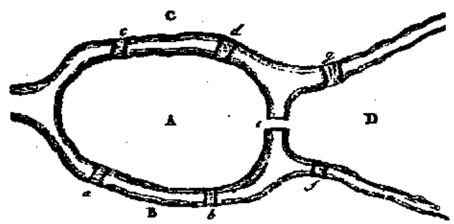
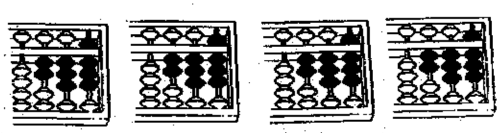
$$\frac{1}{n}\sum_{d|(b_1,\ldots,b_r)} \mu(d) \frac{\left(\frac{n}{d}\right)!}{\left(\frac{b_1}{d}\right)!\cdots\left(\frac{b_r}{d}\right)!}$$





$$\frac{1}{n}\sum_{d|(b_1,\ldots,b_r)} \mu(d) \frac{\left(\frac{n}{d}\right)!}{\left(\frac{b_1}{d}\right)!\cdots\left(\frac{b_r}{d}\right)!}$$

Thirty-Ninth Southeastern International Conference on

Combinatorics, Graph Theory & Computing

Florida Atlantic University

March 3-7, 2008

Program and Abstracts

Invited Talks

Monday, March 3, 2008
Nick Wormald
Methods and results for random regular graphs (9:30 AM)
Properties of graphs of large girth (2:00 PM)

Tuesday, March 4, 2008
TBA

Wednesday, March 5, 2008
Koen Thas
p-Modular cohomology algebras of extra-special *p*-groups, and symplectic forms (10:30 AM)
Prime power conjectures, automorphisms and buildings of rank *2* (2:00 PM)

Thursday, March 6, 2008
Vera Pless
Self-dual and Formally Self-dual Codes Part I (9:30 AM), Part II (2:00 PM)

Friday, March 7, 2008
Donald Kreher
The hypergraph degree sequence problem (9:30 AM)

Monday, March 3, 2008 (9:30 AM)

Methods and results for random regular graphs
Nick Wormald, University of Waterloo

I will discuss properties of random regular graphs, and describe some of the main methods used to derive them. Two of these in particular are the small subgraph conditioning method and the differential equation method. Both methods have been useful very recently in gaining information on the chromatic number of random regular graphs. For instance, you can be pretty sure that a large random 4-regular graph has chromatic number 3. However, the situation even for 5-regular graphs is not quite settled.

Monday, March 3, 2008 (2:00 PM)

Properties of graphs of large girth
Nick Wormald, University of Waterloo

It has long been known that there is a simple procedure for converting deterministic results about graphs of large girth, of appropriate type, into results on random regular graphs. (The girth is the length of the shortest cycle.) One aim of this talk is to show that this procedure is to some extent reversible. Another aim is to present old and new bounds on the size of the largest independent set and smallest dominating set in regular graphs of given degree and large girth. For independent sets, these results easily extend to graphs with given maximum degree and large girth. The proof of the new results involves the "probabilistic method" in an unlikely-sounding way. This achieves the reversibility mentioned above. We find typical properties of random structures in a large set C, and then show separately that the properties "transfer" to all structures in C. Naturally, such an optimistic method can only work under special circumstances. In this case, the proof is strongly related to applications of the differential equation method for random regular graphs.

Tuesday, March 4, 2008

TBA

Wednesday, March 5, 2008 (10:30 AM)

p-Modular Cohomology Algebras of Extra-Special Groups, and Symplectic Forms

Koen Thas, Ghent University, Ghent, Belgium

For a finite p-group P, let $H^*(P) = H^*(P, F_p) = \bigoplus_{i=0}^{\infty} H^i(P, F_p)$ denote the p-modular cohomology algebra of P. A theorem of J.-P. Serre states that if P is a p-group which is not elementary abelian, then there exist non-zero elements $u_1, u_2, \ldots, u_m \in H^1(P, Fp)$

such that $(*) \prod_{i=1}^{m} u_i = 0$ if $p = 2$ and $\prod_{i=1}^{m} \beta(u_i) = 0$ if $p > 2$,

where β is the Bockstein homomorphism. The smallest integer m such that relation $(*)$ is satisfied is referred to as the *cohomology length* of P, and is denoted by **chl**(P). In this talk, I will show how the combinatorics of symplectic polar spaces helps to realize the best known bounds on **chl**(P).

Wednesday, March 5, 2008 (2:00 PM)

Prime Power Conjectures, Automorphisms and Buildings of Rank *2*

Koen Thas, Ghent University, Ghent, Belgium

Except for the (dual) Hermitian buildings **H**$(4, q^2)$, up to duality, translation duality or Payne integration, every known finite building of type B_2 satisfies some seemingly very general synthetic properties, usually put together in the term "skew translation generalized quadrangle". In my talk I want to elaborate on a classification (in progress) of finite skew translation generalized quadrangles. In order to pursue this goal, I will discuss — if time permits:

- The Centrality Conjecture;
- The Payne Conjecture on the parameters of skew translation generalized quadrangles;
- Generalized quadrangles with distinct elation groups (again a problem introduced by Payne);
- Classification of the possible elation groups of skew translation generalized quadrangles.

I will emphasize the importance of certain prime power conjectures in this local theory, and beyond.

Thursday, March 6, 2008 Part I (9:30 AM), Part II (2:00 PM)

Self-dual and Formally Self-dual Codes

Vera Pless, University of Illinois at Chicago

Self-dual codes are very good codes associated to interesting designs and groups. We give a history of the classifications of self -dual codes. Formally self-dual codes are a generalization of self-dual codes. Again we find very good codes and show how to construct them.

Friday, March 7, 2008 (9:30 AM)

The hypergraph degree sequence problem

Mellisa Keranen, William Kocay, Donald L. Kreher*, Ben Li, Michigan Technological University and the University of Manitoba

A 3-hypergraph (3-HG), is a collection of triples on a set of vertices. A 3-hypergraph is simple if each triple occurs at most once. A partial Steiner triple system (PSTS) is a 3-hypergraph such that each pair of vertices is contained in at most one triple. The degree of a vertex is the number of triples containing that vertex. The degree sequence is $D = (d_1, d_2, \ldots, d_n)$, where d_i is the degree of vertex i. The degree sequence problem is: *Given an arbitrary sequence D, determine if there is a simple 3-HG or PSTS with degree sequence D.* New results on this problem are presented.

Monday, March 3, 2008

8:00am	Registration in Grand Palm Room			
9:00am	Conference Opening Session: President Brogan, Provost Pritchett, Dean Perry Presentation of Euler Medal to Nick Wormald			
9:30am	**Nick Wormald**			
10:30am	COFFEE			
	Sessions for Contributed papers in Live Oak Pavilion			
	A	B	C	D
11:00am	1	2 Light	3 Okamoto	4 Emert
11:20am	5	6 Bartha	7 P. Zhang	8 Schiermeyer
11:40am	9	10 Hicks	11 Kemnitz	12 Schmeichel
12:00pm	LUNCH (on your own)			
2:00pm	**Nick Wormald**			
3:00pm	COFFEE			
3:20pm	13	14 Khanvilkar	15 Beasley	16 Tittmann
3:40pm	17	18 Short	19 Buelow	20 Cichacz
4:00pm	21	22 Bhandari	23 Stewart	24 J. Holliday
4:20pm	25	26 Boats	27 Parkerson	28 Holt
4:40pm	29	30 J. Zhang	31 Castellana	32
5:00pm	33	34 Mihnea	35 Keranen	36 El-Hashash
5:30pm	**Reception at Baldwin House**			

Tuesday, March 4, 2008

8:00am	Registration in Grand Palm Room			
	Sessions for Contributed papers in Live Oak Pavilion			
·	A	B	C	D
9:00am	45	46 Scott	47 Seo	48 Abbott, McGuire
9:30am	**TBA**			
10:30am	COFFEE			
10:50am	49	50 Alpert, Feder	51 Bechel	52 Low
11:10am	53	54 R. Gargano	55 Shen	56 Evans
11:30am	57	58 Taksa	59 Hu	60 Killgrove
11:50am	61	62 Tran	63 Guan	64 Tapia-Recillas
12:10pm	65	66 Ginn	67 Shawash	68 Flack
12:30pm	LUNCH (on your own)			
2:00pm	**TBA**			
3:00pm	COFFEE			
3:20pm	69	70 Wooten	71 Sawada	72 A. Jamieson
3:40pm	73	74 Beeler	75 Factor	76 L. Jamieson
4:00pm	77	78 Lipták	79 Stevens	80 van der Merwe
4:20pm	81	82 Isaak	83 Ranto	84 Loizeaux
4:40pm	85	86 Georges	87 Ruj	88 Slater
5:00pm	89	90 M. Gargano	91 Yildirim	92 Roden
5:20pm	93	94 Su	95 Taylor	96 Hedetniemi
6:00pm	**Reception at the Visual Arts Patio**			

Wednesday, March 5, 2008

8:00am	Registration in Grand Palm Room			
	Sessions for Contributed papers in Live Oak Pavilion			
	A	B	C	D
8:20am	97 Fitzpatrick	98	99 Harris	100 Janoski
8:40am	101 Hartnell	102 Langley	103 Hur	104 Matheis
9:00am	105 McColm	106 Beyerl	107 Weaver	108 Grimaldi
9:20am	109 Jacobs	110 Brown	111 Laison	112 Shapiro
9:40am	113 Grolmusz	114 Ortega	115 Abay-Asmerom	116 Nkwanta
10:00am	COFFEE			
10:30am	**Koen Thas**			
11:30am	TICA Meeting and Awards Session			
12:05pm	**Conference Photo**			
12:15pm	LUNCH (on your own)			
2:00pm	**Koen Thas**			
3:00pm	COFFEE			
3:20pm	117 Widulski	118 Ferrero	119 Gao	120 Jamison
3:40pm	121 Gottlieb	122 Yerger	123 Sullivan	124 D. Lipman
4:00pm	125 Leck	126 Walsh	127 Riet	128 M. Lipman
4:20pm	129 Harborth	130 Fowler	131 Gunderson	132 McKee
4:40pm	133 Gardner, Nicolio	134 Levit	135 Ozkan	136 Lewinter
5:00pm	137 Jayawant	138 Lefmann	139 Malerba	140 Chen
5:20pm	141 Lurie	142 Shiu	143 Gera	144 Saccoman
6:30pm	**Conference Banquet at Howard Johnson's Ocean Resort**			

Thursday, March 6, 2008

8:00am	Registration in Grand Palm Room			
	Sessions for Contributed papers in Live Oak Pavilion			
	A	B	C	D
9:30am	**Vera Pless**			
10:30am	COFFEE			
10:50am	157	158	159 Glover	160 Allagan
11:10am	161	162	163 Westlund	164 Bobga
11:30am	165	166	167 Klerlein	168 Anstee
11:50am	169	170	171 Enciso	172 Finizio
12:10pm	173	174	175 Proscurowski	176 Combe
12:30pm	LUNCH (on your own)			
2:00pm	**Vera Pless**			
3:00pm	COFFEE			
3:20pm	177	178 Chopra	179 Chan	180 Yarmish
3:40pm	181	182 Delgado	183 Beavers	184 Korenblit
4:00pm	185	186 Peart	187 Tener	188 Suffel
4:20pm	189	190 Fallon	191 Balakrishan	192 Radziszowski
4:40pm	193	194	195 Ho	196 DeLaVina
5:00pm	197	198	199 Vasudevan	200 Božović
5:20pm	201	202	203 Gagliardi	204 Sehgal
5:45pm	**Informal Party at Coyote Jack's**			

Friday, March 7, 2008

8:00am	Registration in Grand Palm Room			
	Sessions for Contributed papers in Live Oak Pavilion			
	A	B	C	D
8:40am	209	210	211	212 Tiemeyer
9:00am	213	214	215	216 Lyle
9:30am	**Donald Kreher**			
10:30am	COFFEE			
10:50am	217	218 Benevides	219 Rubalcaba	220 S. Finbow
11:10am	221	222 McKeon	223 Prier	224 Wei
11:30am	225	226 Simonelli	227 Jacob	228 S. Holliday
11:50am	229	230 P. Johnson	231 A. Finbow	232 McMahon
12:10pm	233	234 Kohl	235 D. Johnson	236 Scanlon

Coffee and refreshments will be served in the registration area of the Grand Palm Room.

1)

2) Chromatic Spectrum of 2-matching Decompositions of $K_{5,n}$

Robert E. Jamison, J. Bowman Light*, Clemson University

This paper is concerned with the decomposition of the complete bipartite graph $K_{5,n}$ into 2-matchings, where a *2-matching* is a pair of disjoint edges. A *coloring* of such a decomposition assigns colors to the 2-matchings so that any two 2-matchings that share a node get different colors. The *chromatic index* of a decomposition is the minimum number of colors required to color it. This extends work of Jamison and Mendelsohn, who made a substantial investigation of 2-matching decompositions of $K_{4,n}$ and also made inroads into the general case. In this paper we will be concerned primarily with the asymptotic behavior of the chromatic spectrum.

3) A Four Colorings Theorem

Gary Chartrand, Western Michigan University, Stephen T. Hedetniemi, Clemson University, Futaba Okamoto*, University of Wisconsin - La Crosse, Ping Zhang, Western Michigan University

The best known vertex coloring of a graph is a proper coloring. Three related colorings are complete, Grundy, and irredundant colorings. Relationships among these four colorings are studied.

4) Multiple Towers of Hanoi with a Directed Cycle Transition Graph

John W. Emert*, Roger B. Nelson, Frank W. Owens*, Ball State University

The multiple towers of Hanoi puzzle with a directed cycle transition graph is a variation of the classic towers of Hanoi puzzle with three posts to a puzzle with p posts, where $p > 2$. Number the posts so that $0,1,2,. . . ,p - 1, 0$ is a directed cycle called the transition graph of the puzzle. Designate one post to be the source post S and another post to be the destination post D. The remaining posts are called temporary posts. As usual, there are n disks no two of which have the same diameter. Number the disks $1, . . . , n$ in order of increasing diameter. Each disk has a hole in the center so that it will fit over a post. At no time may a disk be placed on top of a disk having a smaller diameter. Thus, there are p^n legal configuration states of the n disks on the p posts. Initially, all n disks are on the source post S. A disk may be moved from the top of post i to the top of post j if and only if the move will not result in the disk being placed on top of a disk having a smaller diameter and post i is the predecessor of post j in the transition graph. The problem is to determine the minimum number of moves required to transfer all the disks to the destination post D.

Monday, March 3, 2008, 11:20 AM

5)

6) Deciding the flexibility of matchings in open graphs

Miklos Bartha, Memorial University of Newfoundland

Vertices in an open graph *G* are grouped into two disjoint subsets: primary vertices and secondary ones. From the matching point of view, primary vertices are expected to be covered by all "good" matchings of G, whereas the status of each secondary vertex being covered or not by any matching is irrelevant. A perfect (maximum) primary matching is one that covers all (respectively, a maximum number of) primary vertices. Such a matching *M* is called flexible if there exists another matching $M' \neq M$ covering the same vertices as *M*. We investigate the problem of deciding if an open graph *G* has a flexible perfect primary matching. If *G* is closed, that is, all vertices of *G* are primary, then the fastest solution to this problem is the unique perfect matching algorithm by Gabow, Kaplan, and Tarjan, which works in $O\ (m\log^4 n)$ time for a graph with *n* vertices and *m* edges. We generalize this algorithm to solve our problem for open graphs, preserving the $O\ (m\log^4 n)$ time complexity. We conjecture that the problem is solvable in $O\ (m)$ time, although we have a working linear-time algorithm for elementary graphs only.

