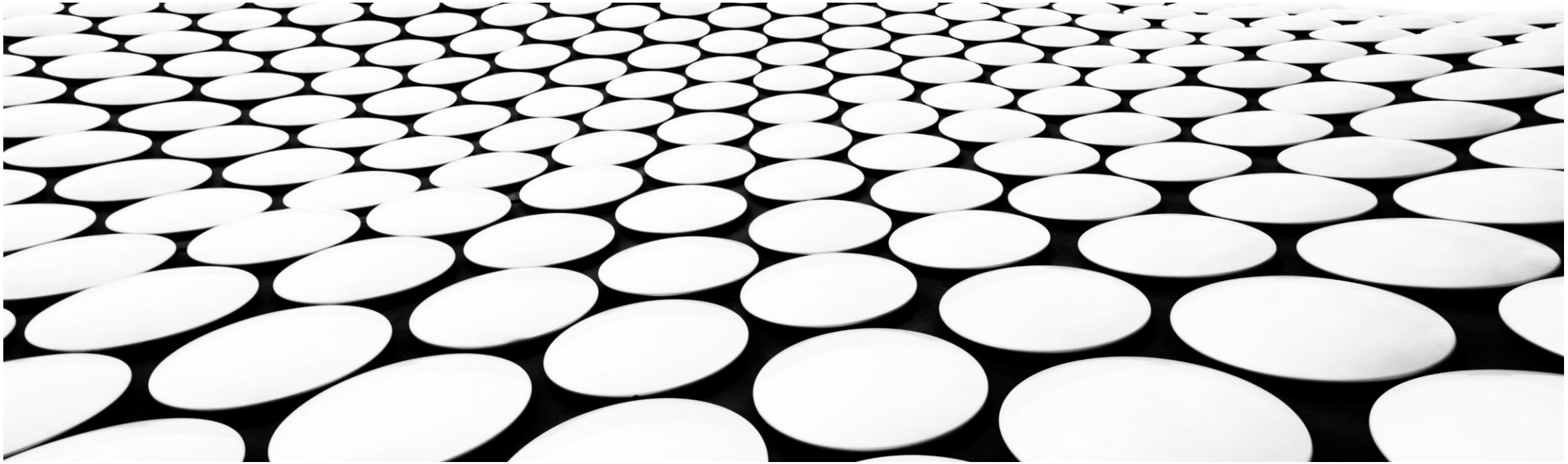
MATH CIRCLE AT FAU

1/11/2025

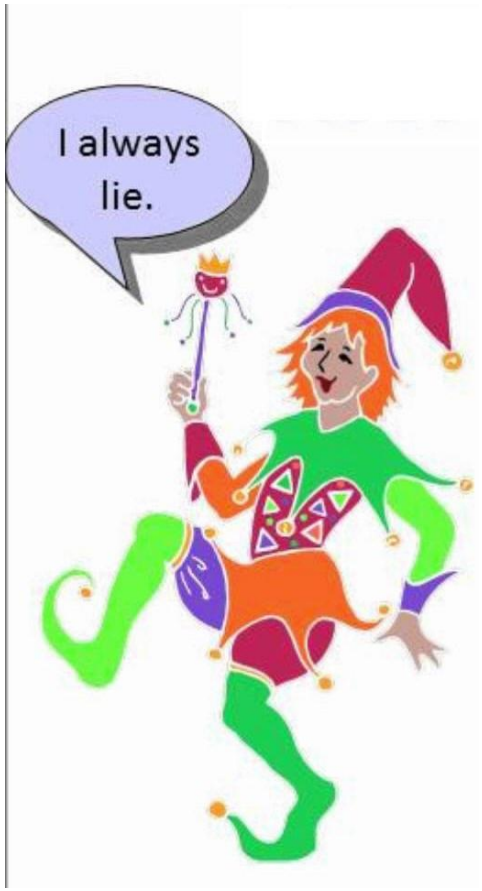


PART 1: PUZZLES

We have discussed many puzzles about the island of knights and knaves last semester. Now we try to working on some different types of puzzle which related to our daily life.

Today's puzzle is called 3X3 logic grid puzzles.

