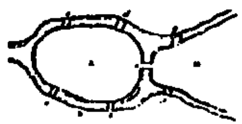
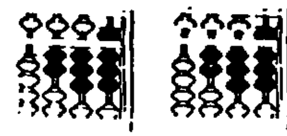
TWENTY-EIGHTH SOUTHEASTERN INTERNATIONAL CONFERENCE



ON



COMBINATORICS GRAPH THEORY COMPUTING

Florida Atlantic University

March 3-7, 1997

CRC PRESS, INC.

Monday, March 3, 1997

	Live Oak Pavillion			Senate Chambers
	Room A	Room BC	Room D	Room 230 UC
8:00 am	Registration until 5:00, downstairs lobby of the University Center (UC), where coffee will be served.			
9:00 am	Opening Ceremonies: FAU President Anthony J. Catenese FAU Provost Richard Osburn FAU College of Science Dean John Wiesenfeld			
9:30 am	Golumbic			
10:30 am	Coffee			
10:50 am	001 Gilpin	002 K.L. Williams	003 Grimaldi	004 K.B. Reid
11:10 am	005 Parker	006 Y.-L. Lai	007 Nostrand	008 Kemnitz
11:30 am	009 Wilmer	010 Carrington	011 P. Zhang	012 Seager
11:50 am	013 Corteel	014 P. Lam	015 Rasmussen	016 Bowser
12:10 pm	017 J. Arkin	018 E. Hare	018 X. Lu	020 Szwarcfiter
12:30 pm	Lunch			
2:00 pm	Golumbic			
3:00 pm	Coffee			
3:20 pm	021 Francel	022 McColm	023 Merz	024 McMorris
3:40 pm	025 Laue	026 Dillon	027 Kennedy	028 Wojciechowska
4:00 pm	029 Wasserman	030 P. Johnson	031 Shull	032 Opatrny
4:20 pm	033 Radziszowski	034 Eggleton	035 S.-R. Kim	036 Przulj
4:40 pm	037 Bluskov	038 Winters	039 Langley	040 Lipman
5:00 pm	041 Kirkpatrick	042 Kokkinos	043 Cho	044 Stueckle
5:20 pm	045 Blake-Wilson	046 Santoro	047 McKenna	048 Qiu
5:40 pm	049	050	051	052
6:00 pm	Transportation back to motels (returns for reception about 6:15)			
6:15 pm	Reception, Board of Regents Room, 3rd floor, Administration Building			
	Transportation back to motels (TBA)			

Tuesday, March 4, 1997

	Room A	Room BC	Room D	Room SC
8:15 am	Registration 8:15-noon and 1:30-4:00, UC, 2nd floor lobby			
8:40 am	053 Albertson	054 Lundgren	055 Kountanis	056 Miyamoto
9:00 am	Book Exhibits 9:00-5:00, Room 229			
9:00 am	057 Simoson	058 Dunbar	059 Kinnersley	060 Sullivan
9:30 am	Mynhardt			
10:30 am	Coffee			
10:50 am	061 D.G. Hoffman	062 Henning	063 Kaneko	064 Killgrove
11:10 am	065 Jaromczyk	066 Levit	067 Dean	068 Grant
11:30 am	069 Hattingh	070 Klostermeyer	071 McRae	072 Ostergard
11:50 am	073 Rinker	074 Laskar	075 McCreary	076 Tanny
12:10 am	077 Fowler	078 Jimenez	079 Kayll	080 Ruskey
12:10 pm	Lunch			
2:00 pm	Mynhardt			
3:00 pm	Coffee			
3:20 pm	081 Gimbel	082 Fraughnaugh	083 Hurlbert	084 Boling
3:40 pm	085 Rosenfeld	086 Hedetniemi	087 Molina	088 Liatti
4:00 pm	089 Juvan	090 Hartnell	091 Bandlow	092 Mishima
4:20 pm	093 Sheng	094 Szabó	095 Bohman	096 Raines
4:40 pm	097 Litiu	098 Parks	099 Liang	100 Di Paola
5:00 pm	101 Gao	102 Markus	103 W.N. Li	104 Haynsworth
5:20 pm	105 R.B. Gardner	106 Domke	107 Rogers	108 Leonard
5:40 pm	109	110	111	112
5:45 pm	Transportation leaves for motels, returns 6:30 to Conference party			
5:50 pm	Transportation leaves UC for Conference party			
6:00 pm	Conference Party, Jack Freeman's home, 741 Azalea St., 395-7921			
	As always, we urge walking or car-pooling, especially with parking spaces scarce near Freeman's. The main parking for the party, the Art Museum of Boca Raton has 25 spaces on the north side of the parking lot only.			
	Transportation back to motels.			

Wednesday, March 5, 1997

	Room A	Room BC	Room D	Room SC
8:15 am	Registration 8:15-noon and 1:30-4:00, UC, 2nd floor lobby			
8:30 am	113	114 Beasley	115 Pekec	116 J. Wu
8:50 am	117 Shareshian	118 Donovan	119 Bergstrand	120 W. Gu
9:00 am	Book Exhibits 9:00-5:00, Room 229			
9:10 am	TICA Session			
9:30 am	Cook			
9:30 am	Coffee			
10:50 am	121 Gordon	122 Cáceres	123 Pinter	124 L.L. Gardner
11:10 am	125 Ahrens	126 Rentería	127 H. Kim	128 Myrvold
11:30 am	129 T.J. Reid	130 Czabarka	131 Kelmans	132 Valdes
11:50 am	133 Gottlieb	134 Carella	135 Cherry	136 Jones
12:10 pm	Conference Photograph			
12:30 pm	Lunch			
2:00 pm	Cook			
3:00 pm	Coffee			
3:20 pm	137 VanderJagt	138 Dobson	139 Slater	140 McLaurin
3:40 pm	141 Guichard	142 Alspach	143 Chartrand	144 Keedwell
4:00 pm	145 Thurmann	146 Starling	147 T.-M. Wang	148 Lewis
4:20 pm	149 Cribb	150 Qin	151 Jamison	152 Mays
4:40 pm	153 Plummer	154 Morris	155 Weinreich	156 Wojciechowski
5:00 pm	157 Anderson	158 J. Liu	159 Savage	160 Hilton
5:20 pm	161 Beard	162 H.-L. Lai	163 Sajna	164 Harborth
5:45 pm	Transportation leaves for motels, returns 6:15 from motels and U.C. to Sheraton for banquet.			
7:00 pm	Conference Banquet at Sheraton. (happy hour runs until 6:30, seating at 6:45.)			
	Transportation back to motels.			

Thursday, March 6, 1997

	Room A	Room BC	Room D	Room SC
8:15 am	Registration 8:15-noon and 1:30-4:00, UC, 2nd floor lobby			
8:40 am	165	166	167	168
9:00 am	Book Exhibits 9:00-5:00, Room 229			
9:00 am	169	170	171	172
9:30 am	Graham			
10:30 am	Selfridge			
10:50 am	Faudree			
11:10 am	Coffee			
11:30 am	173 Gassko	174 J.L. Arkin	175 Lazebnik	176 Bartha
11:50 am	177 Gavlas	178 Soifer	179 P. Yiu	180 C.-S. Peng
12:10 pm	181 Soltes	182 Pfaltz	183 Moon	184 Macula
12:30 pm	Lunch			
2:00 pm	185	von Haeseler	187	188
2:20 pm	189		191	192
2:40 pm	193		195	196
3:00 pm	Coffee			
3:20 pm	197 Gargano	198 C. Wallis	199 Roelants v. B.	200 Krzyzak
3:40 pm	201 Suchenek	202 McKee	203 Finizio	204 Rall
4:00 pm	205 Salzberg	206 Turgeon	207 Kennedy	208 S.-M. Lee
4:20 pm	209 Altman	210 W.C. Shiu	211 Niederhausen	212 T. Johnson
4:40 pm	213 Gross	214 Trenk	215 Goldwasser	216 Fisher
5:00 pm	217 Suffel	218 Ales	219 George	220 Spalding
5:20 pm	221 West	222 Chopra	223 Pagli	224 Shekhtman
5:40 pm	225	226	227	228
5:45 pm	Transportation leaves for motels, returns 6:15 from motels to informal party.			
6:00 pm	Informal conference party until 7:30 in Cafeteria paio area.			
	Transportation back to motels.			

Friday, March 7, 1997

	Room A	Room BC	Room D	Room SC
8:15 am	Registration 8:15-noon and 1:30-3:00, UC, 2nd floor lobby			
8:40 am	229	230	231	232
9:00 am	Book Exhibits 9:00-11:30, Room 229			
9:00 am	233 Sprague	234 Watkins	235 Myers	236
9:30 am	Propp			
10:30 am	Coffee			
10:50 am	237 Reetz	238 Chinn	239 Kubale	240 Rodriguez
11:10 am	241 J. Williams	242 Pachter	243 Alsardary	244 C.C. Lim
11:30 am	245 Jonoska	246 Dreyer	247 Sarvate	248 Cropper
11:50 am	249 Kiaer	250 Wilson	251 Perkel	252 Finbow
12:10 pm	253 Egecioglu	254 Meyerowitz	255 Sritharan	256
12:30 pm	Lunch			
2:00 pm	Propp			
3:00 pm	Coffee			
3:20 pm	257 Waxman	258 Rajan	259 Oellermann	260 Piwakowski
3:40 pm	261 Goldberg	262 N. Kumar	263 Fitzpatrick	264 Jayawardene
4:00 pm	265 Sampath	266 L.-J. Mao	267 J. Shen	268 Moghadam
4:20 pm	269	270	271	272
4:40 pm	273	274	275	276
5:00 pm	277	278	279	280
	Transportation TBA.			
8:00 pm	informal after dinner SURVIVORS' PARTY at the home of Aaron Meyerowitz and Andrea Schuver, 454 NE Third Street. Tell us if you need transportation.			
	Transportation back to motels.			

Invited talks

William Cook, Rice University, *Branch-Width and the Traveling Salesman Problem & Computing Optimal Matchings*.
Wednesday, 9:30 am and 2:00 pm.

R. Faudree, *Paul Erdös and his Mathematics*.
Thursday, 10:50 am.

R. Graham, *Paul Erdös and his Mathematics*.
Thursday, 9:30 am.

Martin C. Golumbic, Bar-Ilan University , *Algorithmic Graph Theory in Temporal Reasoning*.
Monday, 9:30 am and 2:00 pm.

Friedrich von Haeseler, Bremen, *On the Automaticity of Rational Functions*.
Thursday, 2:00 pm.

Christine M. Mynhardt, University of South Africa, *Irredundance in Graphs: An Introduction & Critical Concepts in Domination Theory*.
Tuesday, 9:30 am and 2:00 pm.

James Propp, M.I.T, *When Can You Tile? & How Many Tilings?* .
Friday, 9:30 am and 2:00 pm.

John Selfridge, *Paul Erdös and his Mathematics* .
Thursday 10:30 am.