7) Hamiltonian Colorings of Graphs

Gary Chartrand, Western Michigan University, Ladislav Nebeský, Charles University, Czech Republic, and Ping Zhang*, Western Michigan University

For vertices *u* and *v* in a connected graph *G* of order *n*, the length of a longest u–v path in *G* is denoted by $D(u, v)$. A hamiltonian coloring *c* of *G* is an assignment *c* of colors (positive integers) to the vertices of *G* such that $D(u, v)+|c(u)-c(v)| \geq n-1$ for every two distinct vertices *u* and *v* of *G*. We present some results on Hamiltonian colorings.

8) Closures, cycles and paths

Arnfried Kemnitz, Jochen Harant, Akira Saito, Ingo Schiermeyer*, Technical University Freiberg

In 1960 Ore proved the following theorem: Let *G* be a graph of order *n*. If $d(u) + d(v) \geq n$ for every pair of nonadjacent vertices *u* and *v*, then *G* is hamiltonian. Since then for several other graph properties similar sufficient degree conditions have been obtained, so called "Ore-type degree conditions". In 2000 Faudree, Saito, Schelp and Schiermeyer strengthened Ore's theorem as follows: They determined the maximum number of pairs of nonadjacent vertices that can have degree sum less than *n* (i.e. violate Ore's condition) but still imply that the graph is hamiltonian. In this talk we will show that for some other graph properties the corresponding Ore-type degree conditions can be strengthened as well. These graph properties include traceable graphs, hamiltonian connected graphs, k-leaf connected graphs, pancyclic graphs and graphs having a 2-factor with two components. Graph closures are computed to show these results.

9)

10) Decompositions of Prisms into Matchings

Brandy Hicks*, Robert E. Jamison, Clemson University

If G is any graph, a *G-decomposition* of a host graph $H = (V, E)$ is a partition of the edge set of H into subgraphs of H which are isomorphic to G. The *chromatic index* of a G-decomposition is the minimum number of colors required to color the parts of the decomposition so that two parts which share a node get different colors. The *G-spectrum* of H is the set of all chromatic indices taken on by G-decompositions of H. In this talk we will study the case that the prototype G is a matching and the host H is a prism. In particular, we will be concerned with *complete decompositions* — that is, decompositions in which every two parts share a common node.

11) Circular Total Colorings of Graphs

Arnfried Kemnitz*, Jens-Peter Bode, Andrea Hackmann, Rebecca Klages, Techn. Univ. Braunschweig, Germany

Given positive integers k and d with $k \geq 2d$, a (k, d)-total coloring of a simple and finite graph G is an assignment c of colors $\{0, 1, \ldots, k-1\}$ to the vertices and edges of G such that $d \leq |c(x) - c(x')| \leq k - d$ whenever x and x' are two adjacent edges, two adjacent vertices or an edge incident to a vertex. The circular total chromatic number $\chi_c''(G)$ is defined by $\chi_c''(G) = \inf\{k/d : G$ has a (k, d)-total coloring$\}$. Obviously, a $(k, 1)$-total coloring is just an ordinary total coloring of the graph implying $\chi_c''(G) \leq \chi''(G)$ for every graph where $\chi''(G)$ is the total chromatic number of G. Equality holds for all type-1 graphs. We present some basic properties of $\chi_c''(G)$ and determine exact values for different classes of type-2 graphs. Moreover, we determine infinite classes of graphs G such that $\chi_c''(G) < \chi''(G)$.

12) Strongest Monotone Degree Conditions

Doug Bauer, Stevens Institute of Technology, Nathan Kahl, Seton Hall University, Ed Schmeichel*, San Jose State University

After several authors found degree conditions for a graph to be hamiltonian, Chvátal found a strongest monotone degree condition for a degree sequence to be forcibly hamiltonian graphical. Later, Bondy and Boesch found a similar degree condition for a degree sequence to be forcibly k-connected graphical. We describe a framework in which these degree conditions are seen to be strongest monotone, and then find similar conditions for a degree sequence to be forcibly 2-edge-connected graphical. We also find degree conditions for a degree sequence to be forcibly k-factor graphical for $k = 1, 2$. We then show how this framework can be used to estimate various graph parameters. In particular, we show that a recent lower bound of Murphy for the independence number of a graph G, $\alpha(G)$, is in fact a strongest monotone lower bound for $\alpha(G)$. We also discuss bounds on the clique number and the chromatic number of a graph.

13)

14) Dynamic routing for a static physical network through embedding

Mrinal Khanvilkar*, Dionysios Kountanis, Ala Al-Fuqaha, Western Michigan University

An embedding of the Maekawa network to a physical network with static routing obtains an efficient dynamic routing for the physical network. The Maekawa network is obtained from the Maekawa sets which provide balance and fairness on the workload of the nodes. The routing paths on a Maekawa network are dynamically and distributively computed. The embedding transfers these properties to the physical network. The objective of the mapping is to minimize the average length (in terms of 'n' number hops) of the virtual paths created by the embedding.

15) Extending Partial Tournaments

LeRoy B. Beasley*, David E. Brown, Utah State University, K. Brooks Reid, California State University San Marcos

Let A be a $(0, 1, *)$-matrix with main diagonal all 0's and such that if $a_{i,j} = 1$ then $a_{j,i} = 0$ or $*$. Under what conditions on the row sums of A is it possible to change the $*$'s to 0's or 1's and obtain a tournament matrix (the adjacency matrix of a tournament digraph) with a specified score sequence. We answer this question, and the result is best possible in the sense that no relaxation of any condition will always yield a matrix that can be so extended.

16) Counting Connected Vertex Partitions

Peter Tittmann, University Mittweida, Germany

Let $G = (V,E)$ be a finite undirected graph. A partition π of the vertex set V is called *connected* if each block of π induces a connected subgraph of G. Let $q_i\ (G)$ be the number of connected partitions of G with exactly i blocks. The *partition polynomial* $Q(G, x)$ is the ordinary generating function for the numbers q_i. We show that the polynomial $Q(G, x)$ satisfies nice decomposition and splitting formulae. In order to obtain a polynomial which is more appropriate for applications in social network analysis and reliability theory, we refine the enumeration of vertex partitions as follows. Let $q_{ik}\ (G)$ be the number of vertex subsets of cardinality k that induces a subgraph of G with exactly i components. The corresponding generating function is a polynomial $Q(G; x, y)$ in two variables. The computation of $Q(G; x, y)$ can be performed in polynomial time for graphs of bounded treewidth, whereas the computation of $Q(G; x, y)$ is an #**P**-complete problem in general graphs.

17)

18) A Constrained Minimum Cost Cut-Set Problem

Ramesh Bhandari, Daniel Short*, Laboratory for Telecommunication Sciences, DoD

Given an undirected, unweighted graph, and a pair of vertices, s and t, connected by a path, and a third vertex v not lying on this path, what is the minimum set of edges to cut in the graph so that the path connecting s and t necessarily passes through vertex v? This is an instance of a constrained minimum cost cut-set problem, which is hard to solve but often occurs in the analysis of network flows within the telecommunication world. In this paper, we describe two novel techniques, called the *Path Revival* and *Graph Collapse*, and show how they are combined to solve this problem. The developed algorithm is also shown to be extensible to the case where the shortest path between the given pair of vertices s and t is constrained to pass over a given edge, instead of a given vertex, a scenario that can also occur in telecommunication networks.

19) Local Out-Tournaments with Upset Tournament Strong Components: Real and Nonnegative Integer Ranks of Adjacency Matrices

Zac Buelow*, Kim A. S. Factor and Alicia Crow, Marquette University

A local out-tournament is a digraph in which the outset of every vertex is a tournament. Here, local out-tournaments with upset tournament strong components that are examined. The nonnegative and real ranks of the adjacency matrices of these digraphs depend on the ranks of the components, considered individually, and the connections between the components. For an upset tournament, these two ranks are equal. We look to find the cases when these two ranks are equal for local out-tournaments with upset tournament strong components. We use the fact that nonnegative integer rank of the adjacency matrix is equal to the biclique partition number for any upset tournament. We consider the cases in which the ranks are a maximum value and those in which a minimum value is attained.

20) 2-splittable and cordial graphs

Sylwia Cichacz, AGH U of S and T and University of Minnesota Duluth

E. Miller and G.E. Stevens proved existence of certain families of 2-splittable caterpillars. In this paper we characterize other families of 2-splittable caterpillars. Moreover, we show that for some of them there exists a friendly labeling inducing two isomorphic subgraphs.

21)

22) The Sliding Shortest Path Algorithm with Finite Weight Changes

Ramesh Bhandari, Laboratory for Telecommunication Sciences, DoD

Given an undirected weighted graph, and a pair of vertices, s and t, connected by the shortest path, and a third vertex v not lying on the shortest path, what is the minimal change in the graph weights needed to cause the shortest path between s and t to pass through vertex v? This is the type of a problem often faced by network administrators in the telecommunication world. In this paper, we provide an extension of an existing algorithm called the *Sliding Shortest Path* algorithm to solve this problem; the approach taken is one of the replacement of a set of edges of minimum cardinality with new optimal finite weights, instead of "infinities"; the new weights are derived from a simple, analytic formula. The algorithm is also shown to be extensible to the case where the shortest path between a pair of vertices is constrained to pass over a given edge, instead of a given vertex, a scenario that can also occur in telecommunication networks.

23) Concerning Bicyclic Antiautomorphisms of Mendelsohn Triple Systems

N. Carnes, A. Dye, S. Parkerson, K. Stewart*, McNeese State University

A cyclic triple, (a, b, c), is defined to be the set $\{(a, b), (b, c), (c, a)\}$ of ordered pairs. A Mendelsohn triple system of order v, denoted MTS(v), is a pair (M, β), where M is a set of v points and β is a collection of cyclic triples of pairwise distinct points of M such that any ordered pair of distinct points of M is contained in exactly one cyclic triple of β. An automorphism is a permutation of M which maps β to itself, and an antiautomorphism is a permutation of M which maps β to β^{-1}, where $\beta^{-1} = \{(c, b, a) \mid (a, b, c) \in \beta\}$. We discuss the necessary and sufficient conditions for the existence of a Mendelsohn triple of order v with an antiautomorphism having two cycles of lengths M and N, $1 < M \leq N$.

24) Cutting Numbers of Cycles and Edge Cut Cycles

Brad Bailey, John Holliday*, Dianna Spence, North Georgia College & State University

Let C be a cycle in a connected graph G. We define the cutting number of C to be the number of components in G without the edges of C. Whenever a cycle has cutting number at least 2, we call such a cycle an edge-cut cycle. In this talk, we discuss the motivation for these definitions , characterization of graphs with edge-cut cycles, the necessary and sufficient conditions for a cycle to be an edge-cut cycle with respect to it's graph G as well as explore the relationship between |V(G)| and the cutting number of G.

25)

26) Finding Disjoint Paths in the General Nova Graph

Jeffery Boats*, Lazaros Kikas, John Oleksik, University of Detroit Mercy

For the purpose of large scale computing, we are interested in linking computers into large interconnection networks. Suppose we have k pairs of vertices, $(s_1,t_1),(s_2,t_2),\ldots,(s_k,t_k)$, and wish to find k disjoint paths; each path connecting exactly one pair. If in a graph G we can do this for any k pairs of vertices then we say that G has the "k-disjoint path property." In 2006, Boats, Kikas, and Oleksik introduced the Nova graph, A_4^+, as a new interconnection network. The significance of the Nova graph is that it has the 3-disjoint path property using far fewer edges and vertices than any other graph in the symmetry group family. It is formed by adding only 6 edges to the alternating group graph AG_4, yet this minimal augmentation increases the guaranteed number of disjoint paths from one to three. In this paper, we extend our results to the general Nova graph, A_n^+, by showing it has the (n-1)-Disjoint Path Property. We discuss our proof and we conclude with future research directions.

27) Concerning sufficient conditions for 1,M,N-antiautomorphisms of Directed Triple Systems

N. Carnes, A. Dye, S. Parkerson*, K. Stewart, McNeese State University

A transitive triple (a,b,c) is defined to be the set {(a,b),(b,c),(a,c)} of ordered pairs. A directed triple system of order v, DTS(v), is a pair (D, β), where D is a set of v points and β is a collection of transitive triples of pairwise distinct points of D, called, triples, such that any ordered pair of distinct points of D is contained in precisely one element of β. A permutation of D that maps β to itself is called an automorphism of (D, β). An antiautomorphism of (D, β) is a permutation of D that maps β to β^{-1}, where β^{-1} is defined by $\beta^{-1} = \{(c, b, a) \mid (a, b, c) \in \beta\}$. An antiautomorphism, α, on a DTS(v) is called 1-bicyclic if the permutation consists of two cycles and one fixed point. Considered in this talk are sufficient conditions for the existence of a DTS(v) admitting an antiautomorphism with cycles of lengths M and N, where $1 < M, N = kM$, and a fixed point.

28) Decompositions of the Complete Symmetric Digraph into Orientations of the 4-Cycle with a Pendant Edge

Robert Gardner, Tracy Holt*, East Tennessee State University

There are twenty orientations of the 4-cycle with a pendant edge. We give necessary and sufficient conditions for the decomposition of the complete symmetric directed graph on v vertices into each of these digraphs.

29)

30) Resource Conservation Cluster Routing in Wireless Mobile Ad Hoc Networks

Jing Zhang*, Dionysios Kountanis, Ala Al-Fuqaha, Western Michigan University

N nodes are randomly distributed in an area S. A mobility function is associated with each node. The nodes also have limited resources and they can transmit and receive signals within a range of radius r. The objective is to establish a communication network with minimum use of resources. We use clustering of the nodes with a cluster head chosen from each cluster to form a backbone network. The cluster heads consume more resources than the ordinary nodes. We have developed strategies for the formation of clusters, the selection of cluster heads and a protocol for the network management in order to minimize the use of resources. Subsequently the life of the network is prolonged.

31) On the Spectrum of Minimal Covers By Triples

Vincent Castellana*, Eastern Kentucky University, Dean Hoffman, Auburn University

A Minimal Cover by Triples is an ordered pair (V, T) where V is a finite set and T is a collection of three element subsets of V with the properties that every pair of elements of V appear together in at least one element of T and if any element of T is removed, the first property no longer holds. In this talk we discuss the possible values $|T|$ can take on for a given $|V|$. In addition, some construction techniques will be demonstrated for to constructing a Minimal Cover by Triples for given values of $|V|$ and $|T|$.

32)

33)

34) Clustering Protocols and Algorithms

Piotr Blass, Amy Mihnea*, Florida Atlantic University

In this paper we present some methods of clustering that use reduction of the data to lower dimensions, which simplifies the data and therefore the complexity of the problem. We describe the algorithms involved and give some examples.

35) Small Group Divisible Steiner Quadruple Systems

Melissa Keranen*, Donald Kreher, Artem Zhuravlev, Michigan Technological University

A *group divisible Steiner quadruple system*, is a triple (X, H, B) where X is a v-element set of points, $H = \{H_1, H_2, \ldots, H_r\}$ is a partition of X into *holes* and B is a collection of 4-element subsets of X called *blocks* such that every 3-element subset is either in a block or a hole but not both. We investigate the existence and non-existence of these designs. We settle all parameter situations on at most 24 points, with 6 exceptions. A *uniform* group divisible Steiner quadruple system is a system in which all the holes have equal size. These were called G-*designs* by Mills, and their existence is completely settled.

