On a rank-unimodality conjecture of Morier-Genoud and Ovsienko

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Let $\alpha = (a, b, \ldots)$ be a composition. Consider the associated poset $F(\alpha)$, called a fence, whose covering relations are

$$x_1 < x_2 < \ldots < x_{a+1} > x_{a+2} > \ldots > x_{a+b+1} < x_{a+b+2} < \ldots \ .$$

We study the associated distributive lattice $L(\alpha)$ consisting of all lower order ideals of $F(\alpha)$. These lattices are important in the theory of cluster algebras and their rank generating functions can be used to define q-analogues of rational numbers. In particular, we make progress on a recent conjecture of Morier-Genoud and Ovsienko that $L(\alpha)$ is rank unimodal. We show that if one of the parts of α is greater than the sum of the others, then the conjecture is true. We conjecture that $L(\alpha)$ enjoys the stronger properties of having a nested chain decomposition and having a rank sequence which is either top or bottom interlacing, the latter being a recently defined property of sequences. We verify that these properties hold for compositions with at most three parts and for what we call d-divided posets, generalizing work of Claussen and simplifying a construction of Gansner.

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