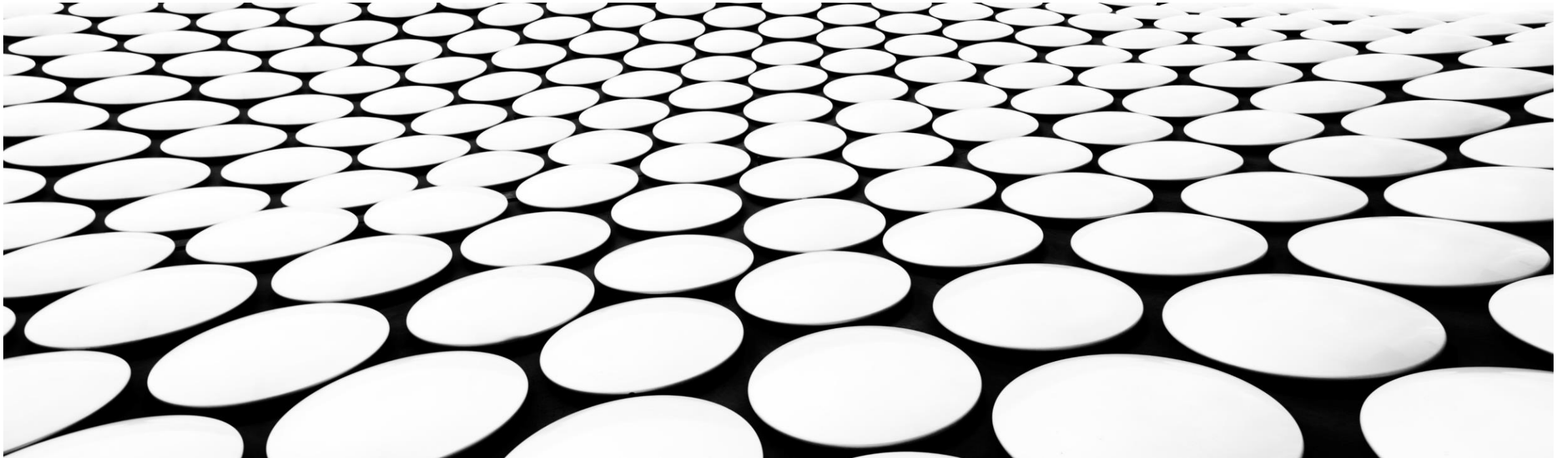

MATH CIRCLE AT FAU

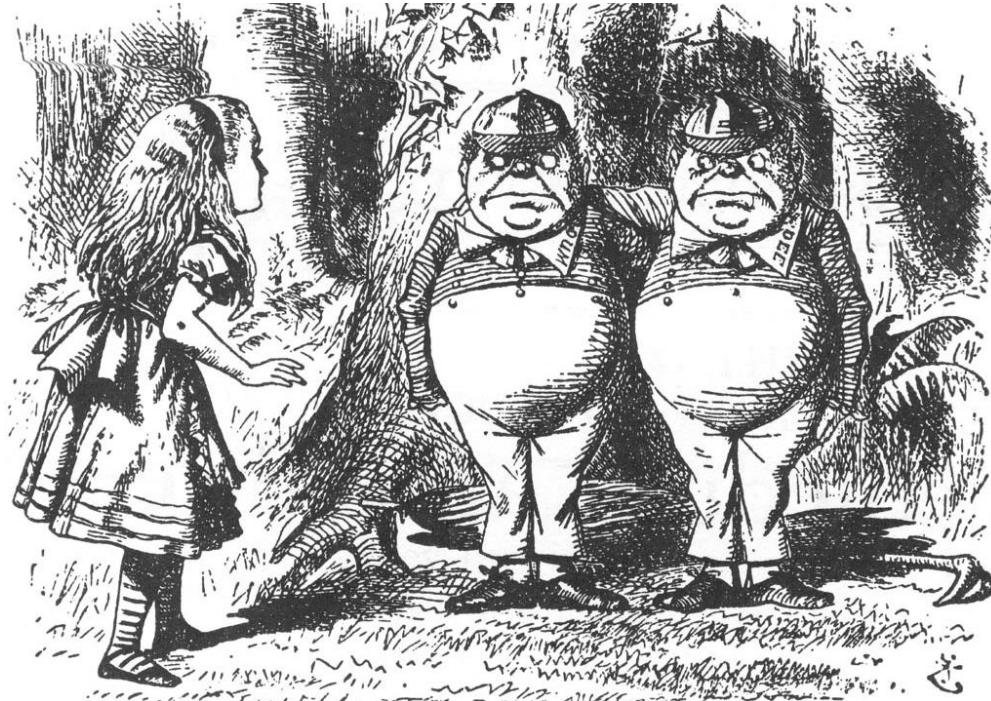
5/11/2024



PROFESSOR SMULLYAN'S ALICE

In *Through the Looking Glass*, Alice meets two strange brothers, Tweedledee and Tweedledum.