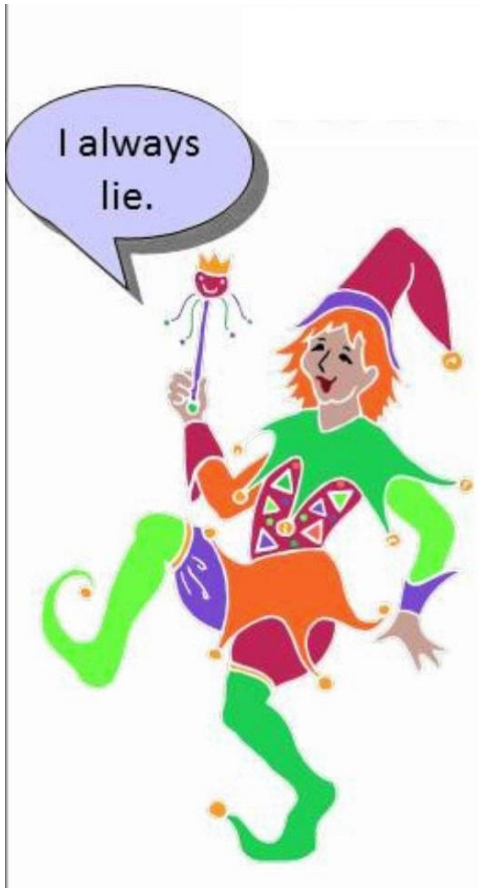
Puzzle 1: Daycare drop offs logic puzzle



PUZZLE: DAYCARE DROP OFFS LOGIC PUZZLE

A local daycare has 3 new attendees today, who have just been dropped off by their slightly anxious parents. Each child was given a toy from home to make them more comfortable. Using the clues provided, can you determine the age of each child and the toy they were dropped off with?
Names: Alice, Bob, Cindy
Ages: 3,4 and 5
Toys: Fire Truck, Helicopter, Bulldozer

Clues:
Bob, the youngest of the three, does not have a Bulldozer.
Cindy was comforted by her Helicopter.
The 4-year-old was not dropped off with a Bulldozer.



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Name	Age	Toy
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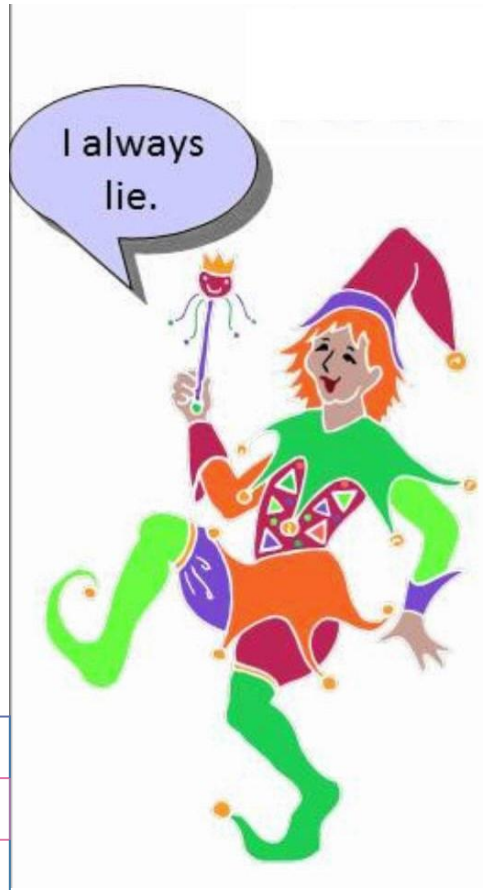
Cindy		