Irredundance in graphs: An introduction

Christine M. Mynhardt, University of South Africa

A vertex subset S of a graph $G = (V, E)$ is said to be irredundant if every vertex $s \in S$ is either isolated in $G[S]$or has a neighbour $s' \in V - S$ such that s' is not adjacent to any other vertex in S. The concept of irredundance is closely related to that of domination in the sense that the property of irredundance is a criterion for a dominating set to be minimal dominating.

In this introductory talk we will attempt to convey an intuitive feeling for the concept of irredundance. The relationships between domination, independence and irredundance in graphs will be discussed and bounds for the upper and lower irredundance numbers will be given. Other concepts such as irredundant Ramsey numbers, irredundance perfect graphs and irredundant sets on chessboards will also be mentioned. No previous knowledge of these concepts will be assumed.

Critical concepts in domination theory

Christine M. Mynhardt, University of South Africa

We discuss graphs which are critical with respect to the parameters associated with domination, independence and irredundance in any of the following senses: the parameter increases when any edge is deleted, decreases when any edge is added, or increases (respectively decreases) when any vertex is deleted. Of these, the best known case is that of domination critical graphs; i.e., graphs for which the domination number decreases when any edge is added to the graph. We discuss recent results concerning two long-standing conjectures on such graphs; namely, that their independent domination numbers equal the domination numbers and that such graphs with domination number equal to three are hamiltonian.

Contributed talks (alphabetical list of authors)

A. Abdalla 262
C. Ahrens 125
M.O. Albertson 053
J. Ales 218
S.Y. Alsardary 243
B. Alspach 142
T. Altman 209
M.S. Anderson157
K.M. Anstreicher 241
J. Arkin 017
J.L. Arkin 174
D.C. Arney 017, 174
N. Azam 055

E. Babson 117
O.B. Bak 035
J. Bandlow 087, 091
E. Barbeau 076
L. Barowski 075
M. Bartha 176
J.T.B. Beard Jr. 161
L.B. Beasley 114
D. Bergstrand 119
D.R. Berman 140
J.W. Berry 067
A. Betten 025
T. Biedl 246
A. Björner 117
S. Blake-Wilson 045
I. Bluskov 037
A.J. Boals 137
F. Boesch 213, 217
T. Bohman 095
J.E. Bolin 129
P. Boling 084
S. Bowser 016
R.C. Brigham 010, 135

C. Cable 016
A. Cáceres 122
E.R. Canfield 013
N.A. Carella 134
J. Carrington 010
G. Chartrand 137, 143
K. Cherry 135
J. Chew 076
P.Z. Chinn 238
H.H. Cho 043, 054
D.V. Chopra 222
S. Chow 080
D. Clark 148
L. Clark 219
T. Clarke 083
S. Corteel 013
D.W. Cribb 149
M.M. Cropper 248
É. Czabarka 130

P. Dankelmann 259
N. Dean 067
N. Deo 258, 262, 266
M. Dillon 026
J.W. Di Paola 100
E. Dobson 138
G.S. Domke 102, 106
T.P. Donovan 118
A.D. Dorris 161
P. Dreyer 246
J. Dunbar 058
J.E. Dunbar 069, 106
R.D. Dutton 010

O. Egecioglu 253
R. Eggleton 034
L. Eroh 143
E. Eschen 255

M. Fampa 241
L. Fern 121
A.S. Finbow 252
N.J. Finizio 203
D.C. Fisher 082, 216, 220
S.L. Fitzpatrick 263
P. Flocchini 046
T. Fowler 077
M. Francel 021, 247
K. Fraughnaugh 082
R. Fuji-Hara 056

S. Gao 101
L.L. Gardner 124
R.B. Gardner 027, 105
M.L. Gargano 197, 207
I. Gassko 173
H. Gavlas 177
J. George 219
K. Giaro 239
M. Gilpin 001
J. Gimbel 081
W.D. Goddard 062, 259
R. Goldberg 257, 261
J. Goldwasser 070, 215
W.L. Golightly 104
G. Gordon 121, 125
E. Gottlieb 133
C. Grant 068
J.R. Griggs 004
R.P. Grimaldi 003
D. Gross 213, 217
W. Gu 120
D. Guichard 141

H. Harborth 164
E. Hare 018
B. Hartnell 086, 090, 204, 252
J.H. Hattingh 069
T.W. Haynes 058,106
R.B. Hayward 255
S.M. Hedetniemi 086
S.T. Hedetniemi 074, 086
M.A. Henning 058, 062
A.J.W. Hilton 030, 156, 160, 248
R. Hochberg 083
D.G. Hoffman 005, 041, 061, 073, 088
C. Huff 027
G. Hurlbert 083

M. Ibel 253
R. Ichishima 055

D.P. Jacobs 074
R.E. Jamison 151
J.W. Jaromczyk 065
C.J. Jayawardene 264
M. Jimbo 092
G. Jimenez 078
P.D. Johnson Jr. 030, 061
T. Johnson 212
D. Jones 136
N. Jonoska 245
M. Juvan 089

Y. Kaneko 063
S.A. Karl 245
P.M. Kayll 079
A.D. Keedwell 144
A. Kelmans 131
A. Kemnitz 008
J. Kennedy 027
J.W. Kennedy 207
J.D. Key 060
L. Kiaer 249
R.B. Killgrove 064
H. Kim 127
S.-R. Kim 035, 043, 054
N. Kinnersley 059
K.S. Kirkpatrick 041
S. Klasa 200
J. Klerlein 198
W. Klostermeyer 235
D.J. Knisley 106
K. Kokkinos 042
M.C. Kong 059
W. Kostermeyer 070
D. Kountanis 042, 055
C. Krattenthaler 211
A. Krzyzak 200
M. Kubale 239
N. Kumar 258, 262, 266
C.-L. Kwok 208

H.-J. Lai 181
H.-L. Lai 162
Y.-L. Lai 006
P.C.B. Lam 014, 210
S.-D. Lang 266
L. Langley 023, 039
R. Laskar 074
R. Laue 025
F. Lazebnik 175
J. Leasure 121
J. Lee 241
S.-M. Lee 208
P.A. Leonard 108
V.E. Levit 066
J.T. Lewis 148
W.N. Li 103
Y.D. Liang 099
M. Liatti 088
A. Liestman 036
A.L. Liestman 032
C.C. Lim 244
Y.X. Lin 014
C.C. Lindner 084
M. Lipman 040
R. Litiu 097
J. Liu 158
F. LoSacco 197
X. Lu 019
F. Luccio 223

J.R. Lundgren 023, 039, 047, 054, 078

J. MacDougall 034
B. MacKay 128
E. MacMahon 125
A.J. Macula 184
N.V.R. Mahadev 147
E. Mandrescu 066
L.-J. Mao 266
B. Maohar 089
L.R. Markus 102, 106
M.E. Mays 152
G. McColm 022
C. McCreary 075
T. McKee 202
P. McKenna 047
S.C. McLaurin 140
J. McNulty 129
A.A. McRae 069, 071, 098
F.R. McMorris 015, 024
A. Meir 183
S. Mertz 023, 039
A. Meyerowitz 254
M. Mishima 092
N. Miyamoto 056
H.S. Moghadam 268
R. Molina 087, 091
J.W. Moon 183
J. Morris 154
J.G. Moser 003
C. Myers 235
W. Myrvold 128

Y. Nam 035, 043
H. Niederhausen 211
B. Nostrand 007
J. Nouh 137
B. Novick 060, 101
R.J. Nowakowski 263
L. Nowitz 142
K.J. Nurmela 072

O.R. Oellermann 259
D.R. Oliver 238
J. Opatrny 032
P.R.J. Ostergard 072

L Pachter 242
L. Pagli 223
C.A. Parker 005
D.A. Parks 098
R. Pargas 071
A. Pekec 115
C.-S. Peng 180
M. Perkel 251
L. Petingi 240
J. Pfaltz 182
K.T. Phelps 045
D. Pillone 074
M. Pinter 123
K. Piwakowski 260
M.D. Plummer 153
T. Porter 219
S. Pronchik 121
N. Przulj 036

Y. Qin 150
K. Qiu 048, 101

S. Radziszowski 033
J. Raglin 086
M. Raines 096
K. Rajan 258
D. Rall 086, 204
C.W. Rasmussen 015, 023, 039
D. Reetz 237
K.B. Reid 004, 024
T.J. Reid 129
C. Rentería 126
R.B. Richter 157
S.S. Rinker 073
F.S. Roberts 044, 093
C.A. Rodger 068, 096, 160
P. Rodney 157
A. Rodríguez 205
J. Rodriguez 240
D. Roelants van Baronaigien 199
R.O. Rogers 107
T.D. Rogers 218
A. Roncato 046
M. Rosenfeld 085
C.C. Rousseau 264
F. Ruskey 080

M. Sajna 163
P.M Salzberg 205
G. Sampath 265
N. Santoro 046
D.G. Sarvate 247
C.D. Savage 013, 159
M. Schultz 143
S. Seager 012, 082
J. Shapiro 257, 261
J. Shareshian 117
B. Shekhtman 224
J. Shen 267
L. Sheng 093
W. Shi 221
W.C. Shiu 014, 210
R. Shull 031
A. Simoson 057
J. Siran 046
R. Skrekovski 089
P.J. Slater 069, 139
D.D. Smith 140
C. Soares 018
A. Soifer 178
L. Soltes 181, 235
A. Spalding 220
J. Spinrad 255
S. Sportsman 198
A. Sprague 127, 233
R. Sritharan 255
A.G. Starling 146
S.W. Sternfeld 064
S. Stueckle 044
M.A. Suchenek 201
C. Suffel 213, 217
F.E. Sullivan 060
H.C. Swart 259
T. Szabó 094
J.L. Szwarcfiter 020

T. Tadao 233
R.G. Tamez 064
S. Tanny 076
H. Tapia-Recillas 126
T. Terry 262
C. Thomassen 081
C. Thurmann 145
A.N. Trenk 031, 214
J.M. Turgeon 206

L. Valdes 132
D.W. VanderJagt 137
R. Venkatraman 159
P.D. Vestergaard 090

M. Wachs 133
C. Wallis 198
D.-W. Wang 019
P. Wang 175
T.-M. Wang 147
A. Wasserman 025, 029
J.J. Watkins 234
J. Waxman 257, 261
D. Weinreich 155
V. Welker 117
D.B. West 221
J. Williams 241
K.L. Williams 002
E.L. Wilmer 009
D.B. Wilson 250
S.J. Winters 038
A. Woo 119
I. Wojciechowska 028
J. Wojciechowski 156
C.K. Wong 019
J. Wu 116

J. Yates 137
P. Yiu 179

M. Zaragoza 032
P. Zhang 011, 143

1

Klee's Identity and Vandermonde's Identity as Bilinear Forms

Michael Gilpin, Michigan Technological University

Klee's Identity and Vandermonde's Identity are interpreted as bilinear forms on rows and diagonals of Pascal's Triangle. Using elementary algebra and matrix theory we prove these two identities—as well as a third identity on diagonals of Pascal's Triangle.