36) On the Hamiltonicity of the Permutahedron

Mahmoud El-Hashash*, Heidi Burgiel, Bridgewater State College, Bridgewater, MA

A Hamiltonian cycle in a graph G is a cycle that contains each vertex of G exactly once, except for the starting and ending vertex that appears twice. The permutahedron $\pi_{n-1} \subseteq \mathrm{R}^n$ is defined as the convex hull of all vertices obtained by permuting the coordinates of the vector $\langle x_1, x_2, .., x_n \rangle$. Its vertices can be identified with the permutations in S_n in such a way that two vertices are connected by an edge if and only if the corresponding permutations differ by an adjacent transposition (the permutation that maps $x_i \mapsto i$ corresponds to the vertex $\langle x_1, x_2, .., x_n \rangle$). We study Hamiltonian cycles on π_n by using a Hamiltonian cycle of π_{n-1} to create a Hamiltonian cycle on π_n. This method yields Hamiltonian cycles in π_n for some small values of n. In addition, by using a Java program we can find Hamiltonian cycles in π_n for large values of n.

45)

46) Happiness Levels in Random Matchings

Hamilton Scott*, Jennifer Woodell, Anant Godbole, East Tennessee State University

Given two Latin squares M and W with entries in $\{1, 2, \ldots, n\}$ that represent the preferences of partners for n men and n women respectively and a random *permutation* $\pi \in S_n$ which represents the matching *actually* assigned by a matchmaker, let the happiness level H be defined as the sum $\sum_{i=1}^{n} M_{i,\pi(i)} + W_{i,\pi(i)}$. We study the behavior of the random quantity H, focusing on feasible values, symmetry of the distribution, and attainable structures.

47) Open neighborhood locating-dominating sets

Suk Jai Seo*, Middle Tennessee State University and Peter J. Slater, University of Alabama in Huntsville

For a graph G that models a facility, various detection devices can be placed at the vertices so as to identify the location of an intruder such as a thief or saboteur. Here we introduce the open neighborhood locating-dominating set problem. We seek a minimum cardinality vertex set S with the property that for each vertex v its open neighborhood $N(v)$ has a unique non-empty intersection with S. Such a set is an *OLD*(G)*-set*. Among other things, we describe minimum density *OLD-sets* for various (infinite) grid graphs.

48) Using Graphs and Games to generate Cap Set Bounds

Josh Abbott*, Trevor McGuire*, New College of Florida

A cap set is defined as a subset of $\mathbb{F}_x^n$, a finite field of order x^n, which contains no lines of length x. If we look at $\mathbb{F}_3^n$, we can interpret the problem of finding maximal sizes of cap sets as finding the largest number of cards from the game *Set* that one can put in play for which no "set" exists. For the bulk of this talk, we will develop a generalized game of *Set* on graphs and illustrate what it means to have no "set" exist in a graph. We show the benefits of interpreting the maximal cap set size problem through *Set* on graphs by generating bounds for these maximal cap set sizes.

49)

50) The Maximum of the Maximum Rectilinear Crossing Numbers of d-regular Graphs of Order n

Matthew Alpert*, Lawrence High School, Elie Feder*, Kingsborough Community College-CUNY, and Heiko Harborth, Technische Universitt Braunschweig

In this research we investigate the maximum of the maximum rectilinear crossing numbers for graphs in the class of d-regular graphs of order n. We present the generalized star drawing of the d-regular graph of order n where $n + d \equiv 1 \bmod 2$ and prove that it maximizes the rectilinear crossing number. We introduce the *star-like* drawing for $n \equiv d \equiv 0 \bmod 2$ and we conjecture that this drawing maximizes the rectilinear crossing number. In the direction of this conjecture, we offer a simpler proof of two results initially proven by Furry and Kleitman for 2-regular graphs. Additionally, we show that this research generalizes known results regarding the maximum rectlinear crossing number of the cycle graph (C_n) and the complete graph (K_n).

51) Universal Cycles of Functions

Ashley Bechel*, Britni LaBounty-Lay, East Tennessee State University

A connected digraph in which the indegree of any vertex equals its outdegree is Eulerian; this baseline result is used as the basis of existence proofs for *universal cycles* of several combinatorial objects. Recall that a universal (or U-, or DeBruijn) cycle is an optimal cyclical representation of a combinatorial object with no repetitions. For example, the sequence 11100010 is a U-cycle of 3-letter words on a binary alphabet. We present new results on existence of universal cycles of certain classes of functions. In each case the connectedness of the graph is the non-trivial aspect to be established.

52) The Group-magic Labeling Problem: Theorems, Counter-examples, and Open Problems

Richard M. Low*, San Jose State University; W.C. Shiu, Hong Kong Baptist University

Let $G = (V, E)$ be a connected simple graph. For any non-trivial abelian group A (written additively), let $A^* = A - \{0\}$. A function $f: E(G) \to A^*$ is called a *labeling* of G. Any such labeling induces a map $f^+ : V(G) \to A$, defined by $f^+(v) = \sum f(u, v)$, where the sum is over all $(u, v) \in E(G)$. If there exists a labeling f whose induced map on $V(G)$ is a constant map, we say that f is an *A-magic labeling* of G and that G is an *A-magic* graph. For the group-magic labeling problem, we present some interesting theorems, some non-intuitive examples and counter-examples, and some open questions.

53)

54) Optimal Eulerian Cycles Using Immune System Genetic Algorithms

Robert M. Gargano*, Roberts Wesleyan, Michael L. Gargano, Edgar G. DuCasse, Louis V. Quintas, Pace University

Given an eulerian graph G and a starting vertex assume each edge is associated with traversal times that depend on the time when that edge begins to be traversed. The problem is to find an eulerian cycle with minimum traversal time. An immune system genetic algorithm (ISGA) solution is explored.

55) An Explicit Formula of the Suarface Area for the Star Graph and Its Correctness Proof

ZhiZhang Shen*, Plymouth State University, U.S.A., Ke Qiu, Brock University, Canada

Given a graph $G = (V, E)$ and a node u, a question one may ask is how many nodes are at distance d from u in G. This quantity is known as the surface area of G with radius d, or the Whitney numbers of the second kind for the poset defined by G. The study of the surface area has applications in data communications. A star graph is a Cayley graph over S_n with the generator set $\{(1, j) \mid 2 \le j \le n\}$. This graph has been proposed to be used for interconnecting processors in a parallel computer. We give an explicit formula for this number for the star graph and its correctness proof.

56) Orthogonal latin squares based on nonabelian groups

Anthony B. Evans, Wright State University

The multiplication table of a finite group G is a latin square called the *Cayley table* of G. An old question is, For which groups G does the Cayley table of G have an orthogonal mate? The answer to this question was conjectured by M. Hall and L. J. Paige in 1955: this conjecture was settled recently by J. N. Bray, A. B. Evans, and S. Wilcox. We will discuss this proof and the related question, How large a set of mutually orthogonal latin squares can we obtain by permuting the columns of the Cayley table of a group? Very little work has been done on this question for nonabelian groups. We will list the few known results.

57)

58) Improving efficiency of a query formulation method

Isak Taksa, Baruch College, CUNY

Multiple query formulations (MQF) have been shown to considerably improve the quality of information retrieval results. However, many proposed schemes ignore the original query's length and its semantic content, and therefore, cause computational costs associated with these schemes to be prohibitively high. To improve the efficiency of the MQF process, each original (unlimited) size query Q is reduced via a semantic reduction algorithm to n significant terms. The terms are then stored in a n-vector T in reverse order of weight (weight of each term is equal to the number of documents in the collection that contain the given term). A sequence of sub-queries ${}_nQ_k = \{q_1,\ldots,q_m\}$ is formed by choosing k terms from the n significant terms of the document stored in T; clearly, $m = C_k^n$ and each q_j consists of a list of k terms. The first goal is to find reasonable bounds for k, $(l \le k \le h)$ to lessen the number of queries. Even $\left|{}_nQ_k\right| = \sum_{k=l}^{h} C_k^n = \sum_{k=l}^{h} \frac{n!}{(n-k)!k!}$ could be quite large and time consuming to execute. To minimize the number of sub-queries q_j not returning relevant results, we calculate the Scaled Cumulative Query Weight (τ) for each sub-query τ (q_j) before submitting it to a search engine. The second goal of this research is to establish bounds L and H such that a sub-query that does not fit the criteria $L \le \tau(q_j) \le H$ can be automatically discarded.

59) Connectivity Properties of Directed Hyperstars

Eddie Cheng, Oakland University, Philip Hu*, Roger Jia, Troy High School

Star graphs were introduced as a competitive model to the n-cubes. Recently, hyper-stars were introduced to be a competitive model to both n-cubes and star graphs. Indeed, a hyper-star can be viewed as a hybrid of an n-cube and a star graph. In this talk, we present a result on the connectivity of the directed version of hyper-stars.

60) Latin Squares of Orders 5, 8 & More

R. Sternfeld, C. Roberts, ISU, D. Koster, UWLC, L. Taylor, CSUB, R. Killgrove*, Ret.

Given an equivalence relation on LS (Latin squares), an *invariant* separates members of different classes. A complete invariant guarantees those with the same invariant are in the same class. John Brown's signature, which describes permutations between rows of LS concerned isotopic classes, J of Comb 5 no 2 (1968) pp. 177–184; by including conjugates this extends to main classes, ours Proc 8th SE conference pp. 433–452. Also for main classes, local motions, a directed graph, ours Cong Numer 55 (1986) pp. 221–233. Then ALMIT (Abrev. Loc. Mot. Inv. w Transversals) simplified a computation, ours Cong Numer (2005) pp. 69–88. To complete any of these often requires additional work. For order 8 ALMIT is complete for all but 49 pairs of main classes of 283657 such. John Brown's extended signature separates 47 pairs and computing the cycle lengths of the directed graphs separates the other two. Flint stone sequences, a, ba, aba, etc. are part of an invariant for isomorphism classes of LS of order 5.

61)

62) Network Security Risk: an approach to Computing Model

Hoa Tran, Fordham University

As the tool to predict the collapse in terms of finance of a company, the probability of ruin plays a crucial role. The interest rate, initial compounding assets, together with ruin time, ruin function will be discussed for the new directions of observing the chance of being collapsed of the company. Random walk, Brownian motion and the connection with Network Security Risk also will be addressed. The models are applied for modeling the Network Security structure to prevent intrusions.

63) On critical square-free subgraphs of hypercubes

Sul-young Choi, Le Moyne College, Puhua Guan*, University of Puerto Rico

A critical square-free subgraph is a subgraph without any four-cycles, however, adding any edge to this subgraph will result in a four cycle. Erdős conjectured that the number of edges of the largest square-free subgraph of an n-dimensional hypercube is bounded above by $((1/2) + \varepsilon)|E|$ for large n, where E is the number of edges in the hypercube and ε is a small positive number. In this talk we present that the number of edges of the smallest square-free subgraph of an n-dimensional hypercube is bounded above by $((1/4) + \varepsilon)|E|$ for large n.

64) Almost-Bent Functions and Secret Sharing Schemes

H. Tapia-Recillas*, J.C. Ku, Dept. de Matemáticas, UAM-I, México

A class of linear codes over the finite field F_{2^h} ($h \geq 1$, an integer) based on almost-bent functions is introduced, the length, dimension and bounds (upper/lower) of the weight of the nonzero codewords of these codes are determined. A secret sharing scheme based on these codes whose secret space is the field F_{2^h} is given.

65)

66) Differentials on Paths and Grids
Mark Ginn, Appalachian State University

Several years ago, Slater, Goddard and Henning defined the differential of a graph $G = (V,E)$, $\partial(G)$ to be the $max_{X \subseteq V}\ |bd(X)| - |X|$. This parameter was explored by Lewis in 2004. We extend his work looking at the differential of grids.

67) The (*n*, *k*)-Bubble Sort Graphs
Eddie Cheng, Nart Shawash*, Oakland University

One of the most popular interconnection networks is the star graph, S_n. It was introduced as a competitive model to the hypercube. However one common complaint is the restriction on the number of vertices, $n!$ vertices for S_n. So one may face the choice of either too few or too many available vertices. There are good graph topologies that generalize the star graphs and address this issue such as the (n, k)-star graph and the arrangement graph. The star graph is a member of a class of interconnection networks that are generated by transpositions. Another famous member of this class of graphs is the bubble-sort graph B_n. Like the star graphs, the bubble sort graphs suffer the same shortcoming regarding the gaps on the number of vertices in the available graphs. In this paper, we introduce the (n, k)-bubble sort graphs to address this issue for B_n.

68) Generation of Good Edit Codes from Classical Hamming Distance Codes
R. Flack*, S. Houghten, Brock University, Canada

The edit distance, also known as Levenshtein distance, between two words is the minimum number of substitutions, insertions and/or deletions required to change one word into another. An $(n, M, d)_q$ edit code is a q-ary code with minimum edit distance d and in which the longest codeword has length n. A code is optimal if it has the maximum number of codewords for any code with a given maximum length and minimum distance. We explore the idea of using families of "good" Hamming distance codes as a starting point for construction of edit distance codes. For some small parameter sets these can produce provably optimal edit codes. For larger parameter sets where a brute force approach is infeasible, we use the Hamming distance codes as an initial population for a Genetic Algorithm. While these techniques cannot guarantee an optimal code, they do in some cases improve upon the previously known lower bounds.

69)

70) Edge-Induced and Vertex-Induced Cycles Within Circulant Graphs

Robert A. Beeler, Trina M. Wooten*, East Tennessee State University

In recent work by Beeler and Jamison, the problem of finding triangle-free subgraphs becomes increasingly important. As many of their examples involve circulant graphs, we are most interested in finding triangle-free subgraphs within circulants. In this talk, we will state necessary and sufficient conditions for the existence of edge-induced and vertex-induced cycles within circulants.

71) A Gray Code for Reflectible Languages

Joe Sawada*, Roy Li, University of Guelph

We classify a type of language called a reflectible language. We then develop a generic algorithm that can be used to list all strings of length *n* for any reflectible language in Gray code order. The algorithm generalizes Gray code algorithms developed independently for *k*-ary strings, restricted growth strings, and *k*-ary trees, as each of these objects can be represented by a reflectible language. Finally, we apply the algorithm to open meanderic systems which can also be represented by a reflectible language.

72) Some Graph Classes and the Wimer Edge Variant

Alan C. Jamieson, St. Mary's College of Maryland

Inspired by work of Thomas V. Wimer and Bern, Lawler and Wong, the Wimer Edge Variant is a methodology designed to help develop linear-time algorithms for solving edge subset problems. Examples of problems solved utilizing the Wimer Edge Variant include determining the minimum cardinality of a maximal induced matching on a tree and the minimum ev-Domination number on a tree. As we further develop the variant, it is important to look at a variety of classes of graphs that the Wimer method has been traditionally able to be applied to. Here we consider three classes of graphs: grids of fixed dimension, unicyclic graphs and generalized series-parallel graphs and the suitability of the Wimer Edge Variant for edge subset problems on those classes of graphs. We also provide some simple algorithms created utilizing the Wimer Edge Variant as an example of the method in use on each of these classes of graphs.

73)

74) Open Labellings on Graphs

Robert A. Beeler*, Trina M. Wooten, East Tennessee State University

A Z_n-labelling on a graph $G = (V,E)$ is an injective function f from the vertex set of G to the elements of Z_n. These labels induce labels on the edges of both G and the complement of G as follows:

$f^*(E) = \{|f(x) - f(y)|n : x, y \in V(G), xy \in E\}$

and

$f^*(\overline{E}) = \{|f(x) - f(y)|n : x, y \in V(G), xy \in \overline{E}\}$.

For $S \subseteq \{1, 2, ..., n/2\}$, we say that a Z_n -labelling is *open* with respect to S if $f^*(\overline{E}) \cap S = \varnothing$. In this talk, we will examine the implications of open labellings with regards to the problem of finding vertex-induced subgraphs. We will also discuss the connection between open labellings and other labellings on graphs.

