Distance-k Labeling of Interval and Unit Interval Graphs

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In this paper we study the distance-k labeling problem, an interesting graph theory problem motivated by the task of assigning frequencies to transmitters that interfere at many (k) levels depending on proximity. Transmitters interfering at level k must receive frequencies which are at least k integers apart. In this graph theoretic analog of the frequency assignment problem, instead of geographical distances between transmitters represented by the vertices x and y of a graph G, we consider the distance $d_G(x, y)$ between vertices x and y. If G is any graph, then in the distance-k labeling problem, [where k ≤ diameter of G] we seek to assign a label f(x), (where f(x) is either 0 or a positive integer) to every vertex x of G such that if $d_G(x, y) = k-i$, [where i= 0,1, . . ., (k-1)] , then $|f(x)-f(y)| \geq i+1$. The span $sp_k(f)$ of a distance-k labeling is the maximum {f(x): x ϵ V(G)} and the minimum span $\lambda_k(G)$ is the minimum {$sp_k(f)$: f is a distance–k labeling of G}. The goal is to find upper bounds for the number $\lambda_k(G)$ along with heuristic algorithms that achieve these bounds. Let G be any strongly chordal graph, where there exists a common perfect elimination order for all powers of G, and k ≤ diameter of G. Let ω_i be the maximum clique size in G^i, i=2, 3, . . . , k, the ith power of G, and ω be the maximum size of a clique. Then, $\lambda_k(G) \leq \omega_k + 2\omega_{k-1} + \ldots + 2\omega_2 + 2\omega - (2k-1)$. We improve this upper bound for interesting subclasses of strongly chordal graphs, particularly interval and unit interval graphs. We show that if G is an interval graph whose diameter is ≥ k, and Δ is the maximum degree of a vertex, then, $\lambda_k(G) \leq \Delta(k-1)^2 + 2\omega - (2k-3)$, and if G is unit interval, then, $\lambda_k(G) \leq k^2\omega - 2k + 1$.