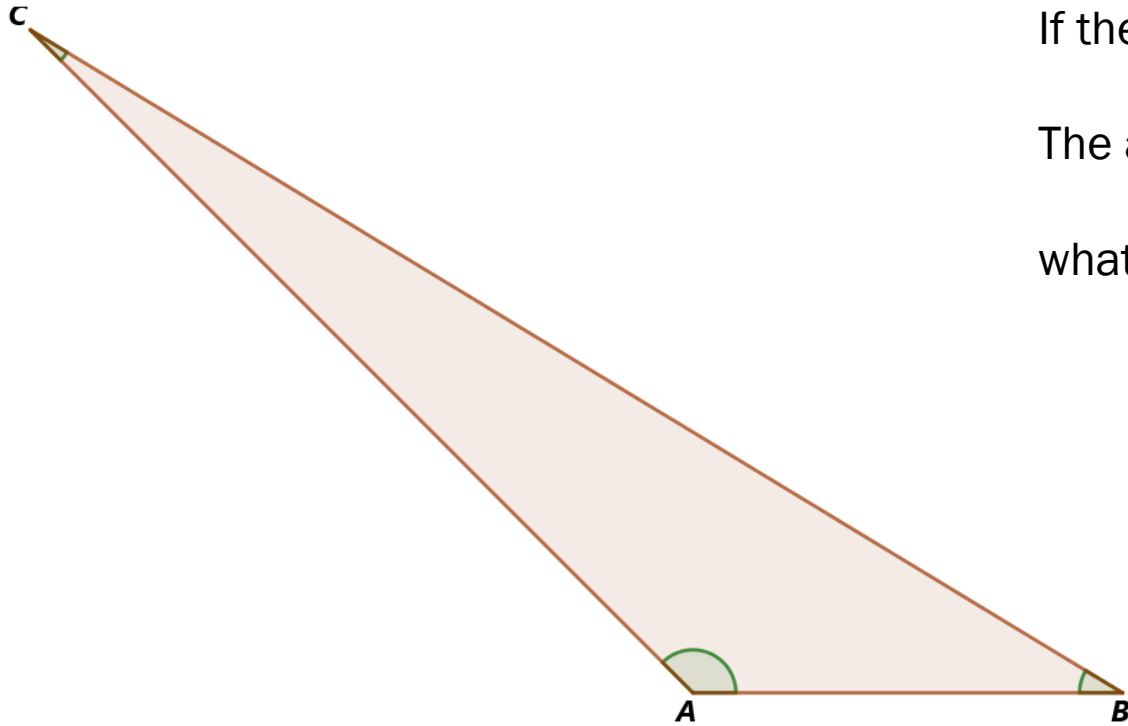
What are Arbie and Barbie?



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# *Geometry Basics, and More*

## SUM OF ANGLES, FIRST TRY.



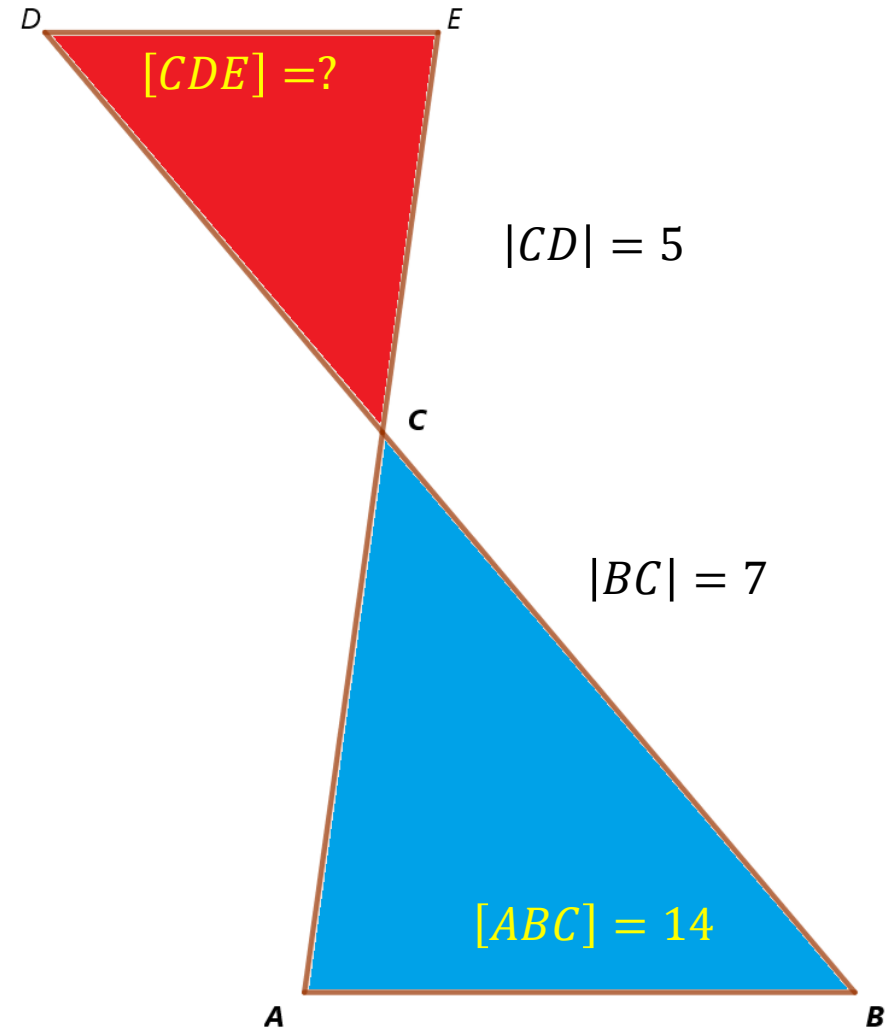
If the angle at  $B$  measures  $32^\circ$ ,