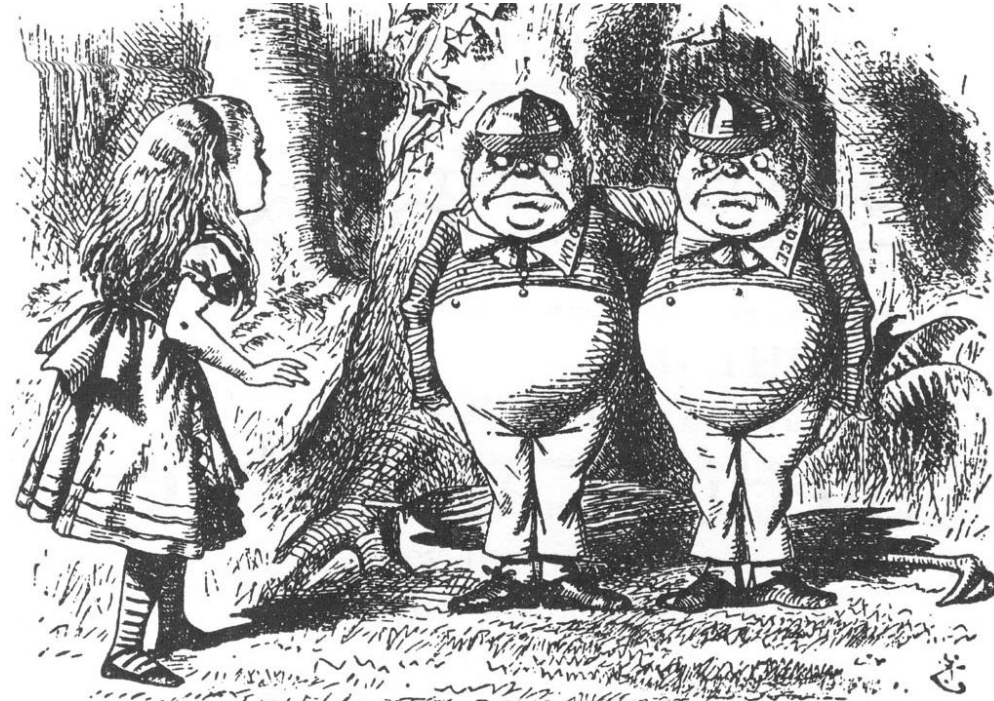
Turns out one of them lies on Mondays, Tuesdays and Wednesdays, tells the truth all other weekdays. The other one lies on Thursdays, Fridays and Saturdays; tells the truth the other days of the week.



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One day Alice meets the brothers and they say:

First one: I am Tweedledum.

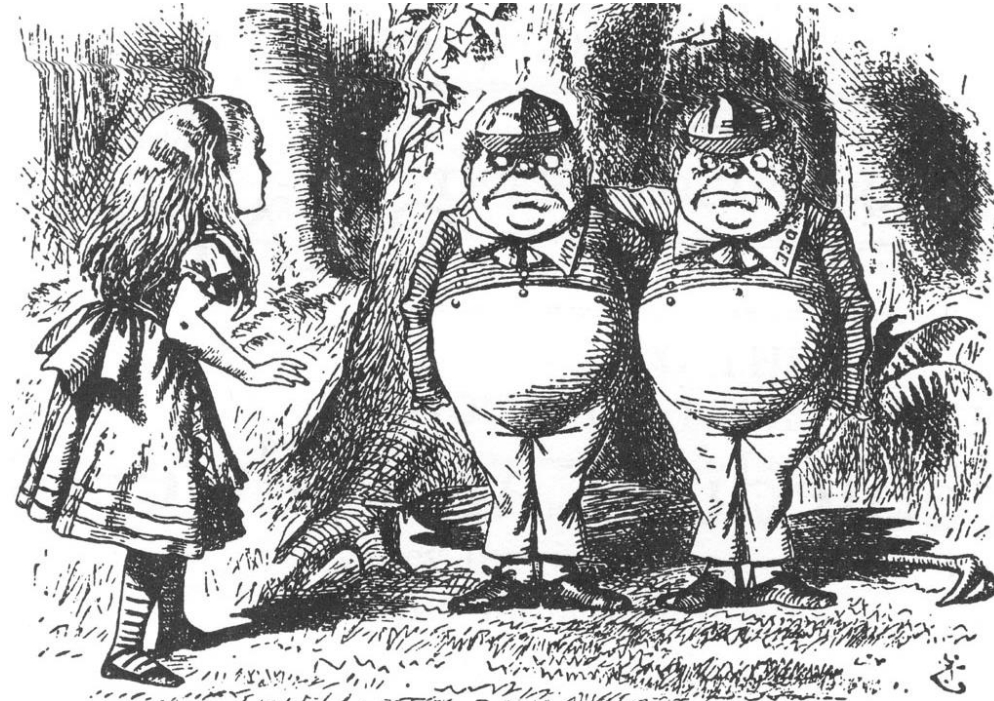
Second One: I am Tweedledee

Which one is really which?