75) Digraphs with Isomorphic Underlying and Domination Graphs: Pairs of Paths

Kim A. S. Factor*, Marquette University, Larry J. Langley, University of the Pacific

A domination graph of a digraph D, $dom(D)$, is created using the vertex set of D and edge $uv \in E(dom(D))$ whenever $(u,z) \in A(D)$ or $(v,z) \in A(D)$ for any other vertex $z \in V(D)$. Here, we consider directed graphs whose underlying graphs are isomorphic to their domination graphs. Specifically, digraphs are completely characterized where $UG^c(D)$ is the union of two disjoint paths.

76) Alliances in Generalized Series Parallel Graphs

Lindsay H. Jamieson, St. Mary's College of Maryland

In general, the extension of Wimer-style algorithms to series parallel graphs does not change the linear run time of the algorithm. However, finding a minimum defensive alliance on a series parallel graph using a Wimer-style algorithm cannot be done in linear time because of an additional look-up table necessary for alliances in series parallel graphs. The lookup tables involved in the algorithms for finding alliances in series parallel graphs contain information regarding the balances between neighbors in the set and neighbors not in the set for each terminal and how the combination of those sets will affect the rest of the alliance. With trees, the information that would have been stored in these tables was able to be stored within the composition table for the classes. With series parallel graphs, all possible combinations of neighbor counts must be stored, which cannot be easily stored within the composition tables for the classes. We will discuss the algorithms for minimum defensive alliances and minimum global defensive alliances in generalized series parallel graphs. A defensive alliance is a non empty set of vertices $S \subseteq V$ such that $\forall v \in S$, $|N[v] \cap S| \geq |N(v) - S|$ (Kristiansen, et.al, 2002). A global defensive alliance is a defensive alliance for which $\delta(S) = V\text{-}S$.

77)

78) A Tree Labelling Problem

Eddie Cheng, Lászlo Lipták*, Oakland University, Lih-Hsing Hsu, Providence University, Cheng-Kuan Lin, Jimmy J. M. Tan, National Chiao Tung University

In this talk, we consider two tree labelling problems. Let T be a tree on $n \geq 3$ vertices. The first is a function $f: E(T) \rightarrow \{0, 1\}$ that satisfies the following properties: (1) $|f^{-1}(0)| = \lceil (n-1)/2 \rceil$, and (2) for every $e_1 \in E(T)$, there is $e_2 \in E(T)$ such that e_1 and e_2 are independent and $f(e_1) \neq f(e_2)$. The second is a more complicated version. These tree labelling problems have their origin in the study of the orientation of Cayley graphs generated by transpositions.

79) Watching *Star Wars* Every Way Possible: The Shortest String Containing All Permutations

Gary E. Stevens, Hartwick College, Oneonta, NY

Suppose you wanted to watch the six *Star Wars* movies in every order possible. How long would it take you? We investigate this question, develop an algorithm for producing a string, hopefully the shortest, containing every permutation of $1,\ldots,n$ as a substring, and determine the length of that string. We also look at the distribution of the numbers appearing in the string.

80) A Family of 4-Critical Graphs with Diameter Three

Lucas van der Merwe*, Marc Loizeaux, Francesco Barioli, University of Tennessee at Chattanooga

Let $\gamma_t(G)$ denote the total domination number of the graph G. G is said to be total domination edge critical, or simply γ_t -critical, if $\gamma_t(G+e) < \gamma_t(G)$ for each edge $e \in E(\overline{G})$. In this paper we study a family H of 4-critical graphs with diameter three, in which every vertex is a diametrical vertex, and every diametrical pair dominates the graph. We also generalize the self-complementary graphs, and show that these graphs provide a special case of the family H.

81)

82) On line rankings and the arank number

Garth Isaak*, Lehigh University, Darren Narayan, Rochester Institute of Technology

A k-ranking of a graph is a proper coloring using colors $\{1,2,\ldots,k\}$ such that every path between two vertices of the same color contains a vertex labeled with a larger color. The arank number is the largest k for which there is a minimal (reducing any label results in a non-ranking) k-ranking. We show that the arank number corresponds to the worst (largest) case ranking produced by a greedy algorithm. Combining this with known arank values for paths yields new bounds for the online ranking number paths.

83) Locating vertices using an identifying code

Geoffrey Exoo, Indiana State University, Ville Junnila, Tero Laihonen, Sanna Ranto*, University of Turku, Finland

A subset of the vertices of a graph is called an *r-identifying* code if for every vertex v the set of the elements of the code within distance r from v is unique and nonempty. The concept of identification was introduced by Karpovsky et al. in 1998. The motivations come from finding malfunctioning processors from multiprocessor systems and sensor networks, such as fire and intruder alarm systems. Identifying codes are closely related to *locating-dominating sets* introduced by P. Slater. In this talk, we consider r-identifying codes in binary Hamming spaces i.e. hypercubes. The problem is to find the smallest possible i.e. optimal cardinalities for r-identifying codes of different dimensions. The optimal cardinality of an r-identifying code of dimension n is denoted by $M_r(n)$. We discuss improvements on known bounds. We consider, for example, the problems inspired by the open question stated by Blass et al. (2001) whether $M_{r+s}(n + m) \leq M_r(n)M_s(m)$ is true.

84) Edge Critical Graphs with Endvertices

Marc Loizeaux*, Lucas van der Merwe, University of Tennessee at Chattanooga

Let $\gamma_t(G)$ denote the total domination number of the graph G. G is total domination edge critical, or simply γ_t-critical, if $\gamma_t(G+e) < \gamma_t(G)$ for each edge $e \in E(\overline{G})$. We provide a sharp upper bound on the number of endvertices in γ_t-critical graphs. We also constructively establish the existence (with one exception) of edge critical graphs with k endvertices, where k is any integer less than this upper bound.

85)

86) Some results on λ_x-invertible graphs

John P. Georges*, David Mauro, Yan Wang*, Trinity College, Hartford

The recent work of Griggs and Jin on distance-constrained graph labelings has prompted the consideration of real number labelings. For graph G and non-negative real number x, an L_x-labeling of G satisfies the conditions that labels of adjacent vertices differ by at least x and labels of vertices distance two apart differ by at least one; for fixed value of x, the minimum span of all L_x-labelings of G is denoted $\lambda_x(G)$. In this paper we introduce the notion of λ_x-invertible graphs: for $x > 0$, G is said to be λ_x-invertible if and only if $\lambda_x(G) = x\lambda_{1/x}(G^c)$. We investigate the properties of λ_x-invertible graphs and identify several classes of graphs with λ_x-invertibility, such as the products of complete graphs, the line graphs of complete graphs and a subfamily of self-complementary graphs.

87) Traitor Tracing Schemes to Identify All Traitors Using Combinatorial Designs

Sushmita Ruj*, Bimal Roy, Indian Statistical Institute, Kolkata

We present several explicit constructions of traceability schemes. These constructions are based on combinatorial designs. Our constructions have the advantage that all colluders can be identified after the pirate decoder is confiscated. We also devise schemes to add more users without redistributing the keys.

88) The Domination Continuum

Miranda L. Roden, Peter J. Slater*, The University of Alabama in Huntsville

For each sequence $c_1, c_2, c_3, c_4, \ldots$ with $c_i \leq c_{i+1}$, we define a domination parameter $\gamma_{X(c_1, c_2, c_3, \ldots)}(G)$. The requirement is for each vertex subset of size i to be dominated at least c_i times. We thus have an (uncountably) infinite lattice set of domination parameters. We focus on the 2-dimensional sublattice of parameters $\gamma_{X(c_1, c_2)}(G)$ and examine the relationships among these parameters.

89)

90) The Edge-Weight Sums of a Graph mod n

Edgar G. DuCasse, Michael L. Gargano*, Louis V. Quintas, Pace University

A *vertex labeling* of G, a graph of order n, is a bijective function $f : V \to \{ 0, 1, 2, \ldots, n-1 \}$ and the *associated edge labeling* induced by f is the function $g : E \to \{ 0, 1, 2, \ldots, n-1 \}$ given by $g(e) \equiv (f(u) + f(v)) \bmod n$ for each edge $e = \{ u, v \}$ in E. The *value of G labeled by f* is defined as $\mathrm{val}_f(G) \equiv \sum_{e \in E} g(e) \bmod n$.

The set $\mathrm{val}(G) = \{ \mathrm{val}_f(G) \mid f \text{ is a vertex labeling} \}$ is called the *value set of G mod n*. The following questions are considered: Given a graph what is its value set? Which subsets of the set $\{0, 1, 2, \ldots, n-1\}$ can be the value set for some graph?

91) A Study of the Sudoku Graph Family

Hilmi Yildirim*, M. S. Krishnamoorthy, Rensselaer Polytechnic Institute, Narsingh Deo, University of Central Florida

This paper describes the properties of a family of Sudoku graphs, defined from valid solutions to Sudoku puzzles. We also analyze backtracking algorithms with hard instances of the problem. Finally, we describe some codes that can be generated from Sudoku solutions. The standard Sudoku board is a 9 x 9 grid of cells divided into 3 x 3 subgrids. The goal of the puzzle is to enter the digits 1 through 9 in cells of a given, partially-filled board so that each row, column, and subgrid contains exactly one instance of each digit. Two examples are given at Section 3 of the paper. Sudoku is based on combinatorics and in particular Latin Squares and projective planes. Solving Sudoku puzzles, construction of Sudoku puzzles and underlying theory have been discussed in a number of recent publications and websites. Solving a Sudoku puzzle may also be viewed as coloring a graph of 81 vertices--each representing a cell--with 9 colors. Vertex pairs corresponding to cells sharing a row, column, or subgrid are adjacent. This can easily be visualized as a regular graph of degree 20 containing 27 K_9 's (one for each row, column and subgrid)--complete subgraphs of 9 vertices. This paper takes a graph theoretic approach in analyzing Sudoku. We define generalized Sudoku graphs and study the underlying properties of these graphs. We obtain bounds on the bandwidth of generalized Sudoku graphs as well as compute the fault diameter of generalized Sudoku graphs. In addition, we define minimality of Sudoku puzzles and obtain exper0imental results showing that most Sudoku puzzles are not minimal. We further relate the labeling of generalized Sudoku graphs with codes and obtain bounds on the size of codes of length 4 with a minimum distance 2.

92) The Domination Continuum on Trees

Miranda L. Roden*, Peter J. Slater, The University of Alabama in Huntsville

For each sequence $c_1, c_2, c_3, c_4, \ldots$ with $c_i \leq c_{i+1}$, we define a domination parameter $\gamma_{X(c_1, c_2, c_3, \ldots)}(G)$. The requirement is for each vertex subset of size i to be dominated at least c_i times. We focus on the $\gamma_{X(c_1, c_2)}(G)$ parameters which are defined for trees. We examine bounds and characterizations of trees which meet these bounds.

93)

94) On the Integer-Magic Spectra of Honeycomb Graphs

Sin-Min Lee, San Jose State University, Hsin-Hao Su*, Stonehill College, Yung-Chin Wang, Tzu-Hui Institute of Technology, Taiwan

For a positive integer k, a graph $G = (V,E)$ is Z_k-magic if there exists a function, namely, a labeling, $l : E(G) \rightarrow Z_k^*$ such that the induced vertex set labeling $l^+ : V(G) \rightarrow Z_k$, where $l^+(v)$ is the sum of the labels of the edges incident with a vertex v is a constant map. Note that 1-magic is Z-magic. The set of all positive integer k such that G is k-magic is denoted by IM(G). We call this set the ***integer-magic spectrum*** of G. In this paper, we investigate the integer-magic spectra of the honeycomb graphs, which is a finite connected union of hexagonal cells. We show that besides N for a single cell honeycomb graph, there are only two possible types of integer-magic spectra N $-$ {2} or N $-$ {2, 3} of honeycomb graphs.

95) Total Perfect Codes in Tensor Products of Graphs

Ghidewon Abay-Asmerom, Richard H. Hammack, Dewey T. Taylor*, Virginia Commonwealth University

A total perfect code in a graph is a subset of the graph's vertices with the property that each vertex in the graph is adjacent to exactly one vertex in the subset. We prove constructively that the tensor product of any number of simple graphs has a total perfect code if and only if each factor has a total perfect code.

96) Secondary Domination in Graphs

Stephen Hedetniemi*, Sandra Hedetniemi, Clemson University

Given a dominating set $S \subseteq V$ in a graph $G = (V,E)$, place one guard at each vertex in S. Should there be a problem at a vertex $v \in V - S$, we can send a guard at a vertex $u \in S$ adjacent to v to handle the problem. If for some reason this guard needs assistance, a second guard can be sent from S to v, but the question is: how long will it take for a second guard to arrive? This is the issue of what we call *secondary domination*. We will focus primarily on dominating sets in which a second guard can arrive in at most two time steps. A (1, 2)-*dominating set* in a graph $G = (V,E)$ is a set S having the property that for every vertex $v \in V - S$ there is at least one vertex in S at distance 1 from v and a second vertex in S at distance at most 2 from v. We present a variety of results about secondary domination, relating this to several other well-studied types of domination. We include NP-completeness results for (1, 2)-domination and a two algorithms for trees.

97) Copwin Edge Critical Graphs

Shannon Fitzpatrick, University of Prince Edward Island

The game of "Cops and Robber", introduced by Nowakowski & Winkler, and independently by Quilliot, is a pursuit game played on a graph. While graphs on which one cop is guaranteed a win (copwin graphs) have been characterized in terms of a decomposition algorithm that compares the neighbourhoods of vertices, no structural characterization exists for graphs that require two cops to win. This is the motivation behind examining **copwin edge critical graph** (introduced by Clarke, Hill & Nowakowski). These graphs are not themselves copwin, but the addition of any edge results in a copwin graph. The graphs themselves require two cops to win. In this talk, I will discuss some basic properties of copwin edge critical graphs, give results for regular graphs and present an infinite family of copwin edge critical graphs.

98)

99) Sierpiński Square Graphs

Elizabeth Harris*, East Tennessee State University, Joshua Berry, Science Hill High School

The focus of this talk is on a problem that emanates in an area at the intersection of graph theory and geometry. The Sierpiński triangle is a well studied object, of interest both due to its fractal and graph theoretic properties. We generalize this structure and define, for the first time, the Sierpiński Square, which is an infinite structure obtained by forming, through iteration, continuously progressive Sierpiński Triangles and an inner square. We define SQ_1 to be the unit square, and subsequent squares, SQ_n, $n \geq 2$, are derived from the first n iterations of the process described above. Some of the properties studied are the order and size of SQ_n; the chromatic number, index, and total chromatic number of the Sierpiński square; and other graph invariants such as the perimeter, diameter, and domination number. Also studied are the Hamiltonicity, pancyclicity, and fractal nature of SQ_n. Finally, several open questions are posed.

100) Matrices of Row and Column Sum 2

Janine E. Janoski*, Neil Calkin, Clemson University

Let $M(n, s)$ be the number of $n \times n$ matrices with binary entries, row and column sum s, and whose rows are in lexicographical order, and let $S(n)$ be the number of $n \times n$ matrices with entries from $\{0, 1, 2\}$, symmetric, with trace 0, and row sum 2. (The sequence $S(n)$ appears as A002137 in N.J.A. Sloane's Online Encyclopedia of Integer Sequences.) We will show $M(n, 2) = S(n)$.

Wednesday, March 5, 2008, 8:40 AM

101) Equi-2-matchable Graphs

Arthur Finbow, Bert Hartnell*, Heather Pickup, Saint Mary's University

Consider the following 2 player game. The players alternate choosing an edge in a graph. The only restriction is that at most 2 edges can be selected at any vertex. Which graphs have the property that the outcome of the game is the same regardless of how the game is played? For instance, a star would always have exactly 2 edges chosen. The situation in which at most one edge can be selected at any node and the outcome is always the same has been examined (such graphs are called equimatchable [Lesk, Plummer and Pulleyblank]).