The angle at  $C$  measures  $13^\circ$ ,

what is the measure of the angle at  $A$ ?

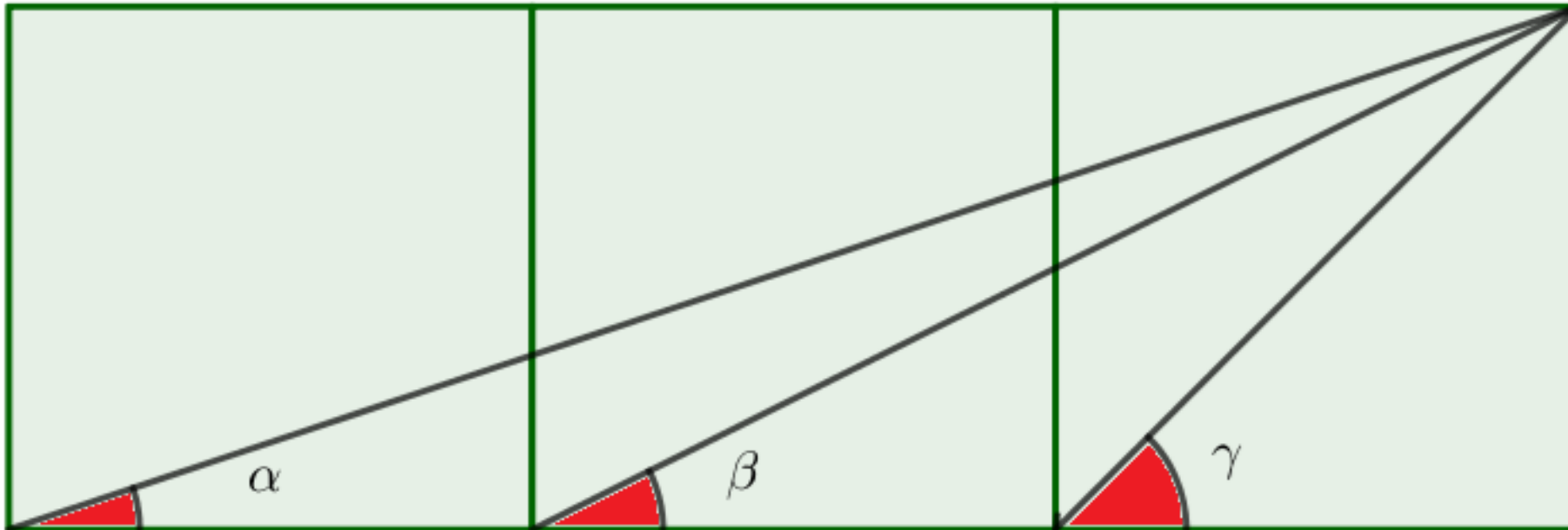
## TWO TRIANGLES

- The area of triangle  $ABC$  is 14 square feet: Side  $BC$  is 7 feet long. Side  $CD$  of triangle  $CED$  is 5 feet long.
- What is the area of triangle  $CDE$ ?