Key words: **Combinatorial Identities, Klee's Identity, Vandermonde's Identity**.

Some bounds on bandwidth, edgesum, and profile of graphs

2

Kenneth L. Williams (williams@cs.wmich.edu)
Western Michigan University

Given graph $G = (V, E)$ of order $n = |V|$ and size $k = |E|$ we consider the parameters: bandwidth of G, $B(G)$; edgesum of G, $s(G)$; and profile of G, $P(G)$. It is well known that the decision problems associated with determining each of these parameters for arbitrary graphs are NP-complete. We consider each of these parameters as functions of n and k. By starting with K_n and removing edges we obtain insight into these parameters. We also obtain a result on the existence of graphs with a given size and a given profile.

Key words: graph, bandwidth, edgesum, profile.

3

Tennis Balls and the Catalan Numbers

Ralph P. Grimaldi*, Rose-Hulman Institute of Technology
and
Joseph G. Moser, Westchester University of Pennsylvania

For $n \geq 1$ consider $2n$ tennis balls numbered 1, 2, 3,..., $2n$. During each of n equal time intervals the following occurs. For $1 \leq i \leq n$, during time interval i, the tennis balls numbered $2i - 1$ and $2i$ are placed in a box and then one ball in this box (containing $i + 1$ balls) is removed. Keeping track of the numbers on the tennis balls removed, one obtains a subset of size n from $\{1, 2, 3, ..., 2n\}$. Further, the number of such subsets of size n turns out to be c_n, the n - th Catalan number. In addition, for $n \leq j \leq 2n$, we examine other results such as $M(n, j)$, the number of these subsets of size n where j is the largest element.

Keyword: Catalan Numbers

4 **Landau's Theorem on Score Sequences Revisited**

Jerrold R. Griggs, Univ. of S. Carolina, K. B. Reid*, Calif. St. Univ. San Marcos

Two new elementary proofs have been found for Landau's Theorem on necessary and sufficient conditions for a sequence of integers to be the score sequence for some tournament. The proof to be discussed in this talk is related to existing proofs by majorization, but it avoids depending on any facts about majorization. The second proof is natural and direct, but a bit more basic than existing proofs. Both proofs are constructive, so they each provide an algorithm

for obtaining a tournament realizing a sequence satisfying Landau's conditions. (The research of the first author was supported in part by NSA Grant MSPF-94S- 202; the research of the second author was supported in part by ONR Grant N00014-92-J-1400.)

Key Words: tournament, score, score sequence

1

Klee's Identity and Vandermonde's Identity as Bilinear Forms

Michael Gilpin, Michigan Technological University

Klee's Identity and Vandermonde's Identity are interpreted as bilinear forms on rows and diagonals of Pascal's Triangle. Using elementary algebra and matrix theory we prove these two identities—as well as a third identity on diagonals of Pascal's Triangle.

Key words: Combinatorial Identities, Klee's Identity, Vandermonde's Identity.

2

Some bounds on bandwidth, edgesum, and profile of graphs

Kenneth L. Williams (williams@cs.wmich.edu)
Western Michigan University

Given graph $G = (V, E)$ of order $n = |V|$ and size $k = |E|$ we consider the parameters: bandwidth of G, $B(G)$; edgesum of G, $s(G)$; and profile of G, $P(G)$. It is well known that the decision problems associated with determining each of these parameters for arbitrary graphs are NP-complete. We consider each of these parameters as functions of n and k. By starting with K_n and removing edges we obtain insight into these parameters. We also obtain a result on the existence of graphs with a given size and a given profile.

Key words: graph, bandwidth, edgesum, profile.

3

Tennis Balls and the Catalan Numbers

Ralph P. Grimaldi*, Rose-Hulman Institute of Technology
and
Joseph G. Moser, Westchester University of Pennsylvania

For $n \geq 1$ consider $2n$ tennis balls numbered 1, 2, 3,..., $2n$. During each of n equal time intervals the following occurs. For $1 \leq i \leq n$, during time interval i, the tennis balls numbered $2i - 1$ and $2i$ are placed in a box and then one ball in this box (containing $i + 1$ balls) is removed. Keeping track of the numbers on the tennis balls removed, one obtains a subset of size n from $\{1, 2, 3, ..., 2n\}$. Further, the number of such subsets of size n turns out to be c_n, the n - *th* Catalan number. In addition, for $n \leq j \leq 2n$, we examine other results such as $M(n, j)$, the number of these subsets of size n where j is the largest element.

Keyword: Catalan Numbers

4 **Landau's Theorem on Score Sequences Revisited**

Jerrold R. Griggs, Univ. of S. Carolina, K. B. Reid*, Calif. St. Univ. San Marcos

Two new elementary proofs have been found for Landau's Theorem on necessary and sufficient conditions for a sequence of integers to be the score sequence for some tournament. The proof to be discussed in this talk is related to existing proofs by majorization, but it avoids depending on any facts about majorization. The second proof is natural and direct, but a bit more basic than existing proofs. Both proofs are constructive, so they each provide an algorithm

for obtaining a tournament realizing a sequence satisfying Landau's conditions. (The research of the first author was supported in part by NSA Grant MSPF-94S- 202; the research of the second author was supported in part by ONR Grant N00014-92-J-1400.)

Key Words: tournament, score, score sequence

5

Bipartite Path Decompositions

D. G. Hoffman and Carol A. Parker*
Auburn University

We investigate partitioning the edges of the complete bipartite graph $K_{a,b}$ into paths of length k.

Key Words: Path decomposition, complete bipartite graph

6

Bandwidth, Edgesum and Profile of Network Architecture Graphs

Yung-Ling Lai (ylai@cs.wmich.edu)
Western Michigan University

For graphs in general it is well known that the decision problems associated with finding bandwidth, edgesum and profile are all NP-complete, but for several of the common network architectures these values have been established. Among the widely used architectures are the butterfly and the hypercube. This paper establishes the bandwidth of the butterfly and provides polynomial algorithms to number the vertices of this architecture to achieve edgesum and profile. A polynomial algorithm to number the vertices of the hypercube to achieve profile is introduced.

Key words: bandwidth, edgesum, profile, butterfly, hypercube.

7

Uniform Polytopes

B. Nostrand, de Moivre Institute

Notions of geometric symmetry have occupied the attention of mathematicians since the time of the ancient Greeks. Great strides have been made by Coxeter and others in understanding the combinatorial properties of such figures. Recently, Ludwig Danzer suggested a program for unifying the theory of polytopes with the theory of buildings. Uniform polytopes are geometric objects with vertex-transitive 1-skeletons. These objects, introduced by Martini and developed by Johnson and Weiss, generalize the notion of local geometric symmetry. We examine properties of uniform polytopes which further Danzer's program of Incidence Complexes.

8

Score Sequences of Multitournaments

Arnfried Kemnitz, Techn. Univ. Braunschweig, Germany

A tournament T_n is a digraph of order n such that each pair of distinct vertices v_i and v_j is joined by exactly one of the arcs (v_i, v_j) and (v_j, v_i). A multitournament T_n^k is a digraph of order n such that each pair of distinct vertices v_i and v_j is joined by exactly k of the arcs (v_i, v_j) and (v_j, v_i), $k \geq 1$ ($T_n^1 = T_n$ by definition). A sequence $(s_1, s_2, ..., s_n)$ of nonnegative integers is called a score sequence of a multitournament if there exists a multitournament T_n^k whose vertices can be labeled as $v_1, v_2, ..., v_n$ such that the outdegree of the vertex v_i equals s_i for $i = 1, 2, ..., n$. For tournaments Landau ('53) gave a necessary and sufficient condition for a sequence to be a score sequence which was generalized by Moon ('63) for multitournaments. We sketch a new, much easier proof of this result.
Let $s(n)$ denote the number of different score sequences $(s_1, s_2, ..., s_n)$ of tournaments such that $s_1 \leq s_2 \leq ... \leq s_n$. An explicit formula for $s(n)$ is not known but some estimations for the order of magnitude are proved. We give some bounds for the number $s(n, k)$ of different nondecreasing score sequences of multitournaments T_n^k.

Random Walks on Simple Digraphs

E. L. Wilmer, Harvard University

9

Studying the convergence to stationarity of random walks on graphs has many applications; for instance, to Monte Carlo algorithms based on random walks. There are extensive results for undirected graphs and for highly symmetric graphs. Using estimates from probability, we give a detailed description of the convergence to stationarity of random walks on certain simple families of digraphs—ones to which previous methods do not apply. Let $X_{m,n}$ consist of a directed m-cycle and a directed n-cycle joined at a single vertex. Let $L_{n,k(n)}$ be a directed n-cycle with self-loops attached to $k(n)$ vertices. For $X_{m,n}$, the total number $m+n$ of vertices determines the convergence behavior. For $L_{n,k(n)}$, only the number $k(n)$ of loops matters, not their positions. We describe the distributions these walks pass through on the way to stationarity. The limits of these distributions prior to stationarity are the same as those for random walk on an (undirected) cycle; they satisfy a heat equation. For $\{X_{m,n}\}$ and $\{L_{n,k(n)}\}$, local periodicity effects slow the rate of convergence. It takes time $c(m+n)^3$ for the walk on $X_{m,n}$ to be near stationarity, and time $cn^3/k(n)$ for the walk on $L_{n,k(n)}$.

Keywords: random walks, Markov chains, digraphs, rates of convergence.

10

A Combinatorial Optimization Problem

Robert C. Brigham and Ronald D. Dutton
Department of Mathematics, University of Central Florida
Orlando FL 32816
Julie R. Carrington* and Richard P. Vitray
Department of Mathematical Sciences, Rollins College
Winter Park FL 32789

In the course of a general study of extremal problems associated with the bandwidth of bipartite graphs, it was found that a subproblem could be mapped to the following combinatorial form. Let $M = 1,2,\ldots,m$ and let A, B, and C be arbitrary subsets of M of cardinalities r, s, and t, respectively. Define the *weight* of each $a \in A$ as the number of elements of C greater than a plus the number of elements of $M - C$ less than or equal to a. Similarly, for each $b \in B$, the weight of b is the number of elements of C less than b plus the number of elements of $M - C$ greater than or equal to b. The problem, given r, s, and t, is to choose A, B, and C such that the sum of all the weights of elements of A plus all the weights of elements of B is maximum. We present a partial solution.

Truncated Boolean Algebras as Subspace Arrangements

11

Ping Zhang, Western Michigan University

The truncated Boolean algebra can be considered as a subspace arrangement embedded in the coordinate hyperplane arrangement. We derive two forms of the characteristic polynomial of this arrangement using two methods. One employs the Blass-Sagan interpretation of certain characteristic polynomials as counting a set of lattice points. The other involves hypergeometric series.