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On a different day of the same week Alice again meets the brothers and they say:

First one: I am Tweedledum.

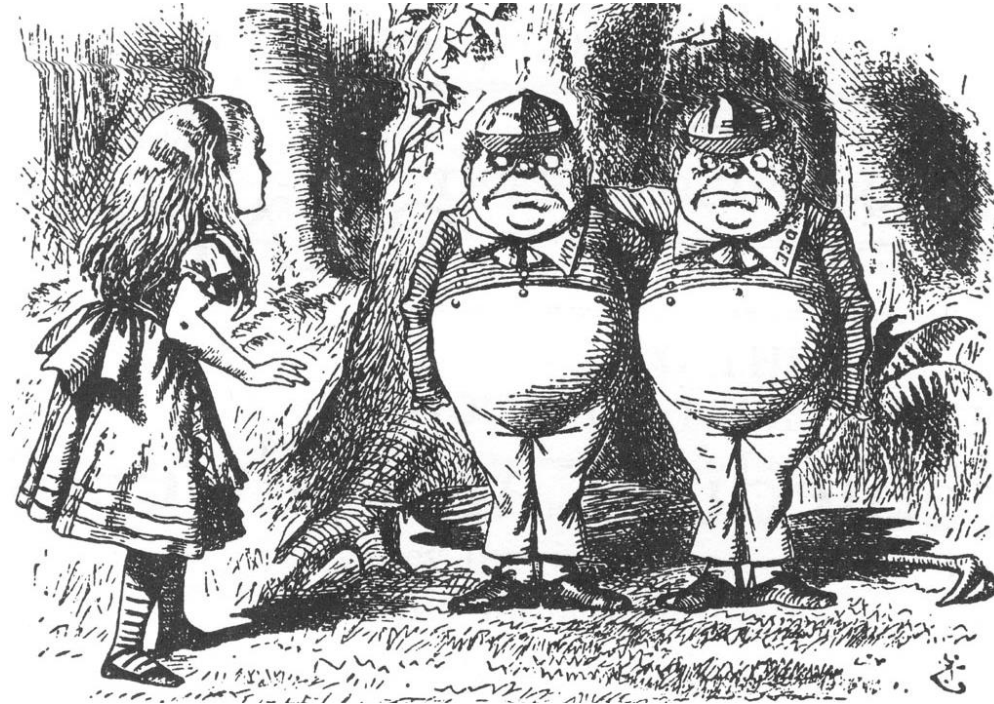
Second One: If that is really true, then I am Tweedledee.

Which one is really which?

PROFESSOR SMULLYAN'S ALICE

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Turns out one of them lies on Mondays, Tuesdays and Wednesdays, tells the truth all other weekdays. The other one lies on Thursdays, Fridays and Saturdays; tells the truth the other days of the week.



One day Alice met only one of the brothers. He said:

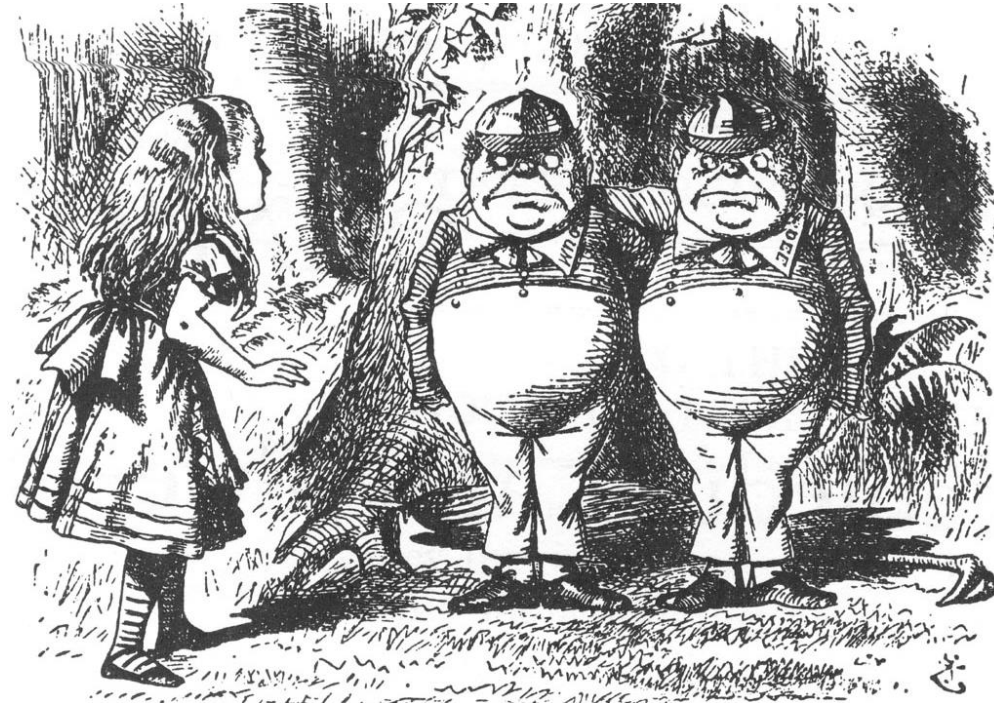
I am lying today, and I am Tweedledee.