102) A Characterization of Bipartite Unit Probe Interval Graphs

David E. Brown, Utah State University, Larry J. Langley*, University of the Pacific

Probe interval graphs are a generalization of interval graphs introduced by Zhang for an application concerning the physical mapping of DNA in the human genome project. Here we examine *unit probe interval graphs* (uPIGs) where the intervals are all of the same length. Brown, Sheng, and Lundgren characterize cycle free uPIGs by forbidden subgraphs, and here we generalize to a forbidden subgraph characterization of bipartite uPIGs.

103) The Kuratowski covering conjecture for graphs of order ≤ 9

Suhkjin Hur, The Ohio State University

Kuratowski proved that a finite graph embeds in the plane if it does not contain a subdivision of either K_5 or $K_{3,3}$, called Kuratowski subgraphs. A generalization of this result to all nonorientable surfaces says that a finite graph embeds in the nonorientable surface of genus $\tilde{g}$ if it does not contain $\tilde{g}+1$ Kuratowski subgraphs such that the union of each pair of these fails to embed in the projective plane, the union of each triple of these fails to embed in the Klein bottle if $\tilde{g} \geq 2$, and the union of each triple of these fails to embed in the torus if $\tilde{g} \geq 3$. We prove this conjecture for all graphs of order ≤ 9.

104) Enumeration of (-1,0,1)-Matrices

Kenneth Matheis*, Shanzhen Gao, Florida Atlantic University

Let r(m,n,s,t) be the number of (-1,0,1)-matrices of size $m \times n$ with each row sum equal to s and each column sum equal to t (sm=nt). The determination of r(m,n,s,t) is an unsolved problem and it is unlikely that a simple formula exists except for very small s,t. We present some rather involved closed formulas and an algorithm for r(m,n,s,t).

105) Thresholds and Achlioptas Games

Gregory McColm, University of South Florida

In the game-theoretic model of evolution of random structures introduced by D. Achlioptas, there are two players, a Radical and a Conservative, and there is a neutral Generator. For each move, the Generator chooses a player at random, and a set of elements for the chosen player to choose from. The sequence of choices is the *play* of the game. The two players are interested in a property of strings of moves, which is a right ideal I of the set of strings: the Radical wants to attain I as soon as possible, while the Conservative wishes to delay as much as possible. Assuming that the players use fixed strategies, we will look at the threshold behavior of the *stopping time*, i.e., the random variable that gives the total number of moves in the play. This problem has application to other structures represented by appropriate semigroups, including Achlioptas's original concern, graphs. It also exhibits some pathologies that do not appear in simpler models like the classical Erdős-Rényi model of random graphs.

106) Interval Graphs Where No Interval Contains Two Others

Jeffrey J. Beyerl*, Robert E. Jamison, Clemson University

An interval graph is *proper* iff it has a representation in which no interval contains another. Fred Roberts characterized the proper interval graphs as those containing no induced star $K_{1,3}$. Proskurowski and Telle have studied q-proper graphs, which are interval graphs having a representation in which no interval is properly contained in more than q other intervals. We are initiating the study of *p-improper interval graphs* where no interval contains more than p other intervals. This paper will focus on the case $p = 1$ and the search for minimal forbidden subgraphs (MFS). Of course, the star $K_{1,4}$ is a MFS, but there are other more complex examples.

107) Constructing all minimum genus embeddings of $K_{3,n}$

Mark Ellingham, Adam Weaver*, Vanderbilt University

Ringel determined the minimum orientable and non-orientable genus for all complete bipartite graphs. For $K_{3,n}$ the orientable genus is $\lceil (n-2)/4 \rceil$ and the nonorientable genus is $\lceil (n-2)/2 \rceil$. Euler's formula implies that almost all of the faces in such an embedding must be 4-cycles. We discovered that these embeddings can be represented by certain edge colorings of n-vertex cubic graphs. Using this correspondence and results of Kotzig, we show that every minimum genus embedding of $K_{3,n}$ can be constructed using simple operations that involve adding one crosscap or one handle at a time.

108) The Distribution of 0's and 1's in Jacobsthal Strings

Ralph P. Grimaldi, Rose-Hulman Institute of Technology

For the alphabet $\sum=\{0,1\}$, let A be the language {0, 01, 11}. For $n \geq 1$, the number of strings of length n in A^* (the Kleene closure of A) is given by $\left(\frac{2}{3}\right)2^n + \left(\frac{1}{3}\right)(-1)^n = J_n$, the n-th Jacobsthal number. As we consider these J_n strings we are now interested in the number of times a 1 occurs in position k of the strings, for $1 \leq k \leq n$. This results in a Pascal-like triangle from which various properties of these distribution numbers are derived.

109) Computing the Inverse of a Tree's Incidence Matrix
David P. Jacobs*, Clemson University, Catia M. S. Machado, Elaine C. Pereira, Fundação Federal Universidade Federal do Rio Grande, Vilmar Trevisan, Universidade Federal do Rio Grande do Sul

Let T be a tree, and assume the root of T is given a loop, and let A be the incidence matrix of T. It is easy to see that A is nonsingular, and it is known that $Ax = b$ can be solved in linear time. We give a simple, explicit construction of A^{-1}, based on paths in the tree. The algorithm has worst case complexity in $O(n^2)$, but on average is in $O(n^{1.5})$. We also give an analogous construction for directed trees.

110) Recognition Algorithms and Structural Characterizations for Bipartite Tolerance and Bipartite Probe Interval Graphs
David E. Brown*, Utah State University, Arthur H. Busch, University of Dayton, Garth Isaak, Lehigh University

A graph G is a *tolerance graph* if and only if each vertex $v \in V(G)$ can be associated with an interval I_v of the real numbers and a positive real number t_v with $uv \in E(G)$ if and only if $|I_v \cap I_u| \geq \min\{t_v, t_u\}$. Graph G is a probe interval graph if there is a partition of $V(G)$ into sets P and N with each vertex associated to an interval of the real number line such that $uv \in E$ if and only if $I_v \cap I_u \neq \emptyset$ and $\{u, v\} \cap P \neq \emptyset$. We give a recognition algorithm for bipartite tolerance graphs that yields a structural characterization in terms of 2-connected blocks. With a few modifications, the same recognition algorithm works for bipartite probe interval graphs and yields a structural characterization for them in terms of 2-edge-connected blocks. The recognition algorithm is $O(|V| + |E|)$ for both classes of graphs.

111) Dual Unfoldings of Polyhedra
Josh Laison*, Willamette University, John Watkins, Colorado College, Luc Wilson, University of Maryland

An unfolding of a polyhedron P is a obtained by cutting P along its edges and flattening it into the plane. Given an unfolding U of P, we define its dual unfolding U^Δ, which is an unfolding of the dual P^Δ of P. Each of these four geometric objects has combinatorial properties which can be encoded in an associated graph. We investigate relationships between properties of an unfolding and properties of its dual.

112) Six Catalan Identities in Search of Good Proofs
Lou Shapiro, Howard University

There are dozens of Catalan generating function identities and many have very pretty combinatorial proofs. These six, though short and sweet, do not (yet, that I know of). Let C, B, and F be the generating functions for the Catalan, central binomial, and Fine number sequences. Two of the six are $F + 2BC = 3BF$ and $1/(1\text{-}4z) = BC + z(BC)^2$. We will also prove a few identities and indicate why these six are of particular interest.

113) Broken Symmetry in Proteins

Vince Grolmusz*, Gábor Iván, Zoltán Szabadka, Eötvös University, Budapest, Hungary, Uratim Ltd, Nyíregyháza, Hungary

An enormous amount of mathematical and combinatorial work was spent on the sequence analysis of DNA and protein molecules in the literature. Perhaps the two main reasons for this is the importance of such studies and also the easy availability of large sequence databases. However, much less work was done analyzing these sequences *together* with the three-dimensional structure of the molecules. One especially important question is the description of the areas on proteins that bind smaller molecules. Note, that the great majority of biochemical processes involve proteins and small molecules, and also note, that almost all pharmaceuticals are small molecules, binding human, bacterial or viral proteins. In this work we performed data mining in the whole Protein Data Bank (PDB, containing more than 47 000 entries), and found striking asymmetries never observed before in protein-small molecule binding patterns.

114) A Class of Interval Digraphs

Shilpa Das Gupta, J. Richard Lundgren, Elena Ortega*, University of Colorado at Denver

Recently tournaments that are interval digraphs have been characterized by Brown, Busch, and Lundgren. They show that a tournament on n vertices is an interval digraph if and only if it has a transitive (n-1)-subtournament. We investigate a broader class of oriented graphs on n vertices that contain a transitive (n-2)-tournament as a subdigraph. If such an oriented graph D is not itself a tournament, then it may be an interval digraph even if it does not contain an (n-1)-transitive tournament as a subdigraph, as long as there are specific restrictions on D. We explore what restrictions we can place on D to guarantee that it is interval. A directed graph D is an interval digraph if for each vertex u there corresponds an ordered pair of intervals (S(u), T(u)) such that uv is an arc of D if and only if the intersection of S(u) and T(v) is nonempty. A bipartite graph G is an interval bigraph if to each vertex there corresponds an interval such that vertices are adjacent if and only if their corresponding intervals intersect and each vertex belongs to a different partite set. We use the equivalence of the models for interval digraphs and interval bigraphs in our investigation of which of these oriented graphs are interval digraphs.

115) A Variation of the Lexicographic Product of Graphs

Ghidewon Abay-Asmerom*, Richard H. Hammack, Dewey T. Taylor, Virginia Commonwealth University

In this paper we introduce a generalization of the lexicographic product, or composition, of graphs along the idea of the complementary cartesian product introduced by Haynes et al. We explore various standard graph invariants for this generalization.

116) Lattice Paths, RNA Matrices, and RNA Secondary Structures

Asamoah Nkwanta, Morgan State University

Two lower-triangular arrays with entries that count RNA secondary structures of a given length are mentioned in this short talk. The array entries also count specific lattice walks. There is a one-to-one correspondence between RNA structures and a subset of the walks. We will discuss various ways in which the walks can be used as a tool to help predict primary RNA sequences.

117) Even-Pancyclic Subgraphs of Meshes

A. Delgado[1], D. Gagliardi[2], M.L. Gargano[3], M. Lewinter[1], W. Widulski[4]*, [1]Purchase College, [2]SUNY Canton, [3]Pace University, [4]Westchester Community College

A graph G of order n is *even-pancyclic* if it contains cycles of all possible even lengths 4, 6, 8, …, $2\lfloor n/2 \rfloor$. The *2-dimensional mesh* $M(m, n)$ is the Cartesian product of the two paths P_m and P_n. We present several results on even-pancyclic subgraphs of meshes.

118) Connectivity of Walk Graphs

Daniela Ferrero, Texas State University

For a given graph G and a positive integer k the W_k-walk graph of G, denoted as $W_k(G)$, has for vertices the set of walks of length k in G in which no two consecutive edges are equal. Two vertices of $W_k(G)$ are adjacent when one of the corresponding walks can be obtained from the other by deleting an edge in one end and adding an edge to the other end. We present conditions in G that guarantee a certain level of connectivity in $W_k(G)$.

119) Enumeration of (0,1,2)-Matrices

Shanzhen Gao*, Kenneth Matheis, Florida Atlantic University

Let t(m,n,s,t) be the number of m×n matrices with entries from {0,1,2} and with each row sum equal to s and each column sum equal to t (sm=nt). The determination of t(m,n,s,t) is an unsolved problem. We will present some rather involved closed formulas and an algorithm for t(m,n,s,t).

120) Decompositions of Prisms into Matchings

Brandy Hicks*, Robert E. Jamison, Clemson University

If G is any graph, a *G-decomposition* of a host graph $H = (V, E)$ is a partition of the edge set of H into subgraphs of H which are isomorphic to G. The *chromatic index* of a G-decomposition is the minimum number of colors required to color the parts of the decomposition so that two parts which share a node get different colors. The *G-spectrum* of H is the set of all chromatic indices taken on by G-decompositions of H. In this talk we will study the case that the prototype G is a matching and the host H is a prism. In particular, we will be concerned with *complete decompositions* — that is, decompositions in which every two parts share a common node.

121) Posets from Fair Division

Eric Gottlieb, Rhodes College

We offer some posets as models for thinking about some questions in fair division. One of these posets, which has been studied earlier by Gardenfors in the context of voting theory, has connections to Coxeter groups.

122) Pebbling graphs of diameter three

Carl Yerger*, Luke Postle, Noah Streib, Georgia Institute of Technology

Given a configuration of pebbles on the vertices of a connected graph G, a *pebbling move* is defined as the removal of two pebbles from some vertex, and the placement of one of these on an adjacent vertex. A graph is called *pebbleable* if for each vertex v there is a sequence of pebbling moves so that at least one pebble can be placed on vertex v. The *pebbling number* of a graph G is the smallest integer k such that G is pebbleable given any configuration of k vertices on G. We improve on the bound of Bukh by showing that the pebbling number of a graph of diameter 3 on n vertices is at most $\frac{3}{2}n + 2$. In addition, this technique allows us to characterize the pebbling number of graphs with diameter 2 in a simple manner.

123) Minimal Sizes of Binary Linear Forms

Shaun Sullivan*, Shanzhen Gao, Joshua Fallon, Florida Atlantic University

A binary linear form is a set of the form $sA + tA = \{sx + ty|x, y \in A\}$ where $s, t \in$ N and A is a finite subset of nonnegative integers. We consider the minimal size of $sA + tA$ for a fixed size of the set A and find all such sets that obtain this minimum. We also give results to enumerate these sets given a bound on the largest element in the set.

124) An algorithm for finding the influence digraph of a Time-Stamped Graph

Drew J. Lipman*, Oakland University, Marc J. Lipman, Indiana University – Purdue University

A Time-Stamped Graph is a graph with multiple edges but no loops, where each edge is labeled with a timestamp. A timestamp is used to represent the time of collaboration between vertices joined by the edge. Given a time-stamped graph H, the associated influence digraph of H is the digraph of the vertex set of H with an arc from X to Y iff there is a path from X to Y in H with non-decreasing timestamps. We say that X influences Y. In this talk we will show the construction of an efficient algorithm for constructing the influence digraph.

125) Maximal flat antichains of minimum weight

Martin Grüttmüller, Thomas Kalinowski, Universität Rostock, Germany, Sven Hartmann, Technische Universität Clausthal, Germany, Uwe Leck*, University of Wisconsin – Superior, Ian Roberts, Charles Darwin University, Australia

We study maximal families A of subsets of $[n] = \{1, 2, \ldots, n\}$ such that A contains only pairs and triples and $A \not\subseteq B$ for all $\{A,B\} \subseteq$ A, i.e. A is an antichain. For any n, all such families A of minimum size are determined. This is equivalent to finding all graphs $G = (V, E)$ with $|V| = n$ and with the property that every edge is contained in some triangle and such that $|E| - |T|$ is maximum, where T denotes the set of triangles in G. The largest possible value of $|E| - |T|$ turns out to be equal to $\lfloor (n+1)^2/8 \rfloor$. Furthermore, if all pairs and triples have weights w_2 and w_3, respectively, the problem of minimizing the total weight $w(A)$ of A is considered. We show that $\min w(A) = (2w_2 + w_3)n^2/8 + o(n^2)$ for $3/n \le w_3/w_2 =: \lambda = \lambda(n) < 2$. For $\lambda \ge 2$ our problem is equivalent to the (6,3)-problem of Ruzsa and Szemerédi, and by a result of theirs it follows that $\min w(A) = w_2 n^2/2 + o(n^2)$.