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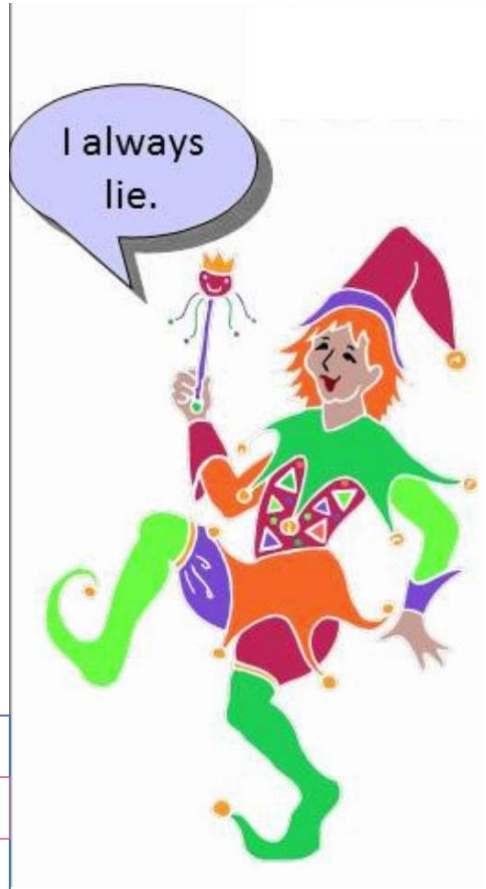
Name	Age	Toy
Alice	5	Bulldozer
Bob	3	Fire Truck
Cindy	4	Helicopter



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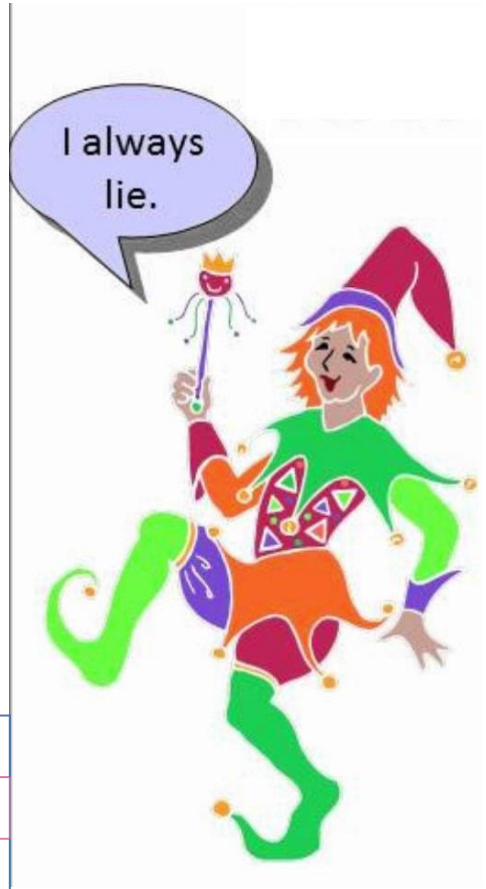
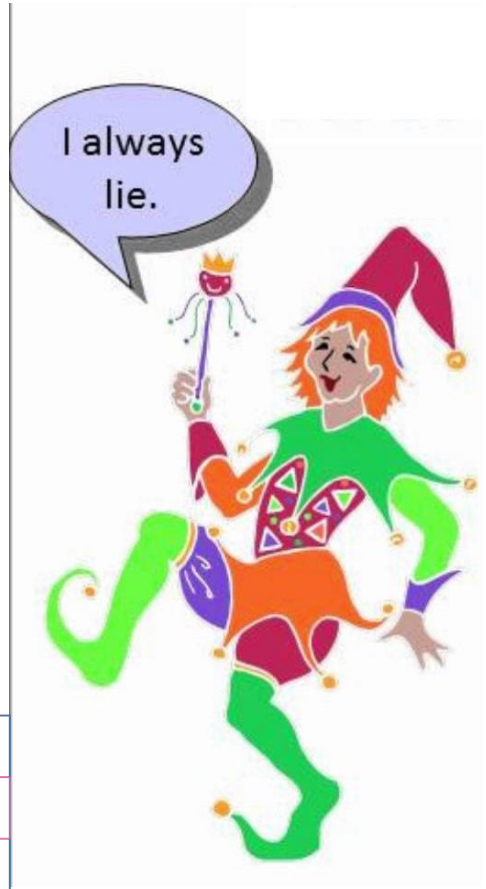


PUZZLE: ANOTHER TRY

Puzzle:
There are three kids: Arnold, Eric, and Peter. Their ages are 7, 8, and 9, but not necessarily in that order. Their birthdays are in January, April, and September, but again, not necessarily in that order.

