On the k-Steiner radius and k-Steiner Diameter of a graph with $4 \leq k \leq 5$

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Given a connected graph $G = (V, E)$ and a vertex set $S \subset V$, the *Steiner distance* $d(S)$ of S is the size of a minumum spanning tree of S in G. For a connected graph G of order n and an integer k with $2 \leq k \leq n$, the k-eccentricity of v of a vertex v in G is the maximum value of $d(S)$ over all $S \subset V$ with $|S| = k$ and $v \in S$. The minimum k-eccentricity $\mathrm{rad}_k(G)$ is called the k-radius of G and the maximum k-eccentricity $\mathrm{diam}_k(G)$ is called the k-diameter of G. In their 1990 paper "On the Steiner Radius and Steiner Diameter of a Graph," Henning, Oellermann, and Swart showed that for each $k \geq 2$, there exists a graph G_k such that $\mathrm{diam}_k(G_k) = \frac{2(k+1)}{2k-1} \mathrm{rad}_k(G_k)$. Additionally, the authors proved that for any connected graph G, $\mathrm{diam}_3(G) \leq \frac{8}{5} \mathrm{rad}_3(G)$ and $\mathrm{diam}_4(G) \leq \frac{10}{7} \mathrm{rad}_4(G)$. In this talk, a related proof that $\mathrm{diam}_4(G) \leq \frac{10}{7} \mathrm{rad}_4(G)$ is presented which can be extended to prove that $\mathrm{diam}_5(G) \leq \frac{12}{9} \mathrm{rad}_5(G)$.

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