Key Words: characteristic polynomial, subspace arrangements, the intersection lattice of a subspace arrangement, hypergeometric series.

12

Cyclic Niche Grids and Prisms

Suzanne Seager Mount Saint Vincent University

A graph G = (V,E) is a cyclic niche graph if there is a digraph D = (V,A) such that xy is in E iff there exists z in V such that either xz and yz is in A or zx and zy is in A. For triangle-free graphs, the cyclic niche property can be characterized in terms of certain coverings by paths and cycles. We apply this characterization to graphs such as grids and prisms.

Keywords: niche graph, grid, prism, competition graph

13

Partitions and Durfee square statistics

E.R. Canfield, Univ. of Georgia, S. Corteel*, C. D. Savage, North Carolina State Univ.

A partition of integer n is a sequence $\pi = (\pi_1, \pi_2, \ldots, \pi_k)$ such that $\pi_1 + \ldots + \pi_k = n$ and $\pi_1 \geq \pi_2 \geq \ldots \geq \pi_k \geq 1$. It has Durfee square size d if $\pi_d \geq d$ and $\pi_{d+1} \leq d$. Let $\mathbf{F}(n)$ be a family of partitions of n and let $\mathbf{F}(n,d)$ be the number of partitions in $\mathbf{F}(n)$ with Durfee square of size d. We investigate $\mathbf{F}(n,d)$ for several families of partitions and show that the identities, generating functions and recurrences for all these families have a common form. We present experiments that lead us to the conjecture that the most likely size of Durfee square is about $c\sqrt{n}$ for n large and we predict the constant c for each family. This conjecture has been recently proven by E. Rodney Canfield who has calculated these constants. Other experiments show that for n from 1 to 5000, the sequences $\{\mathbf{F}(n,d)\}_{0 \leq d \leq \sqrt{n}}$

are (1) log-concave and that (2) the mean and the mode differ at most by 1 and that for n from 1 to 500 (3) the Durfee polynomials $\sum_d \mathbf{F}(n,d)y^d$ have all roots real. A proof of (3) for all n would imply (1) and (2) for all n. Canfield has shown that (1) and (2) hold asymptotically for unrestricted partitions. We conjecture that (1), (2) and (3) hold for all n. We focus on the computational aspects in this talk.

Keywords: combinatorics, partitions of integers, log-concavity, real r oots.

14

Duality in Bandwidth Problems

Peter Che Bor Lam*, Wai Chee Shiu and Yi Xun Lin.
Hong Kong Baptist University.

The bandwidth is an important invariant in graph theory. The bandwidth problem for graphs and matrices originates from sparse matrix computation, circuit layout of VLSI designs and other areas. However, the problem to determine the bandwidth $B(G)$ of a graph G is a minimization problem which is NP-complete. So a direction of research is to find sharp bounds for $B(G)$. It is well-known in combinatorial optimization that minimax relatio ns can usually provide good lower or upper bounds. For bandwidth problem, as a minimization problem, we could establish relations with some maximization problems. In this paper, we study this type of duality in several aspects. First, we illustrate how a duality relation about degrees implies a series of bounds. Second, we discuss the so-called density lower bounds. Finally, we describe a method based on the boundary lower bounds.

Key words and phrases: Bandwidth, Duality.

15

ϕ-Tolerance Upper Bound Graphs of Partially Ordered Sets

F.R. McMorris, University of Louisville
C.W. Rasmussen*, Naval Postgraduate School

Let $V = \{v_1, v_2, \ldots, v_n\}$ be a nonempty set, and let $P = (V, \preceq)$ be a partially ordered set. Let $T : V \rightarrow \mathbf{N}$ be a function, with $T(v_i) = t_i$, and let $\phi : V \times V \rightarrow \mathbf{N}$ be a symmetric function. For $x \in V$, let $U(x) = \{y \in V | x \preceq y\}$. We define the *$\phi$-tolerance-upper-bound graph* $G_{P,T,\phi}$ by $G_{P,T,\phi} = (V, E)$, where, for any $v_i, v_j \in V$, $v_iv_j \in E$ if and only if $|U(v_i) \bigcap U(v_j)| \geq \phi(t_i, t_j)$. We show that every graph arises as a ϕ-tolerance-upper-bound graph, and then give new characterizations for the threshold graphs and the interval graphs in terms of specific choices of ϕ and P.

Keywords: tolerance intersection graphs, interval graphs, threshold graphs .

16

Dense and Sparse Niche Graphs

Steve Bowser*,Allegheny College, sbowser@alleg.edu
Charles Cable,Allegheny College, ccable@alleg.edu

We refer to a graph on n vertices as "dense" if it has a subgraph isomorphic to K_{n-2} and to the complement of a dense graph as "sparse". We provide a complete description of those dense and sparse graphs which are niche graphs.

RESEARCHES ON PARTITIONS, REVISITED

17

Joseph Arkin* and David C. Arney
United States Military Academy,
West Point, New York 10996.

In this paper we shall find explicit formulas for the number of partitions of n into parts not exceeding m for certain values of m. We denote by $p_m(n)$ the number of partitions of n ($n=0,1,2,\ldots$) into parts not exceeding m, where

$$(1-x)(1-x^2)\ldots(1-x^m) = \sum_{n=0} p_m(n)x^n, \text{ and } p_m(0)=1.$$

It is interesting to note that Cayley found formulas for m =1 through 6 inclusive, but did not go on to m=7. To express $p_7(n)$ by Cayley's method requires the use of 420 polynomials, each of degree 6.

In this paper, using Arkin's method, we find explicit formulas for the $p_m(n)$ for values of m that are greater than 6.

On the Bandwidth of Binomial Trees

18

Eleanor Hare*, Clemson University
Carlos Soares, Nortel Corporation

The bandwidth problem for a graph G is to label its n vertices v_i with distinct integers $f(v_i)$ so that the quantity

$$\max \{ \mid f(v_i) - f(v_j) \mid : (v_i,v_j) \in E(G) \}$$

is minimized.

The binomial tree of order zero, is a single vertex. The binomial tree of order k is formed by joining the roots of two binomial trees of order (k-1) such that the root of one of the binomial trees of order (k-1) becomes the root of the binomial tree of order k. We present an algorithm for finding a lower bound for the bandwidth of binomial trees and a heuristic for labeling the vertices of these trees.

key words: bandwidth, labeling, binomial tree

The Strong Hall Property and Symmetric Chain Orders

19

Xiaoyun Lu*[1], Da-Wei Wang[2] and C. K. Wong[1]

Let $G = (X, Y; E)$ be a bipartite graph with $|X| \geq |Y|$. For $A \subseteq X$, write $\phi(A) = |A| - |N(A)|$ and for $a \leq |X|$, define $\phi(a) = \max\{\phi(A) \mid A \subseteq X, |A| = a\}$. The graph G is said to have the *strong Hall property* if $\phi(a) + \phi(b) \leq |X| - |Y|$ for all nonnegative integers a and b with $a + b \leq |X|$. In this paper we discuss the graphs with the strong Hall property, and also some applications to symmetric chain orders. We show that any unimodal self-dual poset has a symmetric chain order decomposition if its level graphs has strong hall property.

Key words: Poset, Strong Hall Property, Symetric Chain Order.

[1] Department of Computer Science and Engineering, The Chinese University of Hong Kong, Shatin, Hong Kong. {xylu,wongck}@cs.cuhk.edu.hk
[2] Institute of Information Science, Academia Sinica Taipei Taiwan wdw@iis.sinica.edu.tw

Recognizing Clique Graphs of Rooted Path Graphs

20

Jayme L Szwarcfiter, Universidade Federal do Rio de Janeiro

The **clique graph** of a graph G is the intersection graph of the maximal cliques of G. A **rooted path graph** is the intersection graph of paths of a rooted tree. Characterizing clique graphs of classes of graphs is relevant in the context of intersection graphs. In the present work, we characterize clique graphs of rooted path graphs. The characterization is in terms of a suitable sequence of (not necessarily maximal) cliques of the graph, together with a proper ordering of the vertices forming each of these cliques. Based on it, we propose a polynomial time algorithm to recognize graphs of this class.

KEY WORDS: algorithms, clique graphs, intersection graphs, path graphs

Monday, March 3, 1997
3:20 p.m.

21

A Basis for Two-Configurations in Balanced Ternary Designs With Block Size Three

Margaret Francel francelm@cit1.citadel.edu, The Citadel

A balanced ternary design, (BTD), with parameters (V,B,R,K, Λ) is a collection of B blocks on V elements such that each element occurs R times in the design; each block contains K elements, where a element may occur 0, 1, or 2 times in a block (i.e. a block is a collection of elements rather than a set of elements); and each pair of distinct elements occurs Λ times in the design. The n-block configurations of a BTD are the distinct templates for the n-block subsets of the design.

In this talk we will discuss two-block BTD configurations for BTDs with block size three. We determine formulae for the number of two-block subsets of a design that are of a particular configuration type. All formulae are shown to be dependent on the design parameters and the number of occurrences of particular configurations. The peculiarities of the cases where lambda is two or three will also be discussed. Here certain two-configurations are shown to be dependent only on the design parameters.

22

Zero-One Laws for Homogeneous Models of Random Graphs

Gregory L. McColm, USF

There are many models of random graphs besides the Erdős-Rényi model, such as random acyclic graphs, random graphs in a metric space, random subgraphs of a complete bipartite graph, and so on. Typically, for each n, we have a probability space (G_n, μ_n), where G_n is a set of finite graphs. A 'zero-one law' for a set P of properties would work on such a model in the usual way: for every $P \in P$,

$$\lim_{n\to\infty} \mu_n[\{G: G \models P\}] \in \{0,1\}.$$

We can use Ehrenfeucht games to establish zero-one laws when P is defined by some nice logic. We show how this is done by looking at zero-one laws for some common logics (First Order, Monadic Second Order, ...) on models of random graphs that are relatively homogeneous. We will also look at what happens when a successor or ordering is built in.

Digraphs with Interval or Chordal Competition and Resource Graph

J. Richard Lundgren, University of Colorado at Denver, Denver, CO 80217-3364
Larry Langley and Sarah K. Merz*, University of the Pacific, Stockton, CA 95211
Craig W. Rasmussen, Naval Postgraduate School, Monterey, CA 93943

23

Given a digraph D, the competition graph of D is the graph on the same vertex set containing edge $\{x,y\}$ if and only if $\mathrm{Out}_D(x) \cap \mathrm{Out}_D(y) \neq \emptyset$. The resource graph of D is the graph on the same vertex set containing edge $\{x,y\}$ if and only if $\mathrm{In}_D(x) \cap \mathrm{In}_D(y) \neq \emptyset$. It was observed that almost all competition graphs arising from digraphs modelling ecosystems are interval graphs. Wang posed the problem of characterizing acyclic digraphs whose competition graph and resource graph are both interval graphs. We show that if G is an interval graph with at least one isolated vertex, then G is the competition graph of an acyclic digraph whose resource graph is also interval. In addition, if G is a strongly chordal graph with at least one isolated vertex, then G is the competition graph of an acyclic digraph whose resource graph is chordal.

