On the Loebl-Komlós-Sós Conjecture, lopsided trees, and certain caterpillars

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Let G be a graph with at least half of the vertices having degree at least k. For a tree T with k edges, Loebl, Komlós, and Sós conjectured that G contains T. It is known that if the length of a longest path in T (i.e., the diameter of T) is at most 5, then G contains T. Since T is a bipartite graph, let ℓ be the number of vertices in the smaller (or equal) part. Clearly $1 \leq \ell \leq \frac{1}{2}(k+1)$. In our main theorem, we prove that if $1 \leq \ell \leq \frac{1}{6}k+1$, then the graph G contains T. Notice that this includes certain trees of diameter up to $\frac{1}{3}k+2$.

If a tree T consists of only a path and vertices that are connected to the path by an edge, then the tree T is a *caterpillar*. Let P be the path obtained from the caterpillar T by removing each leaf of T, where $P = a_1, \ldots, a_r$. The path P is the *spine* of the caterpillar T, and each vertex on the spine of T with degree at least 3 in T is a *joint*. It is known that the graph G contains certain caterpillars having at most two joints. If only odd-indexed vertices on the spine P are joints, then the caterpillar T is a an *odd* caterpillar. If the spine P has at most $\lceil \frac{1}{2}k \rceil$ vertices, then T is a *short* caterpillar. We prove that the graph G contains every short, odd caterpillar with k edges.

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