Ore and Chvátal-type Degree Conditions for Bootstrap Percolation from Small Sets

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Bootstrap percolation is a deterministic cellular automaton in which vertices of a graph G begin in one of two states, "dormant" or "active". Given a fixed positive integer r, a dormant vertex becomes active if at any stage it has at least r active neighbors, and it remains active for the duration of the process. Given an initial set of active vertices A, we say that G r-percolates (from A) if every vertex in G becomes active after some number of steps. Let $m(G, r)$ denote the minimum size of a set A such that G r-percolates from A. Here, we are concerned with degree-based density conditions that ensure $m(G, 2) = 2$. In particular, we give an Ore-type degree sum result that states that if a graph G satisfies $\sigma_2(G) \geq n-2$, then either $m(G, 2) = 2$ or G is in one of a small number of classes of exceptional graphs. We also give a Chvátal-type degree condition: If G is a graph with degree sequence $d_1 \leq d_2 \leq \ldots \leq d_n$ such that $d_i \geq i+1$ or $d_{n-i} \geq n-i-1$ for all $1 \leq i < \frac{n}{2}$, then $m(G, 2) = 2$ or G falls into one of several specific exceptional classes of graphs. Both of these results are inspired by, and extend, an Ore-type result by Freund, Poloczek and Reichman.

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