Who was he?

PROFESSOR SMULLYAN'S ALICE

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Turns out one of them lies on Mondays, Tuesdays and Wednesdays, tells the truth all other weekdays. The other one lies on Thursdays, Fridays and Saturdays; tells the truth the other days of the week.



If the brother had said:

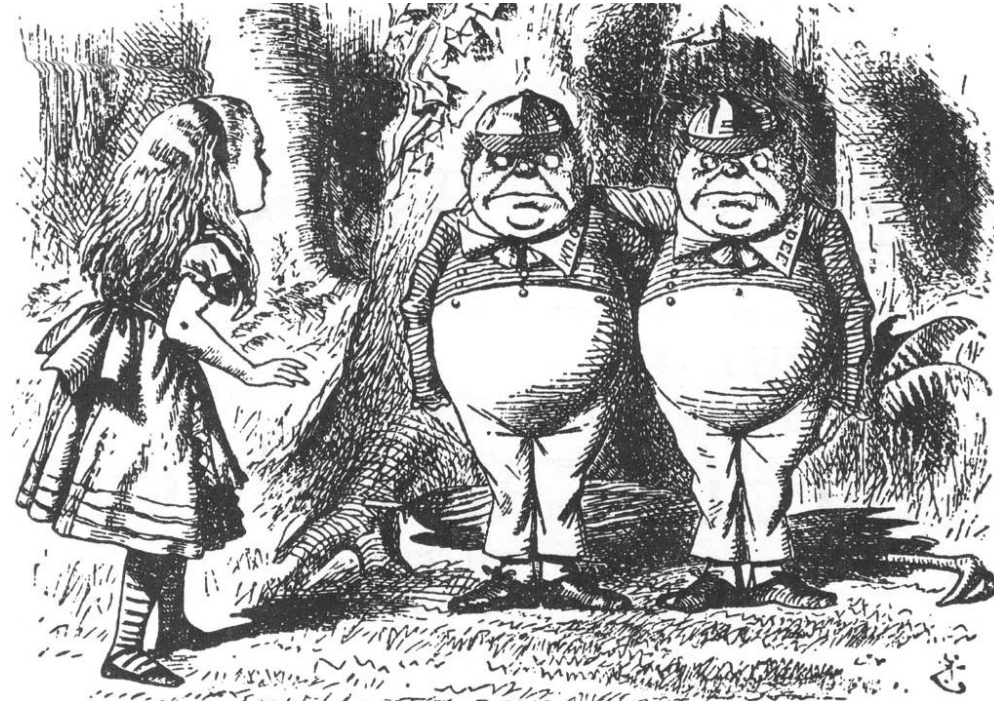
I am lying today, or I am Tweedledee.

Could one determine who he was?

PROFESSOR SMULLYAN'S ALICE

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Turns out one of them lies on Mondays, Tuesdays and Wednesdays, tells the truth all other weekdays. The other one lies on Thursdays, Fridays and Saturdays; tells the truth the other days of the week.



Again Alice meets the brothers, and they say:

First one: If I am Tweedledum, then he's Tweedledee..

Second One: If he's Tweedledee, then I'm Tweedledum.

Can one figure out which one is really which? Can one know what day of the week it is?

CARELESS COURIERS



- A chunk of consecutively numbered pages fell out of a folder. The first page of the chunk has number 463; the last has a number with the same digits 4, 6, 3, in a different order. Each sheet is numbered on both sides, with consecutive numbers.
- How many pages were dropped?

CARELESS COURIERS



- The last page must have an even number, so it is 634.
- If we say that each sheet consists of two pages, front and back, then
$$634 - 463 + 1 = 172$$
- pages were dropped.

86 sheets were dropped.

JACK AND JILL

Jack and Jill ;leave for school from the same spot, at the same time, and follow the same route. Each of Jack' steps is 10%% longer than Jill's, but Jack takes 10% fewer steps per minute than Jill.

Who will get to school first?



JACK AND JILL

Equations help! Let's say that in one minute of walking Jill covers a distance of d . If her steps have length x steps she took d/x steps. In this minute, Jack took only $0.9 \left(\frac{d}{x}\right)$ steps. But his steps were 10% longer than Jill's, so the distance he covered is

$$(1.1x)(0.9) \left(\frac{d}{x}\right) = 0,99 d < d.$$



Jill gets to school first

CHOPPING TREES

A logging company wants to chop down a forest that is 99% pine trees. The Forest Service objects.

The logging company then says it will only cut pine trees, and when it is done the forest will be 98% pine trees.

What part of the forest will be chopped down?