126) Kneser representations of graphs

Peter Hamburger, Attila Por, Western Kentucky University; Matt Walsh*, Indiana-Purdue Fort Wayne

The Kneser graph K$n{:}k$ for positive integers $n \ge k$ has as its vertex set the k-element subsets of some n-set, with disjoint sets being adjacent. Every finite simple graph can be found as an induced subgraph of some Kneser graph; this can be viewed as a way of representing graphs by labelling their vertices with sets. We explore questions of finding the smallest representation (both in terms of n, the size of the label set, and k, the size of the labels) of certain classes of graphs and draw connections to related problems.

127) A generalization of Sidon sets to two sets of natural numbers

Ago-Erik Riet*, Fabricio S. Benevides, Jeffrey P. Wheeler, Jonathan Hulgan, University of Memphis, Nathan Lemons, Cory Palmer, Central European University

A set A of natural numbers is called **a Sidon set** if all pairwise sums of elements of A are distinct (up to the order of summands), i.e. $a + b = c + d$ implies $\{a, b\} = \{c, d\}$. We introduce the notions of coSidon and perfect. We call the pair $\boldsymbol{A}, \boldsymbol{B} \subset$ N **coSidon** if all sums of form $a+b$, $a \in \boldsymbol{A}$, $b \in \boldsymbol{B}$ are distinct. The representation function $R_{A,B}(n) = R(\boldsymbol{A}, \boldsymbol{B}, n)$ is the number of ways to write n as $a + b$. So $\boldsymbol{A}, \boldsymbol{B}$ are coSidon if and only if $R(\boldsymbol{A}, \boldsymbol{B}, n)$ takes only values 0 and 1. We call sets of integers $\boldsymbol{A}, \boldsymbol{B}$ **perfect** if $R^{-1}_{A,B}[N_{\ge 1}]$ is an interval (of integers). We prove results on the size and 'density' of sets $\boldsymbol{A}$ and $\boldsymbol{B}$ if they are coSidon. We characterize perfect coSidon sets, and use them to build a class of pairs of sets of naturals with $R(\boldsymbol{A}, \boldsymbol{B}, n)$ non-decreasing in n.

128) Complete sphere-of-influence graphs of point-sets with large second distance

Marc J. Lipman, Indiana University - Purdue University Fort Wayne

The Sphere-of-Influence Graph of a set of at least two points in the Euclidean plane is the intersection graph of the set of circles centered at the points, each circle with radius equal to the smallest distance to any other point in the set. It is known that K_8 is the sphere-of-influence graph for some point-set and that K_{12} is not. It has been conjectured that K_9 is not. This talk contains a proof that if the sphere-of-influence graph of point-set V is complete, and the ratio of the second smallest distance between points in V to the smallest distance between points in V is at least 2.5, then V contains at most eight points.

129) Turán Numbers for Chessboard Graphs

Heiko Harborth, Techn. Univ. Braunschweig, Germany

For an nxn-chessboard B_n it is asked for the minimum number $B_{n,k}$ of edges (sides of the square cells of B_n) which have to be deleted such that no kxk-chessboard as subboard of B_n is complete. This is equivalent to the minimum number of kx(k+1)-rectangles by which a B_{n-k+1} can be covered. (Common work with Heiko Dietrich.)

130) Single-molecule conductors: a new use for old characteristic polynomials

Patrick W. Fowler*, Barry T. Pickup, Tsanka Z. Todorova, University of Sheffield, UK

Conduction of an electron through a molecular electronic device is modelled by a version of the standard Hückel theory of conjugated hydrocarbons, equivalent to the problem of determining the spectrum of the adjacency matrix of the molecular graph. For conduction we solve the (now continuous) eigenvalue problem under modified 'source-and-sink' boundary conditions. Conductance varies strongly with eigenvalue (representing electron energy) but is fully determined by a combination of four characteristic polynomials: of the molecular graph and three vertex-deleted sub-graphs. Closed-form expressions and properties of the conductance function are deduced for some classes of chemical graphs.

131) On bounds for the van der Waerden number $W(3; r)$

D. S. Gunderson*, University of Manitoba, K. R. Johannson, University of Memphis

For any positive integer r, the number $W(3; r)$ is the least n so that for any r-colouring of $\{1, 2, \ldots, n\}$, there exists a monochromatic 3-term arithmetic progression. The best known bounds on $W(3; r)$ follow from associated density results, with the best known upper bound following from a diffcult analytic proof by Bourgain. Simple combinatorial arguments also give decent upper bounds on $W(3; r)$; one such argument is the standard "block proof". Two variants of this block proof (one well known) and a recent simple argument by Solymosi are given.

132) When Fundamental Cycles Span Cliques

Terry A. McKee, Wright State University

Several papers have studied when a graph G can have a *canonical tree* T, meaning a spanning tree of G such that, for every edge xy of G, the x-to-y path in T spans a complete subgraph of G. Define a *positive boolean subgraph* of G to be a subgraph formed by repeated intersections and unions of maximal complete subgraphs—or (equivalently!) of unit disks ($N[v]$'s). Theorem: A spanning tree T is canonical iff every connected positive boolean subgraph spans a subtree of T. This leads to both known and new characterizations of canonical trees, paired according to this 'duality' between maximal cliques and unit disks.

133) A Maximum Flow Algorithm to Locate Non-attacking Queens on an NxN Chessboard

Leslie Gardner*, Octavian Nicolio*, University of Indianapolis

The N Non-attacking Queens (N Queens) Problem is used to teach backtracking algorithms in introductory computer science courses and discrete math courses because it is engaging for students. We present some network representations of the N Queens problem and an adaptation of a maximum flow algorithm that can be used with each to teach how to represent problems with networks, how to write and uses maximum flow algorithms, and how to assess the effectiveness of algorithms.

134) Graph Operations and Partial Unimodality of Independence Polynomials

Vadim E. Levit*, AUC & HIT, Israel, Eugen Mandrescu, HIT, Israel

If s_k denotes the number of stable sets of size k in G, then the polynomial *I(G; x)* having s_k as its coefficients, is called the independence polynomial of G (I. Gutman and F. Harary,1983). Y. Alavi, P. J. Malde, A. J. Schwenk and P. Erdös (1987) conjectured that *I(T, x)* is unimodal for any tree T. We show that $s_{\lceil (3\alpha=2)/5 \rceil} \geq ... \geq s_{\alpha-1} \geq s_{\alpha}$ and $s_0 \leq s_1 \leq ... \leq s_{\lfloor (2\alpha+2)/5 \rfloor}$ are valid for any graph G obtained from a graph H by appending two pendant edges to each vertex of H, where α is the size of a maximum stable set in G. In particular, when H is a tree, the above partial unimodality phenomenon gives support to Alavi et al.' conjecture.

135) Generalization of the Erdős-Gallai Inequality

Sibel Ozkan*, Florida Atlantic University, Chris A. Rodger, Auburn University

A sequence of non-increasing, non-negative integers is called graphic if it can be realized by a simple graph. P. Erdős and T. Gallai gave necessary and sufficient conditions for a sequence of non-negative integers to be graphic. Here, we modify their result to generalize it for multigraphs.

136) Spanning-tree-center vertices of a graph

[1]A. Delgado, [2]M.L. Gargano, [1]M. Lewinter*, [2]L.V. Quintas, [1]Purchase College, SUNY, [2]Pace University, New York

A vertex in a graph G is a *spanning-tree-center* (STC) if it is in the center of a spanning tree of G. A graph G is a *pancentral graph* (PC), if each vertex of G is an STC vertex. Let STC(G) denote the STC vertices of G and C(G) the center of G. Then, C(G) is a subset of STC(G). We present several theorems and open problems.

137) Combinatorial Analog of Dyson's Theorem

Pallavi Jayawant*, Peter Wong, Bates College

Tucker's Lemma is the combinatorial analog of the Borsuk-Ulam theorem from topology about continuous functions on the sphere. Tucker's Lemma is a statement about labellings of triangulations of the sphere. In 2006 Tucker's Lemma was proven in its full generality. Yang's theorem is a generalization of the Borsuk-Ulam theorem and Dyson's theorem is a special case of Yang's theorem about real valued functions on the sphere. I will talk about the Borsuk-Ulam theorem and Tucker's lemma and then present a combinatorial analog of Dyson's theorem. I will show the equivalence of the analog to Dyson's theorem. I will end with a discussion of possibilities to extend this work to obtain a discrete version of Yang's theorem.

138) Distributions of Points and Small Point Sets with Large Area

Hanno Lefmann, Technical University Chemnitz, Germany

Heilbronn's triangle problem asks for a distribution of n points in the unit square $[0, 1]^2$ such that the minimum area $\Delta_3(n)$ of a triangle is as large as possible. The currently best known lower and upper bounds on $\Delta_3(n)$ are due to Komlós, Pintz and Szemerédi.
Here we consider a generalization of this problem for small point sets with large area/volume. Moreover, for fixed d we consider extensions of this problem in the d-dimensional unit cube $[0, 1]^d$. Also connections to the No-Three-On-A-Line problem will be discussed. The probabilistic existence results can be made constructive by deterministic polynomial algorithms.

139) Degree Symmetric Bipartite Graphs and Equivalent Representations

Michael L. Gargano, Pace University, Marty Lewinter, Purchase College, Joseph F. Malerba*, Pace University

If a degree sequence $d_1 \geq d_2 \geq d_3 \geq \ldots \geq d_m$ is the same for both vertex sets that form the vertex partition of a bipartite graph then the sequence is called *bipartite graphical* and we say that the graph is a *degree symmetric bipartite graph*. We state and prove a Havel/Hakimi type theorem: A sequence $d_1 \geq d_2 \geq d_3 \geq \ldots \geq d_m$ is bipartite graphical if and only if the sequence $d_2 - 1, d_3 - 1, d_4 - 1, \ldots, d_{d_1} - 1, d_{d_1+1}, \ldots, d_m$ is bipartite graphical. We then consider different but equivalent representations of this problem as undirected graphs with loops and square binary (i.e., zero/one) matrices.

140) Yet Another Formula for The Number of Spanning Trees of A Graph

Andrew Chen, Minnesota State University Moorhead

A *chain* is a path whose internal vertices are of degree two and whose end vertices are not of degree two, and the length of a chain is the number of edges in it. To contract a chain is to contract together all the edges in a chain into one edge. The *distillation* of a graph G is denoted by $D(G)$. The distillation is the result of contracting every chain to become a single edge. In previous work it was mentioned that there is a formula for the number of spanning trees of a graph in terms of the lengths of the chains of the graph. Unfortunately, that formula has a number of terms that is equal to the number of spanning trees of $D(G)$. We present a way to represent that formula in terms of a minor of a given square matrix whose size is the number of vertices of $D(G)$. This representation is similar to the classic Kirchhoff Matrix-Tree formula for the number of spanning trees of a graph in terms of a minor of the adjacency matrix, and has a number of advantages which will be presented. We will also present a brief summary of a variety of different ways to determine the number of spanning trees of a graph.

141) Coalition Theory in Teaching Proof, Combinatorics & Set Concepts

Lorraine L. Lurie*, Michael L. Gargano, Pace University, Marty Lewinter, Robert M. Gargano, Roberts Wesleyan, Anthony Delgado, Purchase College

People form coalitions in order to increase their power in certain voting situations. This is a nice practical problem for students to explore using elementary set theory concepts/operations and simple combinatorics. It also presents opportunities to introduce various proof techniques in an interesting political science application.

142) Invariant Factors of Cartesian Product and One Point Unions of Graphs

Wai Chee Shiu, Hong Kong Baptist University

A matrix called Varchenko matrix associated with a hyperplane arrangement was defined by Varchenko in 1991. Matrices that we shall call q-matrices are induced from Varchenko matrices. Many researchers are interested the invariant factors of these q-matrices. In 2004, Shiu transferred this problem to graph model. In this talk, invariant factors of Cartesian product and one point union of some graphs will be discussed.

143) The Independence Number for the Generalized Petersen Graphs

Joseph Fox, Salem State College, Ralucca Gera*, Pantelimon Stanica, Naval Postgraduate School

Given a graph G, an independent set $I(G)$ is a subset of the vertices of G such that no two vertices in $I(G)$ are adjacent. The independence number $\alpha(G)$ is the order of a largest set of independent vertices. We study the independence number for the Generalized Petersen graphs, presenting both sharp bounds and exact results for subclasses of the Generalized Petersen graphs.

144) Spanning Trees of Ideal Non-Proper Split graphs

Laura Helbing, John T. Saccoman*, Seton Hall University

A graph G is a split graph if its node set can be partitioned into a clique and an independent set. While much research has been published regarding the number of spanning trees and All-Terminal Reliability (ATR) of a subclass of split graphs, the threshold graphs, not much has been done with regard to split graphs. We present a new class of split graphs, Ideal Non-Proper Split (INPS) graphs and a formula for the number of spanning trees when the independent set is of order two or three. A surgery that impacts the number of spanning trees is also discussed.

157)

158)

159) Hamiltonian cycles in finite cubic Cayley graphs: the < 2, 4*k*, 3 > case

Henry Glover*, Ohio State University, Klavdija Kutnar, Dragan Marušič, Primorska University, Slovenia

We show that every < 2, 4*k*, 3 > Cayley graph given by a finite group presentation $< a, b \mid a^2 = b^{4k} = (ab)^3 = 1, etc. >$ has a Hamiltonian cycle. We do this by constructing a tree of faces in the Cayley map such that each vertex of the Cayley graph lies on the boundary of at least one of the faces in this tree.

160) The Choice Number of K(4,2,...,2)

Julian Allagan*, Peter Johnson, Auburn University

In 2002 Enomoto, Ohba, Ota, and Sakamoto published a proof that the choice number of the complete k-partite graph K(4,2,...,2) is k if k is odd and k + 1 if k is even. Recently Xu Yang has claimed to detect errors in their proof, and that the choice number is k + 1 in both cases. While Xu Yang's proof is wrong, a certain amount of doubt has been cast on the proof of Enomoto, et al, which, while ingenious, is a proof by induction on k with a complicated induction hypothesis, which should, and evidently does, arouse suspicion. While we judge the original proof to be valid, we offer here a lemma which we hope will be of interest in its own right, and which can be used to simplify radically the proof of Enomoto, et al, to establish beyond the shadow of a doubt that their result was correct in the first place.

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161)

162)

163) Hamilton Decompositions of Cayley Graphs on Finite Abelian Groups

Erik Westlund*, Donald Kreher, Michigan Technological University

If A is a finite abelian group and $S \subseteq A \setminus \{0\}$, such that $s \in S \Leftrightarrow -s \in S$, then the associated Cayley graph ¡= Cay(A, S) is the simple, regular, vertex-transitive, undirected graph with vertex set V(¡) = A and edge set E(¡) = $\{\{x, y\} : x - y \in S$ or $y - x \in S\}$. ¡ is connected if and only if $A = \langle S \rangle$. Alspach conjectured that every $2k$-regular connected Cayley graph on a finite abelian group has a decomposition into k edge-disjoint Hamilton cycles. We discuss some recent progress on this conjecture.

164) More Sporadic Evidence for the Affirmative, in Cropper's Problem

B.B. Bobga*, P.D. Johnson Jr., Auburn University

It is natural to consider the problem of completing a partial latin square to be a list coloring problem, in which the graph is the Cartesian product of a clique with itself and the list assignment is determined in an obvious way by the prescribedentries. Hall's condition is a natural necessary condition on a graph and a list assignment for a proper coloring of the graph from the assigned lists; the condition is a conjunction of inequalities associated with the different induced subgraphs of the graph. Cropper's problem is the question: Is Hall's condition sufficient for the existence of a completion of a given partial latin square? In the past it has been shown that Ryser's theorem on completing latin rectangles to latin squares can be construed as saying that if the cells bearing the prescribed entries of a partial latin square form a subrectangle of the square, then just one of the inequalities required by Hall's condition, the one for which the induced subgraph is the whole graph, suffices to guarantee the completion. And when the prescribed cells form a subrectangle minus one cell, only 3 of the inequalities of Hall's condition suffice to guarantee completion. Here we examine several theorems on completing partial latin squares due to Buchanan and Ferencak, Hoffman, and Rodger, and discover that each of them can be restated in the form: if the prescribed cells form such-and-such a configuration then the satisfaction of the inequality in Hall's condition for just a few special choices of induced subgraph suffices for the existence of a completion.