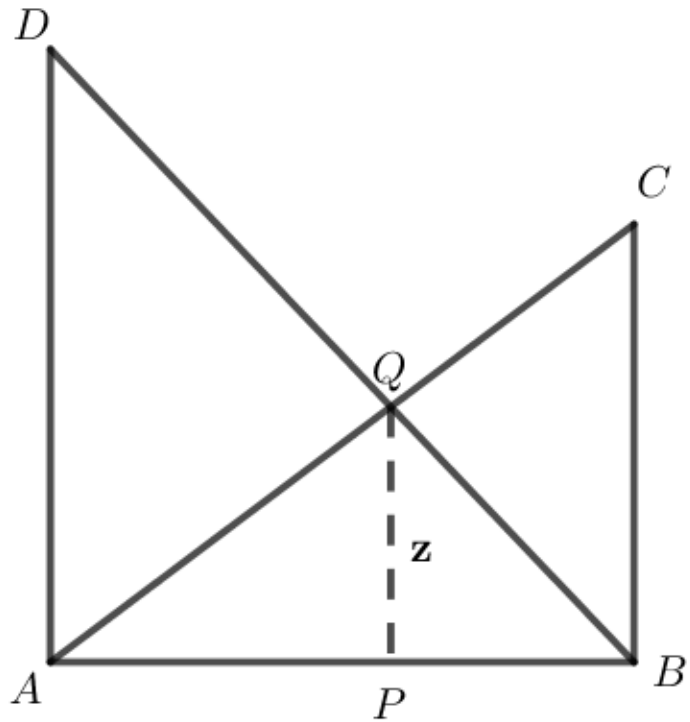
## ANGULAR MYSTERY



Three equal squares are side-by-side. Lines are drawn from the top right vertex of the rightmost square to the bottom left vertices of the three squares, forming angles  $\alpha$ ,  $\beta$ ,  $\gamma$  as shown in the picture. What is

$$\alpha + \beta + \gamma?$$

## WHERE IS PYTHAGORAS, WHEN WE NEED HIM?



$$|AB| = 40$$

$$|AC| = 50$$

$$|BD| = 58$$

Angles  $\angle DAB$  and  $\angle ABC$  are right angles.

Determine  $|PQ|$

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## ANGLES GALORE

In a triangle  $ABC$ , the altitude and the median trisect the angle at  $A$ .

What is the measure of the angle at  $A$ ?

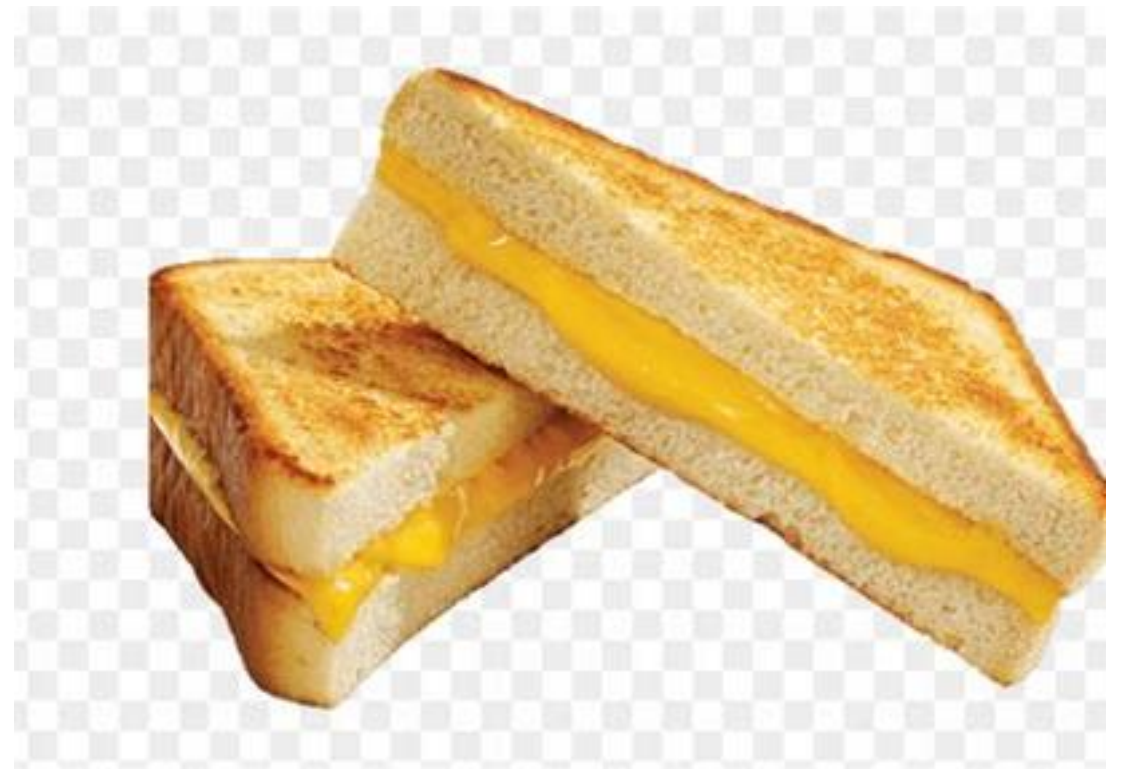
**Justify your answer!!**

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# *The Joys of Counting*

## AT THE SANDWICH SHOP

- Jim is considering what sandwich to buy for lunch. He has a choice of 2 different types of bread, 3 different types of cheese, and 4 different types of meat. For his sandwich he must choose either one type of bread and one, type of cheese or one type of meat, or one type of bread, one type of cheese and one type of meat. How many different sandwiches can he choose?



## STRINGING UP DIGITS

The counting numbers are written as a long string of digits:

**123456789101112131415161718192021 . . .**

What is the 2024th digit of this string?