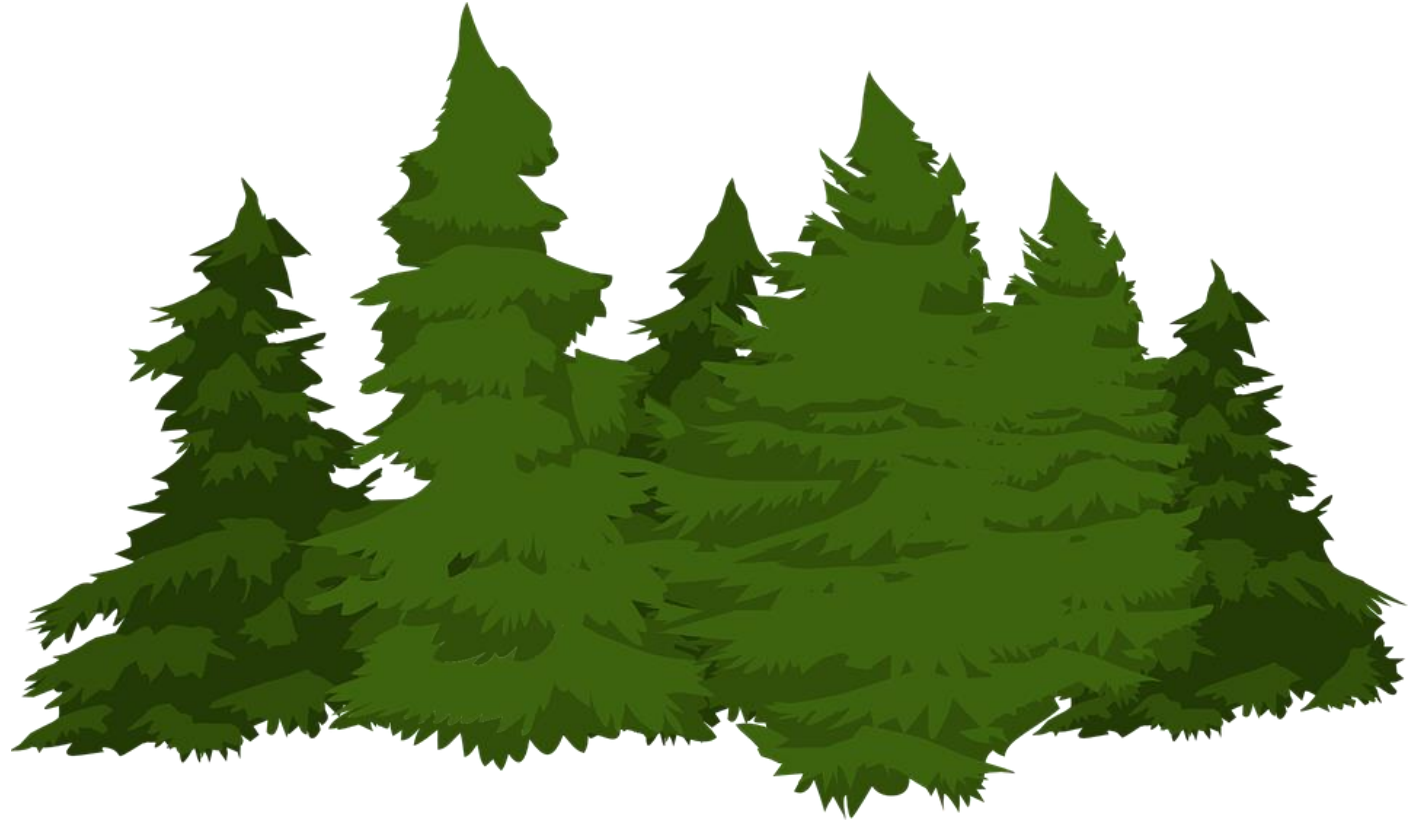


CHOPPING TREES

Let p be the original number of pine trees, n of non-pine trees. Then $p = 0.00(p + n)$. We can solve to get $p = 99n$.

After chopping x pines, we will have $p - x = 0.98(p - x + n)$. Using the value of p from above, we can solve this to get $x = 50n$.

The original number of trees in the forest was $p + n = 99n + n = 100n$.



Half the forest gets chopped down.

A NUTTY STORY

- A merchant brought a bag of nuts to sell at the market.
- The first customer bought 1 nut, the second bought 2 nuts, the third customer bought 4, and so on, with each succeeding customer buying twice as many nuts as the preceding one.
- The nuts the last customer bought weighed 50 lbs.
- The merchant had one nut left over.
- If all nuts weighed the same, how much did the merchant's bag weigh at the beginning of the day?



