Removing Symmetry in Circulant Graphs and Point-Block Incidence Graphs

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We consider a vertex v in a graph G *fixed* if we only consider the automorphism of G that map v to itself. The fixing number of a graph G is the minimum number of vertices that, when fixed, fixes all of the other vertices in G. Fixing numbers were introduced by Laison, Gibbons, Erwin, Harary, and Boutin. A *circulant graph* is a graph in n vertices in which the i-th vertex is adjacent to the $(i+j)$th and $(i-j)$th graph vertices for each j in a list L. We determine the fixing number for multiple classes of circulant graphs, showing in many cases that the fixing number is 2. However, we show that circulant graphs with *twins*, which are pairs of vertices with the same open neighborhoods, have higher fixing numbers. A *point-block incidence graph* is a bipartite graph $G = (P, B)$ with a set of point vertices $P = \{p_1, ..., p_r\}$ and a set of blocks $B = \{B_1, ..., B_s\}$ where $p_i \in P$ is adjacent to $B_j \in B \leftrightarrow p_i \in B_j$. We show that symmetries in certain block designs cause the fixing number to be as high as $\frac{|V(G)|}{4}$. We also present several infinite families of graphs in which fixing any one vertex in G fixes every vertex in G, thus removing all symmetries from the graph.

Keywords: fixing number; circulant graph; point-block incidence graph