Perfect Tournament Digraphs

David Brown*, Utah State University

The **Boolean rank** of a $(0,1)$-matrix M is the minimum number of rank-1 matrices whose sum is M using Boolean arithmetic. The **isolation number** of a matrix is the maximum number of nonzero entries no two of which belong to the same line (row or column) and no two belong to a 2×2 submatrix all of whose entries are nonzero. The isolation number of a $(0,1)$-matrix is therefore a lower bound for its Boolean rank, but both are NP-complete as decision problems. When these parameters are formulated as linear programming problems, they are dual and therefore equal. But in their combinatorial formulation they are not necessarily equal, and the difference is investigated in the case of *tournament matrices* — the adjacency matrices of complete antisymmetric digraphs. The investigation of how large the discrepancy can be uncovers a relationship in a certain graph derived form the tournament matrices; namely that this derived graph must be perfect for the parameters to be equal. This in turn motivates our definition of '*perfect*' in the case of directed graphs. Some analogies between (traditional) perfect (undirected) graphs and perfect directed graphs are developed in the case of tournament matrices.

Keywords: Boolean rank, isolation number, tournament matrix, fractional Boolean rank, biclique cover number, perfect graph.