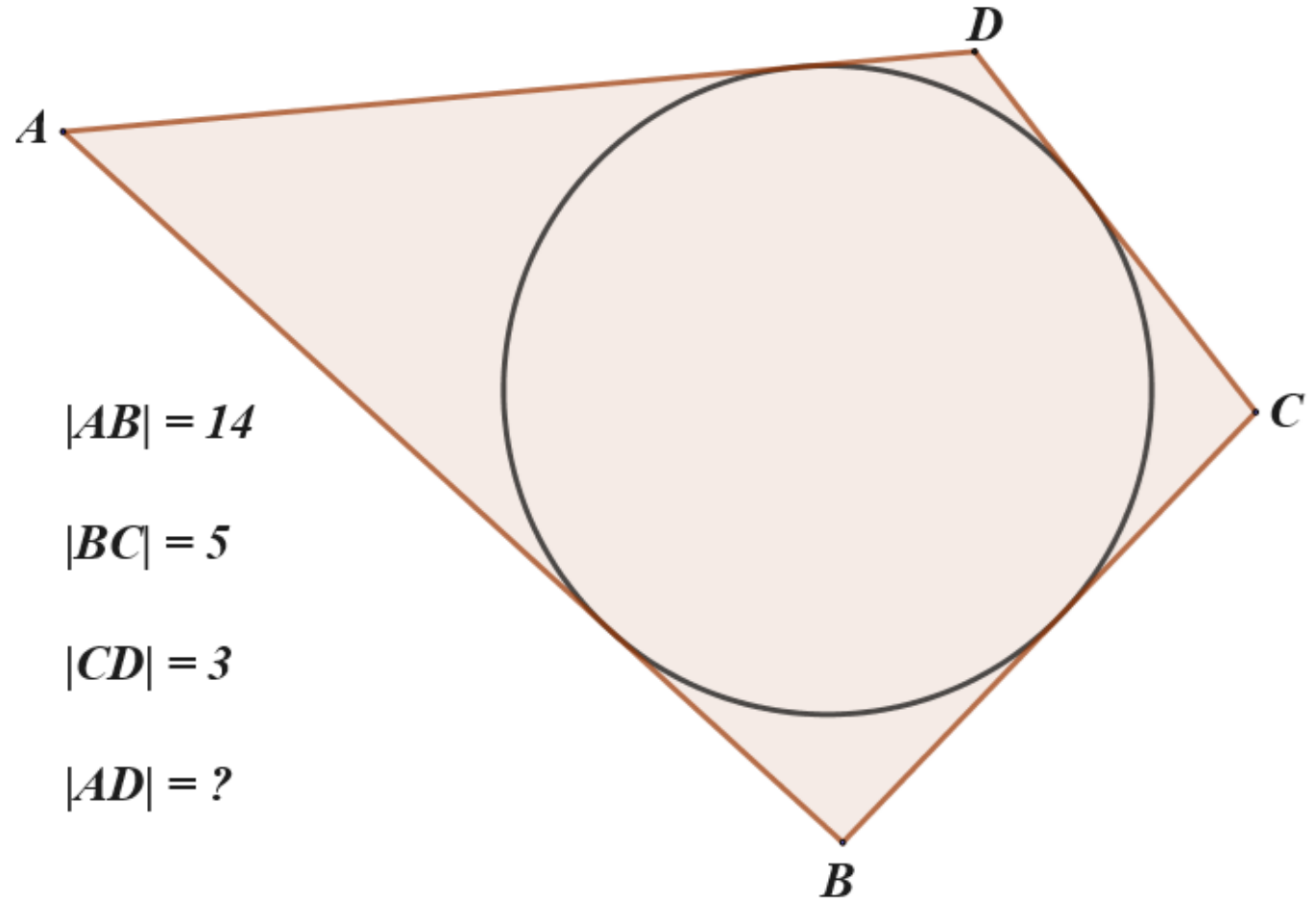
A NUTTY STORY

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 - The merchant had one nut left over.
 - If all nuts weighed the same, how much did the merchant's bag weigh at the beginning of the day?
- Try it out assuming there were only two or three buyers. Or even a single buyer. The number of nuts the last buyer buys exceeds by one (the nut the merchant is left over) the amount of all nuts sold previously. If the last buyer bought 50 lbs., all that was sold before the last buyer, plus the merchant's remaining nut, weighed 50 lbs.

100 lbs.