Key Words: competition graph, resource graph, interval graph, strongly chordal graph

Central k-trees in trees

24

F.R. McMorris*, University of Louisville
K.B. Reid, California State University, San Marcos

A subtree of order k of a tree T of order n is called a central k-tree if it has minimum eccentricity among all subtrees of T with order k. A 'pruning' process is described that yields a central k-tree and is contrasted with a 1985 algorithm of Minieka for tree networks.

25 **Simple 6- and 7-designs on 19 to 33 points**

A. Betten, R. Laue*, A. Wassermann, University of Bayreuth, Germany

Until recently only a few simple 6-designs on small point sets were known [1].Using the program DISCRETA developed by the authors, in the last two years several simple 6- and 7-designs could be constructed. Discreta contains new versions of B. Schmalz's double coset algorithm for computing Kramer-Mesner matrices and of the LLL-algorithm for solving diophantine equation systems. In addition,a backtrack search by B. D. McKay is included. The system has a graphical user interface for a conveniant handling of various features, especially for constructing groups and searching in a database. Some designs were found using the results of DISCRETA and Tran van Trung's construction. The 29 parameter sets of the presently known 7-designs on small point sets are on $20, 24, 25, 26, 27, 28, 30, 33$ points, have block sizes $8, 9, 10$, and λ in the range from 4 to 630. Using group theory the number of isomorphism types of designs with a prescribed large automorphism group could be determined in several cases without isomorphism testing. Many new 6-designs were also found, the smallest having the parameters 6-$(19, 7, 4)$.

Keywords: Simple t-designs, isomorphisms, automorphism groups.

[1] *The CRC Handbook of Combinatorial Designs* C. J. Colbourn, J. H. Dinitz, ed., CRC Press Boca Raton, New York, London, Tokyo(1966).

26 **Conditional Coloring**

Mark Dillon, University of Colorado at Denver

The *conditional chromatic number* $\chi(\mathbf{G}, \mathbf{P})$of a graph G with respect to a graphical property P is the minimum number of colors needed to color the vertices of G such that each color class induces a subgraph of G with property P. The conditional chromatic number of a graph has been studied by various authors since 1968. When P is the property that a graph contains no subgraph isomorphic to a graph F, we write $\chi(\mathbf{G}, -\mathbf{F})$. We focus on two conditional chromatic numbers. We find $\chi(\mathbf{G}, -\mathbf{C}_j)$ for graphs missing at most $j-1$ edges and $\chi(\mathbf{G}, -\mathbf{P}_j)$for graphs missing at most $2j-5$ edges. We determine both conditional chromatic numbers for graphs whose complement is acyclic. We also determine $\chi(\mathbf{G}, -\mathbf{P}_j)$ for cubic graphs. Finally, we determine a lower bound on $\chi(\mathbf{G}, -\mathbf{P}_j)$ in terms of the size of G.

27 **Decompositions and Packings of Digraphs with Orientations of a 4-cycle**

Robert B. Gardner, East Tennessee State University
Coleen Huff, Osceola High School
Janie Kennedy*, Southeast Missouri State University

We present necessary and sufficient conditions for the decomposition of the complete symmetric digraph on v vertices with a hole of size w into each of the orientations of a 4-cycle.

28 **Line broadcasting in grid graphs**

Iwona Wojciechowska, West Virginia University

Broadcasting refers to a process of information dissemination in a communication network whereby one member of the network has one or more messages that are to be transmitted to all members of the network. We model a communication network by a *connected graph* $G = (V, E)$, where V is the set of *vertices* (members) and E is the set of *edges* (communication lines). A message is transmitted by placing a series of *calls* between members. Time is discrete and in one time interval a vertex may participate as a sender or a receiver in at most one call.

In *line broadcasting* a vertex can call any other vertex, provided there is an open line (a path such that every edge in the path is not involved in another call during that time unit) between them. Some line broadcasting algorithms and costs associated with them will be presented.

29

Finding simple t-designs via lattice basis reduction

A. Wassermann, University of Bayreuth, Germany

Since the work of Kramer and Mesner (1976) one way to construct simple t-(v, k, λ) designs with large t is to fix an automorphism group and find the $\{0,1\}$-solutions of the resulting system of linear diophantine equations. This problem is known to be NP-complete.

Kreher and Radziszowski first applied lattice basis reduction to solve these systems. Recent progress in algorithms for lattice basis reduction, the combination with l_∞-norm enumeration and more appropiate lattices make it possible to solve systems with up to several hundred columns and rows which arise by the construction of designs with $t = 7$.

Keywords: t-designs, subset sum problem, lattice basis reduction, linear diophantine systems.

30

Hall's Condition for List Multicolorings and the Fractional Hall Number of a Simple Graph

A. J. W. Hilton
Dept. of Math.
The University of Reading
Whiteknights, Reading RG62AX
England

*P. D. Johnson Jr.
Dept. of Discrete and Statistical Sciences
120 Math. Annex
Auburn University, AL 36849
U.S.A.

Suppose that G is a simple graph, $L : V(G) \to \{\text{finite subsets of S}\}$ is a list assignment, and $k : V(G) \to \{\text{positive integers}\}$ is a function. A (k, L) list multicoloring of G is a choice of sets $C(v) \subseteq L(v)$, $v \in V(G)$, satisfying $|C(v)| = k(v)$ for all $v \in V(G)$ and, if $u, v \in V(G)$ are adjacent in G, then $C(u) \cap C(v) = \emptyset$.

If H is a subgraph of G, for each color $\sigma \in S$, let $\alpha(\sigma, L, H)$ be the independence number of the subgraph of H induced by those vertices $v \in V(H)$ with σ on their lists (i.e., $\sigma \in L(v)$). We will say that G and L satisfy Hall's condition with respect to k if and only if, for each subgraph H of G,

$$\sum_{\sigma \in S} \alpha(\sigma, L, H) \geq \sum_{v \in V(H)} k(v).$$

When k is the constant function 1, this reduces to the previously studied Hall's Condition. The Hall number $h(G)$ of a simple graph G is the smallest positive integer m such that there is a proper L-coloring (a $(1, L)$ list multicoloring) of G whenever Hall's Condition is satisfied and $|L(v)| \geq m$ for all $v \in V(G)$. We know a thing or two about the Hall number; for instance, $h(G) = 1$ if and only if every block of G is a clique.

We define the fractional Hall number $h_f(G)$ of a simple graph G, with reference to Hall's Condition for list multicolorings above, and prove a number of basic results about h_f. Among them: $h_f(G) = 1$ if and only if every block of G is a clique.

Keywords: graph, independence number, list coloring, list multicoloring.

31

Cliques that are Tolerance Digraphs

Randy Shull and Ann N. Trenk*

We discuss directed graph versions of tolerance graphs, in particular, the class of totally bounded bitolerance digraphs and several subclasses. When the underlying graph is complete, we prove that the classes of totally bounded bitolerance digraphs and interval catch digraphs are equal, and this implies a polynomial-time recognition algorithm for the former class. In addition, we give examples (whose underlying graphs are complete) to separate every other pair of subclasses.

Keywords: Tolerance graph; bitolerance digraph; interval catch digraph.

32

Broadcasting in double and triple loop graphs

A. L. Liestman, J. Opatrny*, M. Zaragoza
Simon Fraser Univ., Concordia Univ., Univ. Politècnica de Catalunya

For a positive integer n and a set of positive, pairwise distinct integers $\{s_1, s_2, \ldots, s_k\}$, the *distributed loop graph* $G(n, s_1, s_2, \ldots, s_k)$, (also called a *multiple fixed-step graph*) is a graph whose vertex set equals $\{0, 1, \ldots, n-1\}$ and for any vertex u there is an edge from u to $u + s_i$ for $1 \leq i \leq k$. In this paper we consider the problem of broadcasting in two types of graphs from this family, a double loop graph $G(2D^2 + 2D + 1, D, D + 1)$, and a triple loop graph $G(3D^2 + 3D + 1, D, D + 1, 2D + 1)$. Both of these graphs are of diameter D, and the graph $G(2D^2 + 2D + 1, D, D + 1)$ is the largest double loop graph of diameter D. We determine that the broadcast time of the double loop graph $G(2D^2 + 2D + 1, D, D + 1)$ is equal to $D + 2$, while the broadcast time of the triple loop graph $G(3D^2 + 3D + 1, D, D + 1, 2D + 1)$ is equal to $D + 3$.

33

The Quest for 2–(22,8,4) Designs

Stanisław Radziszowski, Department of Computer Science Rochester Institute of Technology, Rochester, NY 14623, spr@cs.rit.edu

A *design* is a pair $(X,)$ where X is a v-element set of *points* and is a multiset of subsets of X, called *blocks*. A $t-(v,k,\lambda)$ *design* is a design $(X,)$ with $|X| = v$, such that the blocks have size k and $|\{K \in : T \subseteq K\}| = \lambda$ for all $T \subseteq X$ with $|T| = t$. The case $t = 2$ defines an important category called *balanced incomplete block designs*, BIBD's, among which $2-(22,8,4)$ is the smallest design whose existence is unsettled. In this talk we overview previous attacks, and report on our work, done jointly with Brendan McKay, related to these designs.

Using computer algorithms we have shown that in any such design every two blocks have nonempty intersection, every quadruple of points can occur in at most two blocks, and no blocks can be intersected in exactly one point by more than one block. During these computations we constructed above half a million of nonisomorphic "near" $2-(22,8,4)$ designs, which are collections of 33 blocks of size 8, hitting each of the 22 points 12 times, each pair of blocks intersecting in 1, 2, 3 or 4 points, and such that 227 (out of 231) pairs of points are covered exactly 4 times.

Among the various software tools used in this work, the novel ones included an efficient method for eliminating most isomorphs during point by point extensions of partial designs, and splitting the set of points into a few cells and analyzing the possible intersection patterns of all the blocks. The work on this design already required more than a century of cpu time, making it perhaps the second largest computation ever performed (after the solution to the RSA-129 challenge).

34

Hanging planters: a family of primitive minimally triangle-saturated graphs

Roger B. Eggleton* (Illinois State U.) & Jim MacDougall (U. Newcastle)

We introduce a hanging planter construction, which gives a new infinite family of primitive minimally triangle-saturated graphs. A particular instance of this construction is as follows. Let G be any complete bipartite graph of order at least 3, and let H be any spanning subgraph of G in which each component has order at least 3. From the base G, we produce a new graph [G, H, x] which contains G as an induced subgraph, by adjoining for each edge e of H a new support vertex adjacent to both endpoints of e, and finally adjoining a top vertex x adjacent to every support vertex. The hanging planter graph [G, H, x] is a primitive minimally triangle-saturated graph. We shall discuss details of the general construction.