165)

166)

167) Hamiltonicity in C_n X C_m after a single push

Edward C. Carr, North Carolina A & T State University,
Joseph B. Klerlein*, Western Carolina University

Let C_n X C_m be the Cartesian product of two directed cycles C_n and C_m. Pushing a vertex in a directed graph reverses the orientations of all the directed edges incident with the pushed vertex. In this paper we investigate the hamiltonicity of the directed graphs obtained from C_n X C_m after a single vertex is pushed.

168) Some Non-simple Forbidden Configurations and Design Theory

Richard Anstee*, Farzin Barekat, UBC, Vancouver

Let k, l, m, q be given. We let $f(m, k, l, q)$ denote the maximum number of subsets of $\{1, 2, \ldots ,m\}$ in a family F so that we cannot find q sets $A_1, A_2, \ldots, A_q \in F$ and $k + l$ elements $a_1, a_2, \ldots, a_{k+l} \in \{1, 2, \ldots ,m\}$ so that each of the q sets A_i contains the k elements $a_1, a_2, \ldots, a_k$ and does not contain the l elements $a_{k+1}, a_{k+2}, \ldots, a_{k+l}$. We are able to compute $f(m, 1, 1, q)$, $f(m, 2, 1, q)$ and $f(m, 2, 2, q)$ exactly for large m. For example, the natural construction for $k = 2$, $l = 1$ is to take all sets of size 0, 1, 2, m and also the sets of size 3 corresponding to a simple triple system of $\lambda = q-2$ (which exists by a result of Dehon, 1983) and we are able to show for large m that indeed $f(m, 2, 1, q) = \binom{m}{0} + \binom{m}{1} + \binom{m}{2} + \frac{q-2}{3}\binom{m}{2} + \binom{m}{m}$.

169)

170)

171) Parameterized Complexity of the Secure Sets Problem

Rosa I. Enciso*, Ronald D. Dutton, University of Central Florida

A secure set of a graph $G = (V,E)$, is a non empty set $S \subseteq V$ where for every $X \subseteq S$, $|N[X] \cap S| \geq |N[X] - S|$. When $|X| = 1$, the secure set problem reduces to the well known efensive alliance problem. To date, there is no known polynomial time algorithm for dentifying a secure set in a graph. Moreover, the decision problem regarding secure sets is not known to be in the set NP. Unlike the theory of classical complexity, which focuses on whether a problem is hard or not, *parameterized complexity* theory, introduced by Downey and Fellows [3], accepts that a problem is hard and asks the question "What makes the problem computationally difficult?". A problem is said to be *fixed-parameter tractable* (FPT) if there exists an algorithm that correctly solves an input (n, k) in time $f(k)n^{\alpha}$ (or $f(k)+n^{\alpha}$), where α is a constant independent of k, and f is an arbitrary function independent of n. An algorithm is presented for the decision problem on secure sets with a running time of $O(2^{k \log 2k} n)$, showing the problem is fixed-parameter tractable.

172) Splittable Whist Designs

D. Berman, UNCW, N. Finizio*, U of Rhode Island, D. Smith, UNCW

A new specialization of whist designs, namely a **splittable whist** will be introduced. If p is a prime of the form $p = 2^k t + 1$, t odd, $k \geq 3$ then splittable whist designs on p players exist for all such primes p less than 100000 with four possible exceptions. In the case $k = 3$ we establish that splittable whist designs exist for all primes with the possible exception of $p = 41$.

173)

174)

175) Minimum Distortion Embeddings into a Line

Pinar Heggernes and Daniel Meister, University of Bergen,
Andrzej Proskurowski*, University of Oregon

The problem of computing minimum distortion embeddings of a given graph into a line (path) was introduced in 2004 and has quickly attracted significant attention with subsequent results appearing in recent STOC and SODA conferences. So far, all such results concern approximation algorithms or exponential-time exact algorithms.We give the first polynomialtime algorithms for computing minimum distortion embeddings of graphs into a path when the input graphs belong to specific graph classes. In particular, we solve this problem in polynomial time for bipartite permutation graphs and threshold graphs.

176) Generalised Bhaskar Rao designs with block size $k = 3$

Diana Combe, The University of New South Wales, Sydney, Australia

In this talk we define generalised Bhaskar Rao designs (GBRDs). We survey what is known for GBRDs with block size 3 - i.e. we explain the necessary conditions on the parameters, and give a brief general survey of those groups for which these necessary conditions are known to be sufficient. We give details for groups of size 16.

177)

178) On Some Combinatorial Arrays

D.V. Chopra*, Wichita State University; Richard M. Low, San Jose State University; and R. Dios, New Jersey Institute of Technology

An array T with m rows (constraints), N columns (runs, treatment-combinations), and with s levels is merely a matrix T $(m \times N)$ with elements from a set $S = \{0, 1, 2, \ldots, s-1\}$. These arrays assume great importance in combinatorics and the statistical design of experiments, when one imposes some combinatorial structure on them. In this paper, we restrict ourselves to arrays with two elements 0 and 1. An array T is called an orthogonal array (O-array) of strength t $(0 \leq t \leq m)$ if in each $(t \times N)$ submatrix T^* of T, every $(t \times 1)$ vector $\vec{\alpha}$ with i $(0 \leq i \leq t)$ ones in it appears with the same frequency λ (say). Here, λ is called the index set of the O-array. Clearly, $N = \lambda \cdot 2^t$. These arrays have been extensively used in the design of experiments, in coding and information theory, etc. Balanced arrays (B-arrays) are generalizations of O-arrays in the sense that the combinatorial structure imposed on O-arrays is somewhat weakened in that every vector $\vec{\alpha}$ of weight i $(0 \leq i \leq t)$ appears with the same frequency λ_i (say). Clearly if $\lambda_i = \lambda$ for each i, one gets an O-array. In this paper, we will present some results on the existence of some of these combinatorial arrays.

179) The Interchanging graphs associated with sorting by transpositions

Pinglong You, Fujian Agriculture and small Forestry University, Wai Chee Shiu, Wai Hong Chan*, Hong Kong Baptist University, An Chang, Fuzhou University

Sorting a permutation by transpositions is one of important methods of sequence comparison in computational molecular biology for deriving evolutionary and functional relationships between genes. We will establish a new kind of interchanging graphs associated with sorting by transpositions, discuss their structures, and introduce some properties of those graphs.

180) Arithmetic with Very Large Integers Using Parallel Processing

Joshua Yarmish, Pace University, Gavriel Yarmish*, Brooklyn College

We present a practical and efficient procedure for performing operations on very large integers by carrying out operations on components of n-tuples representing these integers. Each integer can be uniquely represented by an n-tuple consisting of its remainders upon division by $m_1,m_2,m_3 \ldots m_n$ where each m_i is an integer greater than 2, and where $\gcd(m_i, m_j)=1$ for $i \neq j$. After computing the value of each component in the result, we can recover its value by multiplying its components by the corresponding basis vectors. These representations are unique so long as $m_1,m_2,m_3 \ldots m_n$ is greater than the results of the arithmetic operation we wish to compute. There are two major advantages to this method: a) Time: The component computation can be done using parallel processing, thereby greatly reducing the execution time, and b) Size: Arithmetic can be performed on integers larger than those that can be ordinarily handled by a computer.

181)

182) On the properties of k-long numbers

[1]A. Delgado*, [2]M. Gargano, [1]M. Lewinter, [2]J. Malerba
[1]Purchase College, SUNY, [2]Pace University, New York

An integer is *oblong* if it can be written $n(n + 1)$ for some positive integer n. An integer is *k-long* if it is of the form $n(n + k)$, where k is nonnegative. A number can be k-long for several values of k. A k-long number is called *strictly k-long* if it is not j-long for any j satisfying $0 \leq j < k$. We present various theorems and generalizations.

183) The Structure of Row-Column Sorting Networks

Gordon Beavers*, A.Gregory Starling, University of Arkansas

We study the structure of the directed graph for the Row-Column parallel sorting network for a permutation of the integers $\{0, 1, 2, ..., k - 1\}$, that is, a network that sorts an element of the symmetric group S_k. Let $k = m \times n$, and shape an element of S_k into an $m \times n$ matrix of m rows and n columns. The row-column sort first sorts the m rows into a sorted up row followed by a sorted down row, continuing this pattern until all rows are sorted, and then the n columns are all sorted up. We refer to this procedure as a phase, Φ, of the network or algorithm. This procedure is repeated on the permutation for $\log(2m)$ times, and then a final sort up of all of the m rows. Define a digraph $G(S_k, E)$ whose set of vertices is S_k, and whose set of directed edges is defined as a relation E, where an ordered pair of vertices $<\pi,\sigma> \in S_k$ is an element of E if and only if $\Phi(\pi) = \sigma$, or the last row sort of the procedure maps π to the identity of S_k. Our purpose in this paper is to study the in-degree structure in this directed graph, and by so-doing show the structure of the row-column parallel sorting network or algorithm.

184) Minimal Cuts in Two-Terminal Directed Acyclic Graphs

Mark Korenblit*, HIT, Israel, Vadim E. Levit, AUC & HIT, Israel

The paper investigates minimal cuts (*mincuts*) in two-terminal directed acyclic graphs (*stdags*). A special st-dag characterized by a nested structure generated by its mincuts is called *nested*. We prove that every nested graph is series-parallel. We demonstrate that the minimum possible number of mincuts in an n-vertex st-dag is $n - 1$. Moreover, if an st-dag is non-series-parallel, then the number of its mincuts must be larger than $n - 1$. Our main observation is that an st-dag of order n has exactly $n - 1$ mincuts if and only if it is nested. It is shown that a nested graph can be obtained by a parallel composition of a nested graph and a single edge or by a series composition of nested graphs. Using the recursive structure of nested graphs, it is possible to present an algorithm for their recognition.

185)

186) Weighted Increasing Trees, Exponential Riordan Matrices and Moments

Paul Peart*, Wenjin Woan, Howard University, Barbara Tenkersley, North Carolina A & T State University

We give a combinatorial representation of the exponential generating functions of Riordan Matrices by Weighted Increasing Trees. We also give an explicit formula for the Stieltjes matrix of Riordan matrix. Then we use moments to count the total number of maximum decreasing chains among all permutations.

187) Efficient Isomorphism for Miyazaki Graphs

Greg Tener*, Narsingh Deo, School of EECS, University of Central Florida

Brendan McKay's software package *nauty* is widely used to solve the isomorphism problem for colored graphs. A colored graph's vertices are partitioned into distinct color classes. Two colored graphs are isomorphic if there exists an isomorphism between the graphs which preserves the colors of the vertices. In 1997, Takunari Miyazaki constructed a family of colored graphs which force *nauty* to run in exponential time. Miyazaki's construction demonstrates that the coloring significantly affects the behavior of the algorithm, and can turn out to be the difference between polynomial and exponential runtime. We present a modification to the canonical-labeling algorithm used by *nauty*. We employ discovered automorphisms to ensure that a canonical labeling is determined in polynomial time for Miyazaki graphs. These automorphisms can affect choices made during the course of execution; so we use a *guide tree* to change the search depending on our partial knowledge of the full automorphism group. The proposed modifications are not specific to Miyazaki graphs. Any graph with a non-trivial automorphism group can benefit from our approach.

188) On Component Order Edge Connectivity of a Complete Bipartite Graph

Daniel Gross, John Saccoman, Seton Hall University; L. William Kazmierczak, Charles Suffel*, Antonius Suhartomo, Stevens Institute of Technology

The traditional parameter used as a measure of vulnerability of a network modeled by a graph with perfect nodes and edges that may fail is the edge connectivity λ. Of course $\lambda(K_{p,q}) = p$ where $p \leq q$. In that case, failure of the network simply means that the surviving subgraph becomes disconnected upon the failure of individual edges. If, instead, failure of the network is defined to mean that the surviving subgraph has no component of order greater than or equal to some preassigned number k then the associated vulnerability parameter, the component order edge connectivity $\lambda_c^{(k)}$, is the minimum number of edges required to fail so that the surviving subgraph is in a failure state. We determine the value of $\lambda_c^{(k)}(K_{p,q})$ for arbitrary $p \leq q$ and k. As it happens, the situation is relatively simple when $p \leq \left\lfloor \frac{n}{k-1} \right\rfloor$ and more involved when $p > \left\lfloor \frac{n}{k-1} \right\rfloor$.

189)

190) Elements of S_n of order dividing a given number

Joshua Fallon*, Shanzhen Gao, Shaun Sullivan, Florida Atlantic University

Let $g(n, k)$ be the set of elements of the symmetric group S_n whose order divides k. We give some closed forms for $|g(n, k)|$ and for $|f(n, k)|$, the number of elements of S_n of order exactly k. We also consider the number of pairs of elements (x, y) of S_n X S_n of order dividing k.

191) Extracting communities in large real-world networks

Hemant Balakrishan*, Narsingh Deo, University of Central Florida

It has been observed that real-world random networks like the WWW, the Internet, neural network, social network, citation network, etc., organize themselves into closely-knit groups that are locally dense and globally sparse. These closely-knit groups are termed *communities*. Most real world networks of interest are enormous in size, varying from a few thousand vertices to a few billion. Extracting communities in such networks would help us mine valuable data embedded in their topology. Most of the existing algorithms are computationally expensive with cubic and quartic time complexities. Scaling these algorithms is a very challenging task. In this paper we present a faster algorithm employing bibliometric techniques that can be used to identify communities in large networks.

192) Reconstruction Numbers of Small Graphs

David Rivshin, Stanisław Radziszowski*, Rochester Institute of Technology

While the *Graph Reconstruction Conjecture* remains open it has spawned a number of related questions. In the vertex graph reconstruction number problem a vertex is deleted in every possible way from a graph G, and then it can be asked how many (both minimum and maximum) of these subgraphs are required to reconstruct G up to isomorphism. For some classes of graphs general answer is known, for others expensive computations are performed to find these numbers. This problem can be extended to k-vertex deletion and to k-edge deletion, for $1 \leq k$. Previous computer searches have computed the 1-vertex deletion reconstruction numbers of all graphs up to 10 vertices, as well as decided 2-vertex deletion reconstructibility of all graphs up to 9 vertices. In this talk we report the computation of reconstruction numbers for all graphs up to 11 vertices for 1-vertex and 1-edge deletion.

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193)

194)

195) A Dynamic Programming Model for k-packing and Other Invariants on Trees

Yiu Yu Ho*, Ronald D. Dutton, University of Central Florida

Many NP-Hard problems on graphs are solvable in polynomial time when restricted to trees. Dynamic programming approaches can be developed to solve a variety of these problems. While each problem requires a different formulation, they all seem to have common properties. In this work, we provide a unifying approach for designing solutions to determine several different graphical invariant values on trees. In particular, we show an $O(k^2n)$ dynamic programming solution for the k-packing problem for trees, which we have not encountered elsewhere.

196) On Some Conjectures About the Maximum Order of Induced Bipartite Subgraphs

Ermelinda DeLaVina*, Ryan Pepper, Bill Waller, University of Houston-Downtown

We discuss some conjectures about the maximum order of induced bipartite subgraphs in simple, connected graphs. First we show that this maximum order is at least twice the radius plus the maximum order of a claw, minus three. This settles a conjecture of the program Graffiti.pc. Using similar techniques, we show that the average distance is less than half the maximum order of an induced linear forest, plus one-half. This partially settles a conjecture of Hansen et al. Finally, we use a result of Gould et al. to show that if the maximum order of an induced bipartite subgraph equals twice the radius, then the graph has a Hamiltonian path, another conjecture of Graffiti.pc.