TANGENTIAL TROUBLES

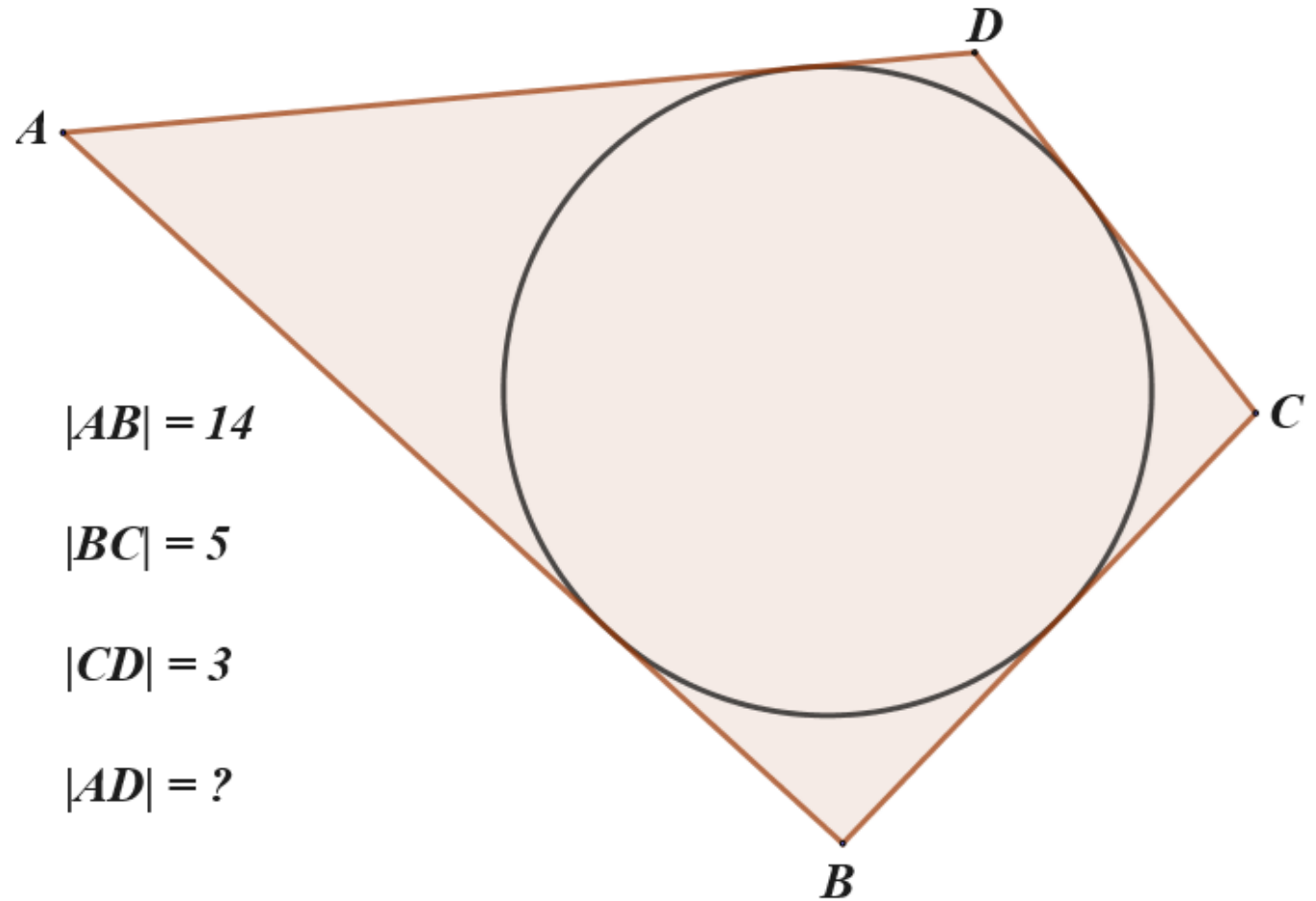
- The quadrilateral $ABCD$ circumscribes a circle.
- If sides AB, BC, CD have the indicated lengths, what is the length of side AD ?



TANGENTIAL TROUBLES

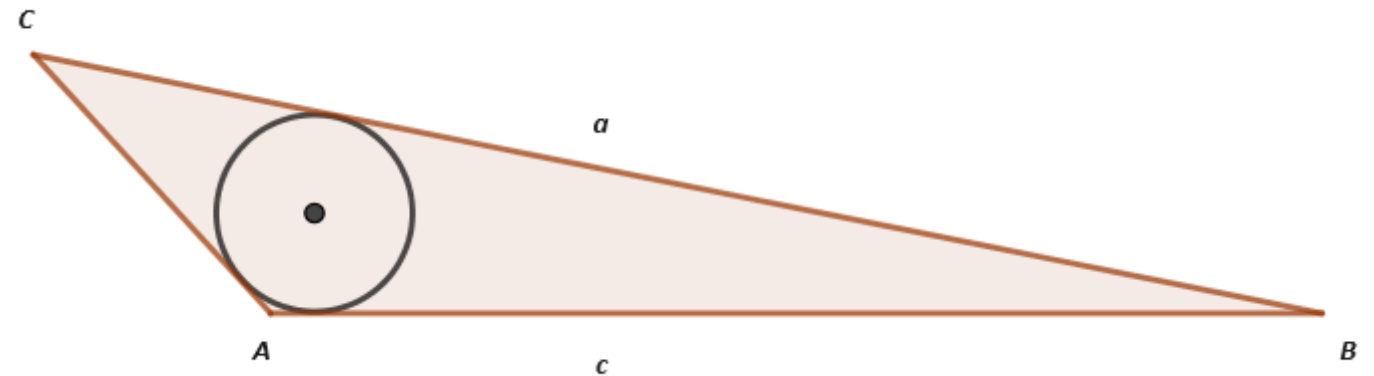
- The quadrilateral $ABCD$ circumscribes a circle.
- If sides AB, BC, CD have the indicated lengths, what is the length of side AD ?

$$|AD| = 12$$



INSCRIBING CIRCLES

- Triangle ABC has sides of lengths:
- $|AB| = c = 29$.
- $|BC| = a = 30$.
- $|CA| = b = 5$.