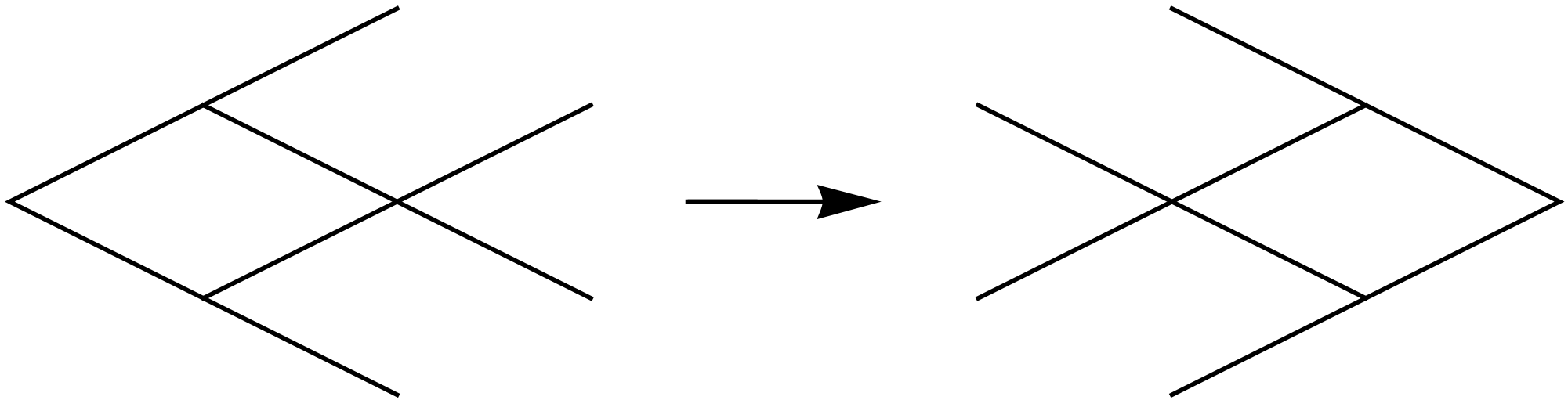
Clues:
1.Peter's birthday is in April.
2.Eric is 7 years old.
3.Arnold's birthday is in September.
4.Peter is 8 years old.

Name	Age	Birthday
Arnold		
Eric		
Peter		



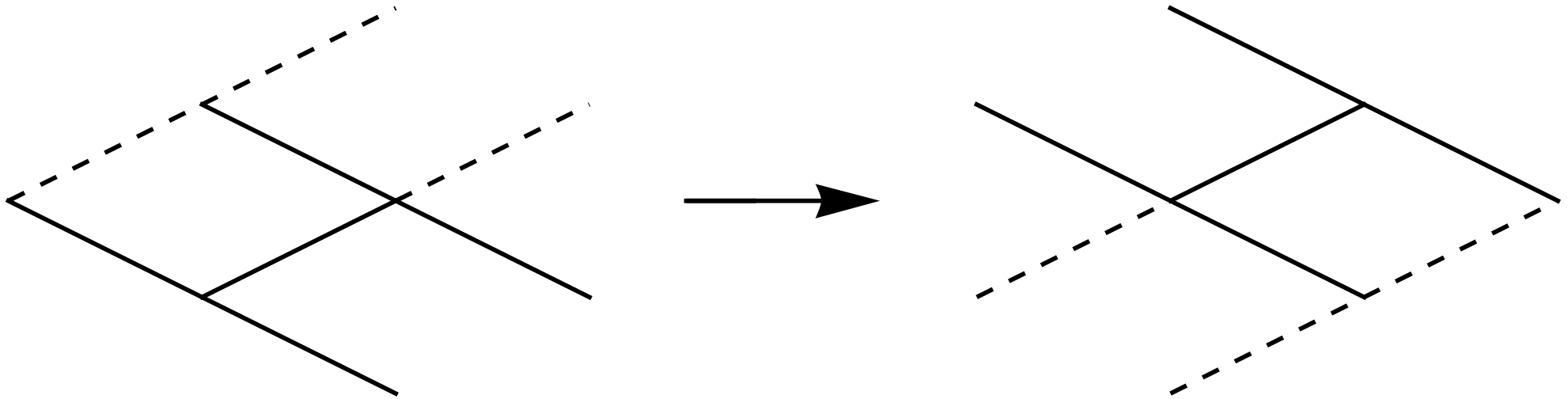
PART 2: ACTIVITY

Moving the three matchsticks shown as dashed lines in the figure



PART 2: ACTIVITY SOLUTION

Moving the three matchsticks shown as dashed lines in the figure



PART 3: ALGEBRA PROBLEM

Please factor $x^5 + y^5$

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Solution:
$(x+y)\ (x^4-x^3\ y+x^2\ y^2-x\ y^3+y^4)$

