New methods to attack the Buratti-Horak-Rosa conjecture

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The conjecture, still widely open, posed by Marco Buratti, Peter Horak and Alex Rosa states that a list L of $v-1$ positive integers not exceeding $\left\lfloor \frac{v}{2} \right\rfloor$ is the list of edge-lengths of a suitable Hamiltonian path of the complete graph with vertex-set $\{0, 1, \ldots, v-1\}$ if and only if, for every divisor d of v, the number of multiples of d appearing in L is at most $v-d$. In this paper we present new methods that are based on linear realizations and can be applied to prove the validity of this conjecture for a vast choice of lists. As example of their flexibility, we consider lists whose underlying set is one of the following: $\{x, y, x+y\}$, $\{1, 2, 3, 4\}$, $\{1, 2, 4, \ldots, 2x\}$, $\{1, 2, 4, \ldots, 2x, 2x+1\}$. We also consider lists with many consecutive elements.

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