**Aggregate Sequences**

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In the game "Triple Town" pieces of grass are placed such that three or more pieces of grass promote to a bush, three or more bushes promote to a tree, three or more trees promote to a red house, and so on until a castle is built. The game is played on a cartesian product of two paths of length six, though we generalize the game to any graph and define a sequence of ones to be promoted to 2, 3, etc according to the recursive promotion rules of the game. There are several parameters to optimize for any given graph. The first is to find the shortest sequence (respectively longest) sequence of ones to be placed on the vertices for the game to terminate, the second is to maximize the sum of the weights of the vertices (1 for grass in the game, 2 for a bush, 3 for a tree, etc) after the game terminates.

We show that finding either the maximum or minimum total weight on any graph is an NP-hard problem, and that the decision versions are NP-complete. We define a greedy algorithm which fixes an ordering of the vertices for each 1 to be placed and show that this greedy algorithm gives an excellent bound on the minimum total weight of the vertices at the end of the game. For graphs with an efficient dominating set (a set that is both dominating and a 2-packing) the greedy algorithm finds the vertices of an efficient dominating set.

This is joint work with the late Dr. Peter J Slater during a visit to his home in Huntsville, AL. We spent the weekend intending to complete another paper, but we stopped that work and stayed up almost the entire night laying the foundation of the theory of aggregate sequences. I wish Dr. Slater would have seen the connection to his favorite topic of efficient dominating sets.

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