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Please factor $x^5 - y^5$

Solution:

$(x-y)\,(x^4+x^3\,y+x^2\,y^2+x\,y^3+y^4)$

PART 4: MATH CONTEST PROBLEMS

The AMC 8 (American Mathematics Competitions 8) is a prestigious math competition designed for middle school students, typically in grades 6-8, though younger students can also participate1. The competition consists of 25 multiple-choice questions that students must complete within 40 minutes.

The AMC 8 aims to promote the development of problem-solving skills and foster a love for mathematics among young students. The questions cover a range of topics, including algebra, geometry, number theory, and probability.

In 2025, the AMC competition will be held at FAU on Saturday, January 25th. Registration is now open on the FAU Math website.

Are you or someone you know planning to participate in the AMC 8?

2016 AMC 8 PROBLEM 5

P5: The number N is a two-digit number.
When N is divided by 9, the remainder is 1.
When N is divided by 10, the remainder is 3.
What is the remainder when N is divided by 11?

2016 AMC 8 PROBLEM 5

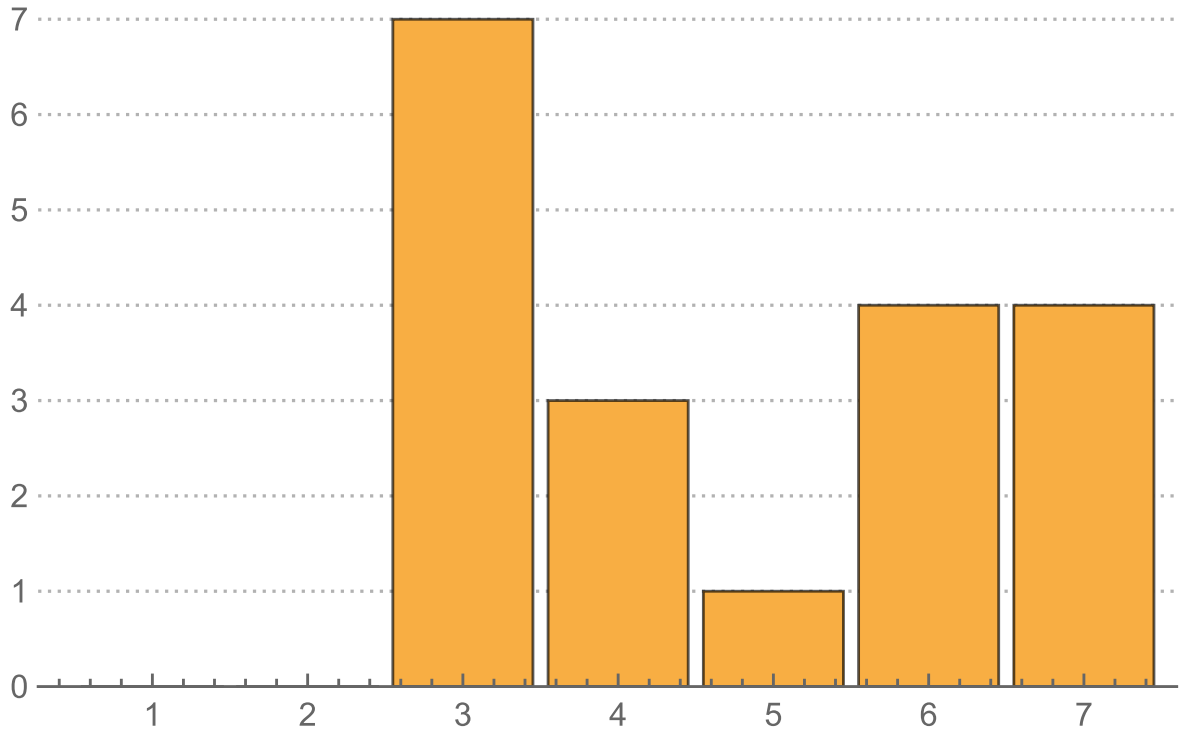
P5: The number N is a two-digit number. When N is divided by 9, the remainder is 1. When N is divided by 10, the remainder is 3. What is the remainder when N is divided by 11?