Key words: triangle-saturated graphs, complete bipartite graphs

ON CCE-ORIENTABLE GRAPHS

35

Ok Boon Bak, Suh-Ryung Kim*, Chang Hoon Park
Kyung Hee University, Seoul, Korea
Yunsun Nam, Seoul National University, Seoul, Korea

Let D be a digraph. The competition-common enemy graph of D has the same set of vertices as D and an edge between vertices u and v if and only if there are vertices w and x in D such that (w,u), (w,v), (u,x), and (v,x) are arcs of D. We call a graph a CCE-graph if it is the competition-common enemy graph of some digraph. We also call a graph $G = (V,E)$ CCE-orientable if we can give an orientation F of G so that whenever (x,u), (x,v), (u,y), and (v,y) are in F, either (u,v) or (v,u) is in F. In this paper, we show that any CCE-orientable graph is a CCE-graph and find various families of graphs which are CCE-orientable graphs. So far we have not found any graph that is not a CCE-orientable graph.

Key words. CCE-graphs, CCE-orientable graphs

Minimum Average Time Broadcast Graphs

36

Arthur L. Liestman and Nataša Pržulj *
School of Computing Science, Simon Fraser University

Given a graph $G = (V,E)$ and a vertex $u \in V$, *broadcasting* is the process of disseminating a piece of information from vertex u to every other vertex in the graph where, in each time unit, any vertex which knows the information can pass the information to at most one of it's neighbors. A *broadcast graph* on n vertices is a graph which allows any vertex to broadcast in time $\lceil \log n \rceil$. A *minimum broadcast graph* on n vertices is a broadcast graph with the minimum number of edges over all broadcast graphs on n vertices.

We are interested in broadcasting under a slightly different time constraint. In particular, we wish to minimize the *average time* at which a vertex is informed during a broadcast. We initiate the study of *minimum average time broadcast graphs* - those graphs on n vertices with the fewest edges in which every vertex can broadcast in minimum average time. We find minimum average time broadcast graphs for all even n, for odd n in the range $2^k \leq n \leq 2^k + 2^{k-1}$, and for some additional small values of n. In addition, we give some upper and lower bounds on the number of edges in such graphs for all n.

KEYWORDS: broadcast, network communication.

37 New Designs from Codes and Geometry

ILIYA BLUSKOV, Simon Fraser University, Canada

Keywords: Codes, Designs, Geometry.

Let $D = \{B_1, B_2, ..., B_b\}$ be a finite family of k-subsets (called blocks) of a v-set $X(v) = \{1, 2, ..., v\}$ (with elements called points). Then D is a t-(v, k, λ) design if every t-subset of $X(v)$ is contained in exactly λ blocks of D. Let $p = \max_{1 \le i < j \le b} |B_i \cap B_j|$. We call p the maximal intersection number of D. We discuss an application of designs with small maximal intersection number for finding new designs. We study the maximal intersection number of known designs. We prove that designs obtained from codes in works of Assmus and Matson; MacWilliams, Odlyzko and Sloan; and van Lint and MacWilliams are designs with small maximal intersection number. We then use a result of Driessen and a previously unknown corollary to this result to prove the existence of many new 3,4, and 5-designs.

We also find a simple 3-(26,8,14) design (one of the designs with small λ whose existence was still unestablished) from the circle geometry of order 5. We then study the intersections between its blocks and the blocks of different designs obtained via Driessen's Theorem and corollaries from the same circle geometry to obtain some other new designs.

38 The Relative Location of the Median within a Graph with Respect to the Center, Annulus and Periphery

Steven J. Winters, University of Wisconsin Oshkosh

The eccentricity of a vertex v in a connected graph G is the distance between v and a vertex furthest from v in G. The center of G is the subgraph induced by those vertices of G having minimum eccentricity, the periphery is the subgraph induced by those vertices of G having maximum eccentricity, and the annulus is the subgraph induced by the remaining vertices. The distance of a vertex v in G is the sum of the distances from v to the vertices of G. The median of G is the subgraph induced by those vertices having minimum distance. It is known that every graph can be the median of some connected graph, and this graph can be arbitrarily far from the center or periphery. In this paper, we show that every graph can be a median where the median is arbitrarily far from both the center and periphery at the same time. In addition, we show that the median can also be arbitrarily far from the annulus of a graph where the median is contained within the center or the median is contained within the periphery.

Key words: median, annulus, center, periphery

39 Posets with Interval Upper and Lower Bound Graph

J. Richard Lundgren, University of Colorado at Denver, Denver, CO 80217-3364
Larry Langley* and Sarah K. Merz, University of the Pacific, Stockton, CA 95211
Craig W. Rasmussen, Naval Postgraduate School, Monterey, CA 93943

Given an ordered set $(P, <)$, the upper bound graph of P is the graph whose vertices are the elements of P containing edge $\{x, y\}$ if and only if x and y have a common upper bound. The lower bound graph of P is analogously defined. Given a digraph D, the competition graph of D is the graph on the same vertex set containing edge $\{x, y\}$ if and only if $\mathrm{Out}_D(x) \cap \mathrm{Out}_D(y) \neq \emptyset$. The resource graph of D is the graph on the same vertex set containing edge $\{x, y\}$ if and only if $\mathrm{In}_D(x) \cap \mathrm{In}_D(y) \neq \emptyset$. If D is the digraph associated with P under $<$, then the upper and lower bound graphs of P equal the competition and resource graphs of D, respectively. Wang posed the problem of characterizing acyclic digraphs whose competition and resource graphs are both interval graphs. We examine posets whose upper and lower bound graphs are both interval.

Key Words: competition graph, interval graph, chordal graph, upper bound graph, series parallel order

40 The Degree Matrix of a Graph

Marc J. Lipman, Office of Naval Research

The degree matrix of a graph is a natural generalization of the degree sequence. For a graph G, the i,j entry of its degree matrix is the number of vertex sets of G of order i whose neighborhood has order j. The degree matrix is a computationally inconvenient object. It is shown that the matrix in general contains more information that the degree sequence, but that it does not in general uniquely determine a graph. The matrix of the union of graphs is described. The matrices for regular graphs of degree two are determined. Ditto for graphs on n vertices that are regular of degree n-3. Some rudimentary results are given for trees.

keywords: degree matrix, degree sequence

41

G–DESIGNS USING GROUP DIVISIBLE DESIGNS

D. G. Hoffman and Kimberly S. Kirkpatrick*
Auburn University

A G-design of order n and index λ is a decomposition of the complete multigraph, λK_n, into a graph G. We use quasigroups and group divisible designs to find such designs for many graphs G on five vertices.

Key Words: G-design, Quasigroup, Group Divisible Design

42

Approximations to Rectilinear Steiner Trees on a Z–Cube

Konstantinos Kokkinos*, Dionysios Kountanis
Computer Science Department, Western Michigan University

Z–Cube is a hypercube–like interconnection network with one fourth fewer node connections. This paper deals with the problem: Given a Z–Cube on the m–hyperplane and a set of n randomly distributed processors on the nodes of the Z–Cube, find a Steiner tree that minimizes the communication cost among the processors. Three algorithms for finding an approximation to the optimal Steiner tree are presented, analyzed and compared. The algorithms are based on the following heuristics: Gravity oriented, Direction oriented and Hyperplane subdivisions. The time complexities of the algorithms are of the order $O(n^2m)$. Experimental data obtained on Pentium PC implementations show the improvements on the communication cost relative to the minimum Spanning tree connections.

keywords: Z–Cube, Approximation Algorithms, Hypercube, Steiner tree, Communication cost

43

A sufficient condition for a tree belonging to $\mathcal{T}(2,2,n)$

Han Hyuk Cho* and Yunsun Nam *Seoul National University, KOREA*
Suh-Ryung Kim, *Kyunghee University, KOREA*

Given a digraph D, a vertex z of D is a 2-step common prey for x and y if there are two paths of length two, one of which is from x to z and the other from y to z. The 2-step competition graph of D has the same vertex set as D and an edge between vertices x and y if and only if x and y have a 2-step common prey in D. Given a graph G, the 2-step competition number of G is the smallest number k such that G together with k isolated vertices is the 2-step competition graph of an acyclic digraph. Cho *et al.* characterized a graph which is the 2-step competition graph of some digraph and found the 2-step competition numbers of paths and cycles. In this paper, we give a sufficient condition for a tree belonging to $\mathcal{T}(2,2,n)$, where $\mathcal{T}(2,k,n)$ denotes the collection of all the trees on n vertices with 2-step competition number k, and conjecture that this is also a necessary condition.

Key Words: 2-step competition graph, 2-step competition number

44

Completion in Graphs with Bounded Degree

Fred Roberts, Rutgers University
Sam Stueckle*, Trevecca Nazarene University

Motivated by problems in molecular biology, the authors characterize those graphs which can be completed through edge addition to chordal graphs with maximum degree at most 4.

Monday, March 3, 1997
5:20 p.m.

45

Constant-Weight Codes and Group Divisible Designs

Simon Blake-Wilson* and Kevin T. Phelps
Discrete and Statistical Sciences, Auburn University

This work investigates codes over arbitrary alphabets with minimum distance 3 in which every codeword has Hamming weight 3. In the binary case, it is well known that optimal weight-3 distance-3 codes correspond to Steiner triple systems. Over an arbitrary alphabet, such codes correspond to group divisible designs with an additional 'distance' property.

We demonstrate new constructions for these group divisible designs, which employ basic combinatorial primitives such as orthogonal latin squares and maximal packings. In particular we show that the obvious necessary conditions for the existence of these designs are asymptotically sufficient.

Keywords: constant-weight code, group divisible design.

46

Minimal Sense of Direction in Regular Graphs with Asymmetric Labeling

P. Flocchini, Université de Montréal
A. Roncato, Università di Venezia
N. Santoro*, Carleton University
J. Siran, Slovak Academy of Sciences

We consider distributed systems modeled by (edge)-labeled graphs. Properties of the labeling can be used in the design of efficient algorithms for such systems. In particular, the property of *Sense of Direction* has a strong impact on the communication complexity of many distributed problems.

We consider sense of direction in networks modeled by d-regular graphs that uses exactly d labels (*Minimal* sense of direction). It has been shown that, for symmetric labelings, a labeled graph has a minimal sense of direction if and only if it is a Cayley graph. An interesting question is whether there exist minimal senses of direction on graphs with asymmetric and, thus, non Cayley, labeling.

In this paper we provide a positive answer; in fact we show an infinite family of asymmetric labeled graphs with minimal sense of direction.