197)

198)

199) Efficient Intrusion-Detection using Programmable Agents based on Attack Graph Patterns

Mahadevan Vasudevan*, Narsingh Deo, University of Central Florida

Intrusion Detection Systems (IDS) are essential for security in a computer network infrastructure. The dynamic nature of such networks calls for a detection system that has the ability to handle intrusions with precision. Agent-based IDS provide the flexibility in handling such dynamic environments. But the existing techniques fail to satisfactorily address issues such as false positives and irrelevant alerts. Any prior hint of the possible attacks in a given network would serve as a great resource to maximize the accuracy of the alerts raised by the IDS. Recent work in network security focuses on the fact that combinations of exploits are the typical means by which an intrusion takes place. Attack graphs or attack trees provide a succinct way of representing the vulnerabilities and their corresponding attacks in a typical network. In an attack graph (or tree) different vulnerabilities in the system are represented as vertices, and a directed edge from one vertex to another denotes the possible transition taken by an intruder because of an exploit existing in the system. In this paper, we aim at providing an agent-based intrusion-detection architecture which uses the patterns provided by the attack graphs to generate alerts with reduced false positives.

200) Bipartite Graphs with No Isolated Vertices And k-Tuples Of Discrete Intervals

Vladimir Božović*, Daniel Socek and Shanzhen Gao, Florida Atlantic University, Boca Raton, Florida

Let $(I_1, I_2, \ldots, I_k)$ be a k-tuple of subintervals of the discrete interval $[1, n]$. The enumeration of k-tuples of intervals with fixed size of their intersection leads to interesting relation to bipartite graphs with no isolated vertices, certain classes of incidence matrices and some urn model distributions. We derive formula for certain type of numbers that appear to be of special importance in establishing connection between those topics.

201)

202)

203) Subcubes of the hypercube Q_n

A. Delgado, M. Lewinter,, Purchase College, SUNY, D. Gagliardi*, SUNY Canton, M.L. Gargano, Pace Univ., NYC, W. Widulski, Westchester Community College

The n-dimensional hypercube Q_n is defined recursively, by $Q_1 = K_2$ and $Q_n = Q_{n-1} \times K_2$. We show that if $d(x, y) = k < n$, then there is a unique copy of Q_k in Q_n containing x and y. When $d(x, y) = k < r < n$, we find the number of copies of Q_r in Q_n containing x and y and obtain additional similar results.

204) 4–cycle decompositions of the line graphs of complete multipartite graphs

C. A. Rodger, Nidhi Sehgal*, Auburn University

In this paper we investigate the necessary and sufficient conditions for the existence of 4-cycle decompositions of the line graphs of complete multipartite graphs.

217)

218) The 3-colored Ramsey Number of Even Cycles

Fabricio Siqueira Benevides*, Memphis University, Jozef Skokan, London School of Economics

Denote by $R(L, L, L)$ the minimum integer N such that any 3-coloring of the edges of the complete graph with N vertices K_N contains a monochromatic copy of a graph L. Bondy and Erdős conjectured that for an odd n-cycle C_n, $R(C_n, C_n, C_n) = 4n - 3$ for $n > 3$. This is sharp if true. Luczak proved that if n is odd, then $R(C_n, C_n, C_n) = 4n + o(n)$, as $n \to \infty$. Kohayakawa, Simonovits and Skokan proved that the exact Bondy-Erdős conjecture holds for sufficiently large values of n. Figaj and Luczak determined an assintotic result for the 'complementary' case where the cycles are even: they showed that for n even $R(C_n, C_n, C_n) = 2n + o(n)$ (Actually their result is much stronger than that because the cycles may be of slightly different sizes). Now we prove that there is n_0 such that for $n \geq n_0$ even $R(C_n, C_n, C_n) = 2n$.

219) If Vizing had lived in the Roman Empire

Robert R. Rubalcaba

V.G. Vizing conjectured that for all graphs G and H, $\gamma(G\Box H) \geq \gamma(G)\,\gamma(H)$, where $G\Box H$ denotes the Cartesian product of G and H and $\gamma(G)$ is the domination number of G. In the following the 2-packing number of G is denoted by $\gamma(G)$. It is known that Vizing's conjecture will hold if G satisfies at least one of the following: $\gamma(G) \leq 3$; $|\gamma(G)-\rho(G)| \leq 1$; G is chordal; G is the a spanning subgraph of K with $\gamma(G) = \gamma(K) = \chi(\overline{K})$. In this talk, we outline failed attempts to construct a counter-example to Vizing's conjecture. The iterated Mycielski construction of a graph (and other graphs with $|\gamma(G)-\rho(G)| > 1$) are investigated. Related bounds on the Roman domination number of the cartesian product are presented. Finally, the use of open-source software SAGE (http://www.sagemath.org) in this investigation will be discussed.

220) Seepage in directed acyclic graphs

Stephen Finbow, Saint Francis Xavier University

In the firefighting and the graph searching problems, a contaminate spreads relatively quickly. We introduce a new model, on directed acyclic graphs, in which the contamination spreads slowly. The model was inspired by the efforts to stem the lava flow from the Eldfell volcano. The contamination starts at a source, only one vertex at a time is contaminated and for some fixed k, k vertices are protected. The slowness is indicated by the name 'seepage'. The object is to protect the sinks of the graph. We show that if a sink of the graph can be contaminated then at most one directed path need be contaminated. We will investigate the Cartesian product of directed paths. This is joint work with N. E. Clarke, S. L. Fitzpatrick, M. E. Messinger and R. J. Nowakowski.

221)

222) The Generalized Chromatic Number of Graphs
Kathleen A. McKeon, Connecticut College

The *generalized chromatic number* $\chi(G,\neg H)$ of a graph G with respect to a subgraph H is the minimum number of colors needed to color the vertices of G such that no color class of vertices of G contains an induced copy of H. H is called a *forbidden subgraph.* We investigate the parameter $\chi(G,\neg H)$ for certain families of graphs G while forbidding complete graphs, K_n, cycles, C_n, and wheels, W_n.

223) Variants of a prize problem of Steve Hedetniemi
Peter Johnson, David Prier*, Auburn University; Matthew Walsh, Indiana-Purdue at Fort Wayne

The inverse domination number of a graph is the least number of vertices in a dominating set which is in the complement of a minimum dominating set; it is well defined if the graph has no isolated vertices. Some years ago it was noticed that the inventors of the inverse domination number, Kulli and Sigarkanti, had claimed that it is never greater than the vertex independence number, but their purported proof was invalid. At that time Steve Hedetniemi offered a prize—a copy of the large and expensive book Topics in Domination--to whomever could settle the matter. Here we consider approaches based on the question: When does a graph with no isolated vertices have a minimum dominating set whose complement in the vertex set contains an independent dominating set? (Or, more to the point, when does a graph have no such minimum dominating set?) In addition we survey what is known about 4 different fractional analogues of the problem.

224) Lower Bounds for the Numbers of SEG Labellings of Some Tree Families
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A simple and connected (p, q)-graph $G = (V, E)$ is called super edge-graceful (SEG), if there exists a bijection f from E to $L(q)$ such that the induced mapping f^* defined by $f^*(u) = \sum_{\{u,v\} \in E} f(u, v)$ is a bijection from V to $L(p)$, where $L(s) = \{z \in Z : |z| \leq s/2, z \neq 0\}$ when s is even, and $L(s) = \{z \in Z : |z| \leq (s-1)/2\}$ when s is odd. Let u_1-u_2-. . .-u_n be a path of length n. Let $T(n; (a_1, a_2, ., a_n))$ denote the tree obtained by amalgamating $u_k (1 \leq k \leq n)$ with the path of length a_k. It is conjectured that $T(2m + 3; (02^{2m+1}0))$ and $T(2m + 2; (02^{2m}1))$ are super edge-graceful, where the former stands for $T(n; (a_1, a_2, \ldots, a_n))$ when $n = 2m + 3$, $a_1 = a_{2m+3} = 0$, $a_2 = a_3 \ldots = a_{2m+2} = 2$, and the latter when $n = 2m+2$, $a1 = 0$, $a_{2m+2} = 1$, $a_2 = a_3 \ldots = a_{2m+1} = 2$. Let $N(G)$ denote the number of different SEG labellings of the graph G. We prove in this article the large lower bounds: $N(T(2m+3, (02^{2m+1}0))) \geq 2^3(m+1)\ m!$ and $N(T(2m+2, (02^{2m}1))) \geq 2^{3m+1}\ m!$. The super edge-gracefulness of $T(2m+3, (02^{2m+1}0))$ and $T(2m+2, (02^{2m}1))$ is an immediate consequence of these bounds.

225)

226) Optimal graphs for chromatic polynomials
Italo Simonelli, McDaniel College

Let $F(n, e)$ be the collection of all simple graphs with n vertices and e edges, and for $G \in F(n, e)$ let $P(G; \lambda)$ be the chromatic polynomial of G. A graph $G \in F(n, e)$ is said to be optimal if another graph $H \in F(n, e)$ does not exist with $P(H; \lambda) \geq P(G; \lambda)$ for all λ, with strict inequality holding for some λ. We discuss the problem of upper bounds for chromatic polynomials via the characterization of optimal graphs and present some recent results, open questions and conjectures.

227) Generalized Dominator Partitions of Graphs
Jobby Jacob*, Wayne Goddard, Clemson University

A vertex $v \in V$ in a graph $G = (V, E)$ *dominates* a set $S \subseteq V$ if it is adjacent to every vertex $w \in S$. A partition $\Pi = \{V_1, V_2, \ldots, V_k\}$ of $V(G)$ is called a *total dominator partition* if for every vertex $v \in V$ there exists $V_j \in \Pi$ such that $v \notin Vj$ and v dominates V_j. The *total dominator partition number* of a graph G, denoted $\pi_t(G)$, is the minimum order of a total dominator partition of G. Similar definitions of dominator partition parameters result from other dominating parameters. We discuss bounds for these parameters along with their properties.

228) Enumerating Labelled Graphs with Certain Neighborhood Properties
L.H. Clark, J.P. McSorley, and T.D. Porter, Southern Illinois University, S. Holliday*, University of Tennessee

Properties of (connected) graphs whose open or closed neighborhood families are Sperner, anti-Sperner, distinct or none of the proceeding have been extensively examined. We examine 24 properties of the neighborhood family of a graph in this paper. We give asymptotic formulas for the number of (connected) labelled graphs for 12 of these properties. For the other 12 properties, we give bounds for the number of such graphs. We also determine the status (a.a.s. or a.a.n.) in Gn, 1/2 of all 24 of these properties. Our methods are both constructive and probabilistic.

229)

230) Edge-colorings of cliques that forbid rainbow cycles
Adam Gouge, Truman University; Dean Hoffman and Peter Johnson*, Auburn University; Laura Nunley, Columbus State University; Luke Paben, Illinois College

A subgraph H of a graph G is rainbow in an edge coloring of G if all of H's edges are different colors. An edge coloring of G forbids rainbow blurgum if no instance of blurgum in G is rainbow. In this paper we do the following. 1. Characterize the edge colorings of the complete graph K(n) with n -1 colors that forbid rainbow triangles (and thus all rainbow cycles). 2. Show that for any edge coloring of K(n), n > 1, for which rainbow triangles are forbidden, the edges of some color class must induce a connected spanning subgraph of K(n). 3. Show that there is an edge coloring of K(n), n > 1, with t colors forbidding rainbow triangles in which the color frequencies are as nearly equal as possible if and only if t is a positive integer no greater than (n + 1)/2. 4. Show that the maximum number of colors that can be used in a rainbow-triangle-forbidding edge coloring of K(n)^z, the multigraph obtained by multiplying each edge of K(n) by z, is the largest integer no greater than n - 1 + (z -1)(n/2), and characterize those colorings. 5. Show that the maximum number of colors that can be used in an arc coloring of the complete directed graph on n vertices forbidding rainbow transitive triples is the largest integer no greater than n - 1 + (n/2), but that one can forbid rainbow cyclic triples using around (7/4)n colors.

231) Total Well-Dominated Trees
Arthur Finbow*, Saint Mary's University, Allen Frendrup, Preben Dahl Vestergaard, Aalborg University

Let $G = (V, E)$ be a graph with no isolated vertex. A set D of vertices is called a *total dominating set* for G if each vertex in G is adjacent to a vertex in D. If all minimal total dominating sets in G have the same cardinality then G is said to be a *total well-dominated graph*. In this work we study the total well-dominated trees.

232) Color-Permuting Automorphisms of Cayley Graphs
Elizabeth McMahon, Lafayette College, et.al.

Let G be a finite group with generators Δ. The full automorphism group of the Cayley color digraph $Aut(Cay_\Delta(G))$ has several significant subgroups, including the color-preserving automorphisms $Aut_G(Cay_\Delta(G))$ (isomorphic to G) and the color-permuting automorphisms $Aut_P(Cay_\Delta(G))$ (that send all edges corresponding to a given generator to edges corresponding to another generator). These subgroups have been studied in various contexts, and we continue that study with another way of looking at these subgroups. We also look at other subgroups that illuminate the structure of these groups.

233)

234) On the Minimal Order of k–chromatic K_{r+1}–free Graphs

Anja Kohl, Technical University Chemnitz, Germany

Let $n_r(k)$ denote the smallest possible number of vertices of a graph G with chromatic number $\chi(G) = k$ and clique number $\omega(G) \leq r$. It is known that $n_2(k)$ has order of magnitude $k^2 \cdot \log k$. Moreover, for $k \leq 5$ the exact values for $n_2(k)$ are already determined: $n_2(2) = 2$, $n_2(3) = 5$, $n_2(4) = 11$ and $n_2(5) = 22$. Only few is known about the case r $\geq$ 3, e.g. $n_3(5) = 11$ that was verified by Jensen and Royle through computer search. For convenient values of r and k one can obtain upper bounds on $n_r(k)$ by considering circulant graphs, Ramsey graphs, or joins of complete graphs and odd cycles. In this talk we will determine relations between the four values $n_r(k)$, $n_{r+1}(k)$, $n_r(k+1)$ and $n_{r+1}(k+1)$ for general k and r. After this we will focus on integers k and r such that $3r \geq 2k$. For these cases we will give lower and, by construction of appropriate graphs, upper bounds on $n_r(k)$. At last we will consider the case $k = r + i$ for small integers i. So far we know the exact value of $n_r(r + i)$ for $i \leq 10$ and $r \geq \min\{2i, 8\}$.

235) On the Independent Domination Number of a Random Graph

Darin Johnson*, Lane Clark, Southern Illinois University

We prove a two point concentration for the independent domination number of the random graph $G_{n,p}$ provided $p^2 \ln(n) \geq 64\ln((\ln n)/p)$.

236) Network Communities Based on Maximizing Average Degree

James M. Scanlon*, Narsingh Deo, University of Central Florida

In the last decade, researchers have realized that graph models for an apparently unrelated group of networks share some structural properties. For example, network models of coupled oscillators, protein interaction networks, the Internet router graph, the world wide web, have all been found to have heavy-tailed degree distributions. Moreover, the sparse graphs of these network models are locally dense. Neither the degree distribution nor the local density would be expected based on the classical Erdös-Rényi theory of random graphs. A new definition for communities in general complex networks is proposed and several of its characteristics are developed. The proposed definition captures the locally dense yet globally sparse nature of non-random structures found in these networks. Example networks illustrate the proposed definition and provide contrast with existing definitions.