Solution: 7

Explanation: Let N=10a+3, then by the condition 1, we have mod(a+3,9)=1. Let a+3=9+1=10.
We can solve for a=7.
So back substitution gives N=73.
Last step, mod(73,11)=7

2016 AMC 8 PROBLEM 6

The following bar graph represents the length (in letters) of the names of 19 people. What is the median length of these names?

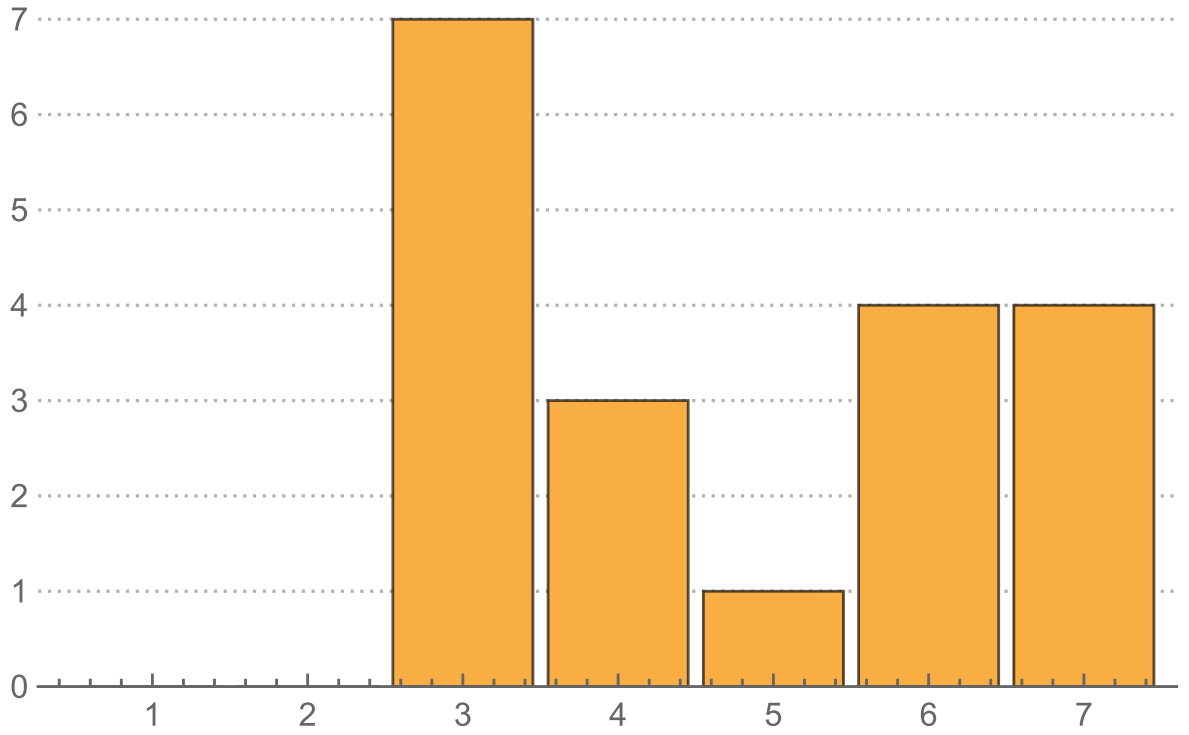


2016 AMC 8 PROBLEM 6

The following bar graph represents the length (in letters) of the names of 19 people. What is the median length of these names?

Solution: 4

Explanation: The middle position of 19 numbers is 10^{th} position.
There are 7 names length 3 with 2 more names of length 4. The next is a name of length 4.



2016 AMC 8 PROBLEM 7

P7: Which of the following numbers is not a perfect square?