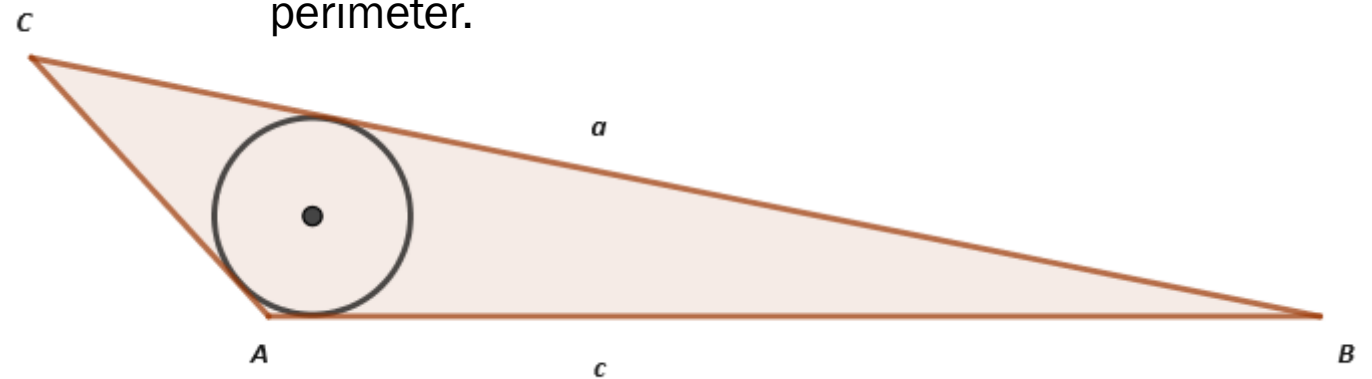
The picture is NOT to scale.

INSCRIBING CIRCLES

- Triangle ABC has sides of lengths:
- $|AB| = c = 29$.
- $|BC| = a = 30$.
- $|CA| = b = 5$.

- What is the radius of the inscribed circle?

For example, by Hero's formula the area is 72.
Then one uses $A = rs$, s being the semi-perimeter.



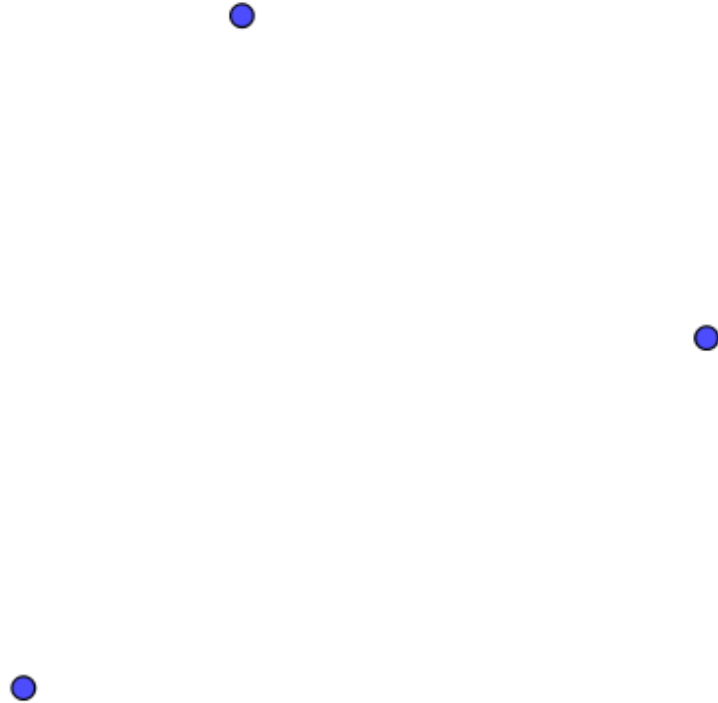
$$r = \frac{9}{4}$$

The picture is NOT to scale.

GHOST TRIANGLES

The midpoints of all three sides of a triangle have been marked. The triangle is then erased leaving only the marked midpoints. How can the triangle be recreated if **the only tools you are allowed to use are a straightedge and a compass?**

Here is how things may look.



GHOST TRIANGLES

The midpoints of all three sides of a triangle have been marked. The triangle is then erased leaving only the marked midpoints. How can the triangle be recreated if **the only tools you are allowed to use are a straightedge and a compass?**

C



Through B draw a line m parallel to AC ; through C a line n parallel to AB , through A a line p parallel to BC . Lines m and n will intersect at a point D , lines n and p at a point E , lines p and m at a point F . The triangle DEF is the triangle that was erased.

B



A

Drawing a line through a given point parallel to a given line is a basic straightedge and compass construction

Why does this construction do the job? Can you justify it?