47

(2) - Neighborhood Graphs of Interval Graphs

J. Richard Lundgren
University of Colorado at Denver (lundgren@carbon.cudenver.edu)

Patricia McKenna *
Kenyon College (mckennap@kenyon.edu)

The (p)-neighborhood graph $N_p(G)$ of a graph G is a graph on the same vertex set as G, with $[x,y] \in E(N_p(G))$ if and only if x and y have at least p common neighbors in G ($x \neq y$). Constructing the (p)-neighborhood graph is a simple matter; we use $(A(G))^p$, where $A(G)$ is the adjacency matrix of G. However, determining whether or not a given graph is a (p)-neighborhood graph is considerably more difficult. A question of interest in applications is "when is the (p)-neighborhood graph of G an interval graph?" In this talk, I will focus on the case $p = 2$, discussing a necessary and sufficient condition for an interval graph to have an interval (2)-neighborhood graph. Further, I will present results on the properties of interval graphs whose (2)-neighborhood graphs are not interval.

48

Notes on Hamiltonicity of Middle Cubes

Ke Qiu, Acadia University, Nova Scotia, Canada

It has been conjectured that all middle cubes are Hamiltonian. In this short note, we observe an interesting property of some of the Hamiltonian cycles in middle cubes. This leads to a sufficient condition for middle cubes to be Hamiltonian.

Tuesday, March 4, 1997
8:40a.m.

53

Extending Colorings of Subgraphs

Michael O. Albertson, Smith College

Recently Thomassen posed the following problem: Suppose G is a planar graph and W is a subset of V(G) such that the distance between any two vertices in W is at least 100. Can a coloring of W be extended to a 5-coloring of G? It is known that no 4-coloring result of this nature can hold. We provide a best possible solution to Thomassen's problem and consider generalizations.

Key words: graph coloring, planar graphs, Kempe chains

54

Domination Graphs of Regular Tournaments

Han Hyuck Cho, *Seoul National University*
Suh-Ryung Kim, *Kyunghee University*
J. Richard Lundgren*, *University of Colorado at Denver*

Vertices x and y dominate a tournament T if for all other vertices $z \neq x, y$, either x beats z or y beats z. Let dom(T) be the graph on the same vertices of T with edges between pairs of vertices that dominate T. We show that for regular tournaments T, dom(T) is either an odd cycle or a forest of paths. In addition, we determine which of these graphs are the domination graph of some regular tournament. Since dom(T) is the complement of the Competition Graph of a tournament formed by reversing the arcs of T, complementary results are obtained for the Competition Graph and the Niche Graph of a tournament.

Key Words: Tournament, Domination Graph, Competition Graph, Niche Graph

55

An Efficient Parallel Sorting Network

Dionysios Kountanis, Rikio Ichishima, and Md. N. Azam, Western Michigan University*

Muller and Preparata proposed a two level parallel sorting network for sorting a sequence of length n. The first level consists of $n(n-1)$ comparators arranged on the vertices of an $n \times n$ mesh with one bit of memory per comparator. The second level consists of $n(n-1)$ adders arranged on the internal vertices of full binary trees that compute the rank of each element in $O(\log_2 n)$ steps. In this paper, we substitute the first level with an $(n-1) \times (n-1)$ triangular mesh whose vertices are $n(n-1)/2$ comparators, having one bit of memory per comparator and achieving the same complexity of $O(\log_2 n)$. The modification is based on the observation that, for a sequence of distinct elements, the contents of the lower-left-triangular mesh are the complement of the transpose of the upper-righ t-triangular mesh. The modified sorting network ranks the same value elements according to their order in the initial list of element, that is, the sorting is stable. Finally, we descri be how lower order sorting networks are combind to form higher order ones.
keywords: Decomposition, Network, Mesh, Parallelism, Stability, Sorting.

56

Hermitian forms and Balanced Arrays

Ryoh Fuji-Hara and Nobuko Miyamoto*
Management Science and Engineering, University of Tsukuba,
Tsukuba, Ibaraki, Japan 305

A balanced array with strength t is an $k \times N$ array $\boldsymbol{A}$ with entries from $S = \{0, 1, \ldots, s-1\}$ such that
(i) in any t-rowed subarray $\boldsymbol{A}_0$ of $\boldsymbol{A}$, any t-dimensional column vector $\boldsymbol{x} \in S^t$ appears $\lambda(\boldsymbol{x})$ times in $\boldsymbol{A}_0$.
(ii) for any permutation σ on the coordinates of the vector $\boldsymbol{x} \in S^t$, $\lambda(\sigma(\boldsymbol{x})) = \lambda(\boldsymbol{x})$.
If $\lambda(\boldsymbol{x}) = \lambda(\boldsymbol{y})$ for every $\boldsymbol{x}, \boldsymbol{y} \in S^t$, then the array is called an orthogonal array of strength t

There is a famous construction of orthogonal arrays by R.C. Bose (1947). The construction uses linear transformations over a finite field. Fuji-Hara and Miyamoto generalized this method by considering non-linear functions in stead of linear transformations and a subset of the vectors as their domains. In previous work, we constructed combinatorial arrays such as orthogonal arrays and balanced arrays by using quadratic forms over finite fields. We use here Hermitian forms instead of quadratic forms for the construction.

Key words: Hermitian form, balanced array, orthogonal array

Tuesday, March 4, 1997
9:00 a.m.

57

Antimagic Pick-Up-Sticks

Andrew Simoson, King College, Bristol, TN 37620

The child's game *Pick-Up-Sticks* is played by casting a number, m, of slender sticks onto the floor in a random heap, and then attempting to pick up the sticks one after the other so that no remaining stick is disturbed. We play a mathematical version of this game by imagining that the m sticks are arrows numbered 1 through m. Since the game might best be played on a computer, we limit the play to a rectangle (the computer screen) and extend the arrows to be directed line segments (the *sticks*) which meet the edges of the screen. These m oriented labeled line segments induce a labeling of the regions formed by the lines: the label of a region is the sum of the boundary labels taken in a counterclockwise direction. In computing the label of a region, if counterclockwise motion within the region is against the direction of a boundary line, then that label is counted negatively. After a player removes a stick, if the *duplicity number*—the number of regions for which there are other regions of the same label—is zero, then that same player is allowed to remove an additional stick, otherwise play passes to the next person. When all the sticks are gone, the person with the most sticks wins. Since the tension/excitement in this game consists in moving so as to achieve minimal duplication in region labels, we call this game *Antimagic Pick-up-sticks*. We first of all demonstate the play of the game, contrast it with the original game, and then present a repertoire of line configurations from which game play can conveniently begin, starting with duplicity value of zero. **Key Words:** Graceful graphs, anti-magic graphs.

58

The Domatic Number of a Graph and its Complement

Jean E. Dunbar, Converse College*
Teresa W. Haynes, East Tennessee State University
Michael A. Henning, University of Natal at Pietermaritzburg

The maximum cardinality of a partition of the vertex set of a graph G into dominating sets is the *domatic number* of G, denoted $d(G)$ and $d()$ denotes the domatic number of its complement. In this paper we consider those graphs which satisfy $d(G) = d()$. We characterize trees, cycles, regular bipartite graphs and cubic graphs which have this property.
Key words: domatic number, domination number

59

On Computing Geodetic Bases

Nancy Kinnersley* and Man C. Kong, The University of Kansas

Let G =3D (V,E) be an undirected graph. A set of vertices S in V is a geodetic cover of G iff every vertex v in V is either in S or on a shortest path betwen two vertices of S. A minimum geodetic cover is called a geodetic basis and its cardinality is called the geodetic number of the graph. It is well-known that the problem of determining whether a given graph G has a geodetic cover of size k,2 =BE k =BE —V—, is NP-complete. In t= his paper, we present computational results for classes of graphs in which geodetic bases can be computed effectively. Experimental results in computing geodetic basis of general graph are also presented and analyzed.
Key words: Graph, geodetic cover, geodetic basis, algorithm, complexity.

BINARY CODES OF STRUCTURES DUAL TO UNITALS

60

J. D. Key, B. Novick and F. E. Sullivan*, Clemson University

Recently Baartmans, Landjev and Tonchev have made some study of the binary codes generated by the points by blocks incidence matrices of Steiner triple systems. We look here at this approach to some other Steiner 2-designs, and in particular for unitals with even block size. We show that in the case of unitals on 28 points, the minimum weight of these codes is 8 or 9.

KEY WORDS: codes, designs

Tuesday, March 4, 1997
10:50 a.m.

61

Worst Possible Greedy Colorings

*D. G. Hoffman and P. D. Johnson Jr.
Dept. of Discrete and Statistical Sciences
120 Math. Annex
Auburn University, AL 36849

Suppose G is a simple graph and $v_1, \ldots, v_n$ are the distinct vertices of G, in some order. The greedy coloring of G with respect to this ordering is the assignment of positive integers to $V(G)$ in which v_1 is assigned 1 and, after $v_1, \ldots, v_{i-1}$ have been assigned integers, v_i is assigned the smallest positive integer not appearing as a color on neighbors of v_i among $v_1, \ldots, v_{i-1}$. The cost of such a coloring is the largest positive integer appearing on the vertices, which is also the number of colors used.

The *greedy chromatic number* of G, $\chi_g(G)$, is the maximum cost of a greedy coloring of G. The *slightly sophisticated* greedy chromatic number of G, $\chi_{sg}(G)$, is defined similarly, but the maximum is taken over orderings $v_1, \ldots, v_n$ satisfying $d_G(v_1) \geq \cdots \geq d_G(v_n)$. It is elementary that

$$\chi(G) \leq \chi_{sg}(G) \leq \chi_g(G) \leq \Delta(G) + 1,$$

and that $\chi_{sg}(G) = \chi_g(G)$ if G is regular.

We reveal some unpleasant truths about these numbers. For instance: for every positive integer r, $\chi_g(Q_{2^r-1}) = 2^r$. (Q_n is the n-cube.)

Keywords: graph, greedy, chromatic number.

62

Generalized Domination and Independence in Graphs

Wayne Goddard and Michael A. Henning*
University of Natal, South Africa

In this paper, we introduce a new parameter which serves to generalize many well known parameters, including the domination number, the total domination number, the independence number, and the vertex covering number. We introduce the scheme, present examples of parameters in the scheme, discuss monotonicity, present Gallai-type results, and give a linear algorithm on trees.

63

A graph theoretic approach to file compression transfer in computer networks

Yoshihiro KANEKO*, Gifu University, Japan,

Recently computer networks are widely popularized, where necessary files or data can be easily transfered and and duplicated. In this situation, large sized files are often compressed in ahead of transfer. This paper deals with the total transfer time including comperssing time from some computer(source) to other computer(sink). This time depends on the ability of those computers and the capacity of communication links. If other computers can compress or decompress except source and sink, files might be transfered faster. On this assumption, a problem of file optimal compression scheduling is to choose two vertices(two machines) to compress and decompress files in

the shortest time. This papers first shows that the problem can be reduced to a shortest path problem. Next, for networks with path graph structure, a linear time algorithm is proposed, which is different from familiar algorithms such as Dijkstra's.