$(A) 1^{2016}$
$(B) 2^{2017}$
$(C) 3^{2018}$
$(D) 4^{2019}$
$(E)\ 5^{2020}$

2016 AMC 8 PROBLEM 7

P7: Which of the following numbers is not a perfect square?
$(A)1^{2016}$ $(B)2^{2017}$ $(C)3^{2018}$ $(D)4^{2019}$ (E) 5^{2020}

Solution: (B)

Explanation:
(A)=1. It's a perfect square $1=1^2$
(C)=$(3^{1009})^2$
(D)=$(2^2)^{2019}$ =$(2^{2019})^2$
(E)= $(5^{1010})^2$

2016 AMC 8 PROBLEM 8

P8: Find the value of the expression 100-98+96-94+92-90+...+8-6+4-2

2016 AMC 8 PROBLEM 8

P8: Find the value of the expression 100-98+96-94+92-90+…+8-6+4-2

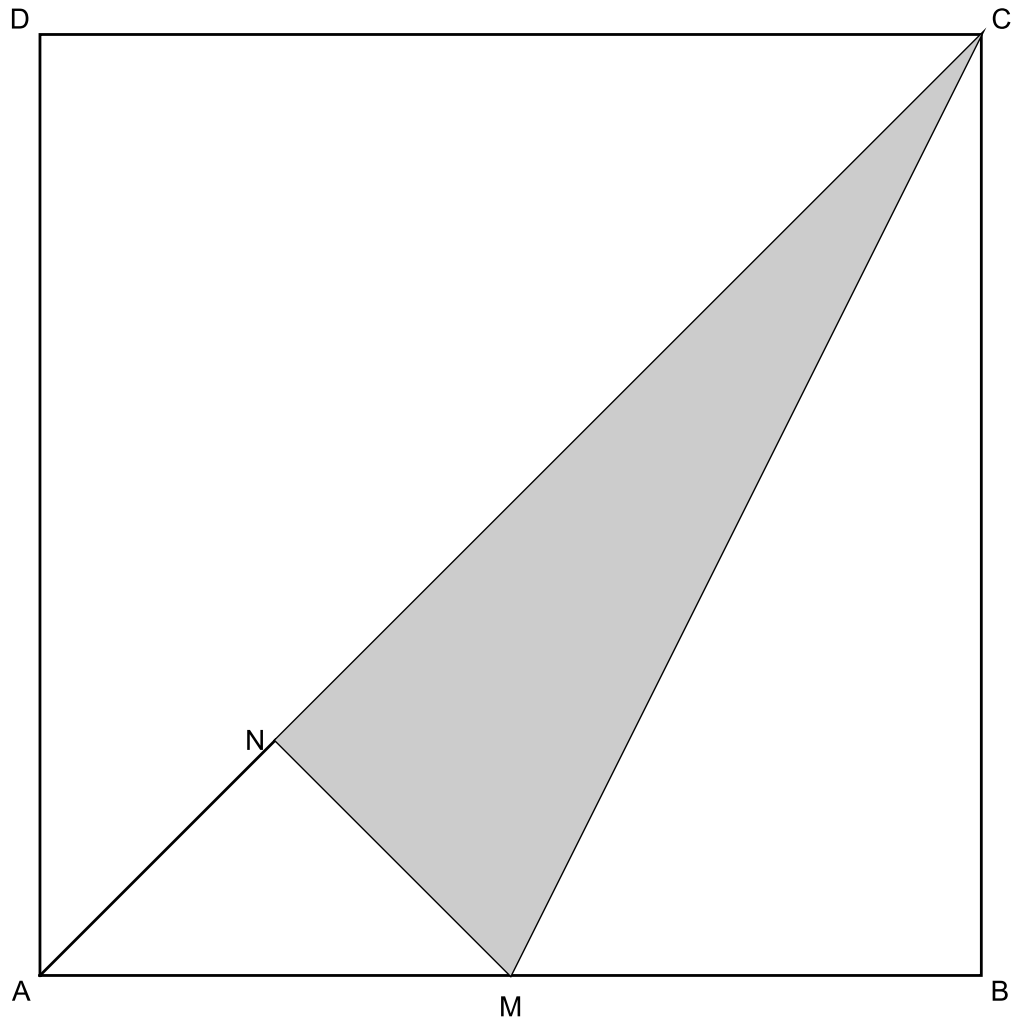
Solution: 50

Explanation: We can regroup the calculation as (100-98)+(96-94)+(92-90)+…+(8-6)+(4-2).
It's the sum of 2's.
Noticed that in (4-2), 4=4*1, in (8-6), 8=4*2, …, in (100-98), 100=4*25. There are 25 of 2's.
Hence the result will be 25*2 = 50.

