Distinct Partial Sums in Cyclic Groups

Jacob Hicks (U. Georgia), Matt Ollis (Marlboro College), John R. Schmitt* (Middlebury College)

Let $(G, +)$ be an abelian group and consider a subset $A \subseteq G$ with $|A| = k$. Given an ordering $(a_1, \ldots, a_k)$ of the elements of A, define its *partial sums* by $s_0 = 0$ and $s_j = \sum_{i=1}^{j} a_i$ for $1 \leq j \leq k$. Alspach conjectured that for any cyclic group $\mathbf{Z}_n$ and any subset $A \subseteq \mathbf{Z}_n \setminus \{0\}$ with $s_k \neq 0$, it is possible to find an ordering of the elements of A such that no two of its partial sums s_i and s_j are equal for $0 \leq i < j \leq k$.

We address this conjecture (and a weakening of it due to Archdeacon) in the case that n is *prime* and do the following. We show how Noga Alon's Combinatorial Nullstellensatz can be used to frame the conjecture. Further, in the case that n is prime, we verify computationally that Alspach's Conjecture is true for small values of $|A|$. In the case that n is prime, we show that a sequence of length k having distinct partial sums exists in any subset of $\mathbf{Z}_n \setminus \{0\}$ of size at least $2k - \sqrt{8k}$ in all but at most a bounded number of cases.

Keywords: Alspach's Conjecture, Combinatorial Nullstellensatz