An Equivalence Class Construction of Clifford Graph Algebras

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A Clifford graph algebra $GA(G)$ is a useful structure for studying a simple graph G with n vertices. Such an algebra associates each of its n generators with one of the n vertices of G in a way that depicts the connectivity of G in that any two generators anti-commute or commute depending on whether their corresponding vertices share or do not share an edge.

In recent talks we developed these algebras for special classes of graphs; specifically path graphs, star graphs, and windmill graphs by selecting a special set of generators from a basis for a classical Clifford algebra. These constructions prompt the question as to whether or not $GA(G)$ will always exist. In this talk we will prove this conjecture by modifying an idea due to A. Macdonald and construct the Clifford graph algebra for any given simple finite graph G. In our approach each monomial in the basis for $GA(G)$ belongs to a special equivalence class of finite sequences of vectors from an orthonormal basis $\{e_1, \ ..., \ e_n\}$ for the real vector space $\mathbb{R}^n$.

Keywords : Clifford algebra, equivalence class, vector space, orthonormal basis.