PART 5: GEOMETRY PROBLEM

In a square ABCD, M is the midpoint of AB. MN is the perpendicular to AC. Determine the ratio of the area of the grey triangle to the area of the square ABCD.

Possible answers are:
(A) 1:6
(B) 1:5
(C) 7:36
(D) 3:16
(E) 7:40



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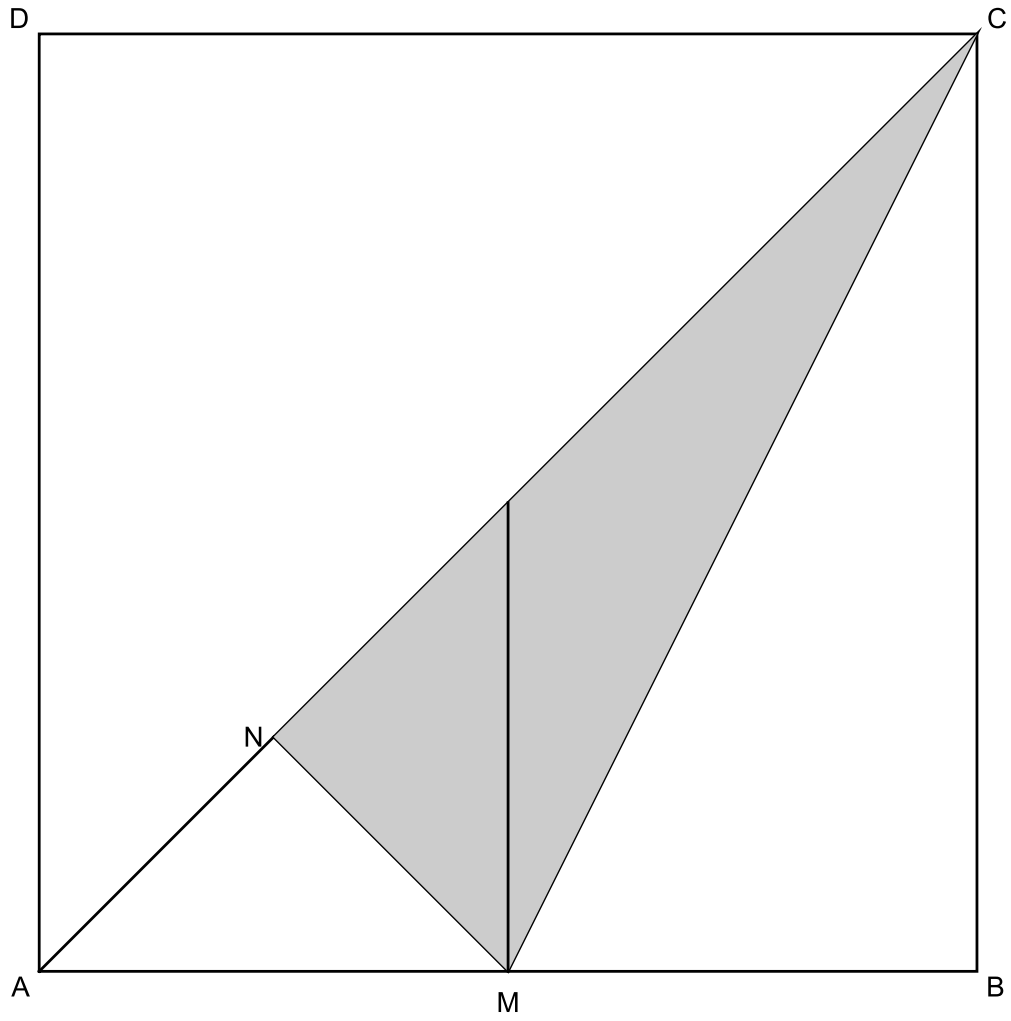
Solution: 3/16



PART 6: A CALCULUS PROBLEM

Inside the square ABCD, there is an equilateral triangle ADE. Now we let the bottom point of the equilateral triangle moving through the line AB.

What is the minimum distance of the moving point E to the fix point C?