64

QUADRANGLE COMPLETIONS AND ANTI-DESARGUES' CONFIGURATION

R. B. Killgrove *, SDSU, R. W. Sternfeld, ISU, R. G. Tamez, CSULA

We revisit quadrangle completions in the projective planes of order 9. In particular we look at the "symmetric" completions. Then we do the same for Pappian projective planes. For order 11, this leads to 4 rows for the transpose of the multiplicative loop table. We postpone completions to potential loops for Coxeter-free Fano-free planes as well as completions to these planes, if any, besides the known plane. Then we give an example of the anti-Desarguesian configuration which resides in a complete affine ordered plane.

65

On size of neighborhood graphs

Jerzy W. Jaromczyk, Dept. Of Computer Science, U of Kentucky, jurek@cs.engr.uky.edu

As many spatial graphs, i.e., graphs defined for points in Euclidean spaces, the Relative Neighborhood Graphs play an important role in computational morphology and pattern analysis. In the RNG of a set of n points, two points p, q are connected with an edge is their distance $d(p,q)$ is not bigger than either of $d(p,s)$, $d(q,s)$, $s \neq p, q$. It is known that in R^2, the size of the RNG is $O(n)$ and in R^d for $d \geq 4$ this size is $O(n^2)$, with simple examples achieving these bounds. The best known upper bound for the RNG in R^3 is $O(n^{4/3})$ obtained with the help of bichromatic graphs. No matching lower bound is known and the exact upper bound for the RNG is an open question. In this talk, we will review results regarding the RNG in R^3 and discuss techniques and properties used in estimating their size.

keywords: spatial graphs, upper bounds, computational morphology, relative neighborhood graphs, bichromatic graphs, unit distance graphs

66

On α-Stable Graphs

V.E. Levit[1] and E. Mandrescu,Center for Technological Education Holon

This work considers the effect of arbitrary edge deletion and/or addition on the stability number $\alpha(G)$ - the cardinality of a stability system (a maximum set of pairwise non-adjacent vertices in G). Let $G = (V, E)$ be called: α^--stable if $\alpha(G - e) = \alpha(G)\ \forall\ e \in E$In this paper we show that: a chordal graph is α^--stable if and only if it has a unique stability system; any bipartite graph, which has an α^--stable spanning tree with the same stability number, is α^--stable. We prove that for any connected bipartite graph G the following conditions are equivalent: G is α^+-stable; G admits a perfect matching; G has two stability systems that partition its vertex set; G has an α^+-stable spanning tree; $\cap\{S : S$ is a stability system of $G\} = \emptyset$. It is shown that if a bipartite graph G has a vertex cover consisting of some pairwise

key words: *chordal graph, bipartite graph, α^--stable graph, α^+-stable graph, stability number, stability system, spanning tree, (bipartite) complement.*

67

Exploring Hypergraphs with Link

Jonathan W. Berry, Computing Sciences, Elon College; Nathaniel Dean*, Bell Labs

Link is a tool for exploring combinatorial objects. It provides a graphical interface, a functional language interface, an algorithm animation system, and a detachable set of C++ libraries. Link allows the user to interactively explore the structure of combinatorial objects such as collections, graphs, digraphs, and hypergraphs. Link makes it easy to define, create, edit, visualize, and manipulate combinatorial objects. We examine the use of Link in the context of a clustering problem, a natural application for the use of hypergraphs. Real-world problems requiring the manipulation of hypergraphs or heavily attributed graphs are often oversimplified to avoid complicated implementations. Link allows the user to explore such combinatorial objects at a very high level.

Key words: graphs, hypergraphs, combinatorics, software, tools, clustering, visualization, applications, language

68

AN n TO $2n$ EMBEDDING OF PARTIAL IDEMPOTENT LATIN SQUARES FOR SMALL VALUES OF n

C. Grant* and C. A. Rodger
Auburn University

In 1983, necessary and sufficient conditions were obtained for a partial idempot ent latin square of order n to be embedded in an idempotent latin square of or der $2n$, providing $n > 16$. In this paper we consider the case where $n \leq 16$.

Key Words: Embedding, Latin Squares

69 Efficient coverage of edge sets in graphs

J.E. Dunbar, Converse College, J.H. Hattingh(*), Rand Afrikaans University,
P.J. Slater, University of Alabama, Huntsville, A.A. McRae, Appalachian State University

Let $G = (V, E)$ be a graph. The efficient covering number of G, denoted $F^{01}(G)$, equals the maximum number of edges covered by a vertex set S with no edge in $E(G)$ covered more than once. The redundance covering number $R^{01}(G)$ equals the minimum total amount of coverage done by a cover. We relate $F^{01}(G)$ and $R^{01}(G)$, compute $F^{01}(G)$ where $G = P_k \times P_h$, $P_k \times C_h$ and $C_k \times C_h$, and bound $F^{01}(G)$. We prove that if G is a graph of order n, then $n-1 \leq F^{01}(G) + F^{01}(\overline{G}) \leq \frac{(n-1)(n+2)}{3}$, characterize the graphs G of order n for which $F^{01}(G) + F^{01}(\overline{G}) = n-1$ and characterize the graphs G of order n for which $F^{01}(G) + F^{01}(\overline{G}) = \frac{(n-1)(n+2)}{3}$. Also, we show that if G is a graph of order n, then $0 \leq F^{01}(G) \cdot F^{01}(\overline{G}) \leq (\frac{(n-1)(n+2)}{6})^2$, characterize the graphs G of order n for which $F^{01}(G) \cdot F^{01}(\overline{G}) = 0$ and characterize the graphs G of order n for which $F^{01}(G) \cdot F^{01}(\overline{G}) = (\frac{(n-1)(n+2)}{6})^2$. Finally, we show that the decision problem corresponding to the computation of $F^{01}(G)$ is NP-complete.

70 Maximization Versions of "Lights Out" Games in Grids and Graphs

John Goldwasser and William Klostermeyer*
West Virginia University

Complexity results and algorithms are given for the problem of maximizing the number of off vertices (switches) in graphs and $m \times n$ rectangular grids. When a switch is toggled, it and its neighbors change state. It is shown that the problem is NP-complete in graphs and a simple approximation algorithm is given as well as a non-approximability result. Fixed-parameter problems are studied in grids and graphs and an algorithm given for $m \times n$ grids that turns at least $mn - \frac{m}{2}$ vertices off, $m \leq n$. It is shown that $m \times n$ grids exist for which at most $mn - \frac{m}{log_2 m}$ vertices can be turned off and a large class of grids is characterized for which at least $mn - 2$ vertices can be turned off. Coding theory and Fibonacci polynomials over $GF(2)$ are two tools used in achieving the results.
Keywords: grid graphs, parity dominating set, NP-completeness, coding theory, finite fields

71 A Permutation GA for Vertex Graph Parameters

Alice McRae* Appalachian State University
Roy Pargas Clemson University

We will discuss a simple genetic algorithm that can search for minimum or maximum property P-sets in graphs. Although the genetic algorithm rarely finds optimal solutions, it can find good solutions quickly for some NP-hard problems on graphs with over a thousand vertices. The genetic algorithm has produced counterexamples to many conjectures that had been made based on results for small graphs.The algorithm has been used on many domination and coloring problems. To change the implementation of the algorithm to work with a new property P usually requires no more than writing a simple one-page function.
Keywords: genetic algorithms, domination, coloring

72 New Covering Designs with Nontrivial Automorphism Groups

Kari J. Nurmela and Patric R. J. Ostergard*, Helsinki University of Technology

A $t-(v,k,\lambda)$ *covering design* is a pair $(X, \mathcal{B})$, where X is a set of v points and $\mathcal{B}$ is a family of k-subsets of X, called blocks, such that every t-subset of X is contained in at least λ blocks. The minimum number of blocks in such a family $\mathcal{B}$ is denoted by $C_\lambda(v,k,t)$. In an earlier paper [K. J. Nurmela and P. R. J. =D6sterg=E5rd,=20 Upper bounds for covering designs by simulated annealing, *Congr. Numer.* **96** (1993), 93-111.],=20 the authors used a stochastic optimization heuristic, simulated annealing, to find good covering designs without any imposed=20 structures for $v \leq 13$. In this work, the search range is extended to $v \leq 28$. To enhance the search, automorphisms of the coverings designs are predetermined and another optimization heuristic, tabu=20 search, is used. Many new upper bounds on $C_1(v, t+1, t)$ are=20 presented.
Keywords: automorphism group, covering design, tabu search.=20

Tuesday, March 4, 1997
11:50 a.m.

Decomposition of 2G into paths of length two

D. G. Hoffman and Susan Serrano Rinker*
73 Auburn University

We prove that if G is a simple graph, then the edges of 2G can be partitioned into paths of length two if and only if no component of G is a tree with a perfect matching. Key Words: Path decomposition

74 Open Perfect Neighborhood Sets in Graphs

S. T. Hedetniemi, D. P. Jacobs, R. Laskar*, D.Pillone, Clemson University

Let S be a set of vertices in a graph. The open boundary of S is the set of vertices that are adjacent to exactly one member of S. When the open boundary of S is a dominating set we say S is an open perfect neighborhood set. Our paper is primarily concerned with the parameter $\theta_o(G)$ and $\Theta_o(G)$ that give the minimum and maximum cardinalities of open perfect neighborhood sets. We compare these parameters to more well-known parameters, and show that the decision problem for open perfect neighborhood sets is NP-complete, even when restricted to chordal graphs.

Keywords: graphs, dominating set, packing

Visualizing Graphs - Techniques and Implementation

75 Carolyn McCreary* and Larry Barowski
Department of Computer Science and Engineering, Auburn University, AL

Drawing graphs by hand can be tedious and time consuming, especially if the number of nodes and edges is large. Much time can be spent just trying to plan how a graph should be organized on the page. A research group at Auburn University has been developing an automated system VGJ, capable of converting a textual description or a drawing of a graph into a well organized and readable layout of the graph. VGJ (visualizing graphs with Java), includes a graph editor and a set of algorithms that will automatically layout and draw a graph. The graph editor provides an intuitive user interface that supports insertion/deletion of nodes and edges, graph editing, node repositioning, and three-dimensional views of the graph. Node shape, size and color can be specified.

Algorithms to layout different categories of graphs such as trees, planar graphs, directed and undirected graphs, and series-parallel graphs are being developed and implemented. For the directed graphs, the drawing method employs a unique graph-grammar decomposition to determine intrinsic substructures (clans) in the graph. The method parses a graph to produce a parse tree. The parse tree is given attributes that specify the node layout and edge routing. The innovative strategy of clan-based graph decomposition is the first digraph drawing technique to analyze locality in the graph in two dimensions. The typical approach to drawing digraphs uses the single dimension, level, to arrange the nodes.

The graph drawing package can be accessed through the world wide web at:
http://www.eng.auburn.edu/department/cse/research/graph_drawing/graph_drawing.html.

A Matrix Dynamics Approach to Golomb's Recursion

76 Edward J. Barbeau, John Chew and Stephen Tanny*
Dept of Math, U of Toronto, Toronto, ON M5S 3G3

In an unpublished note Golomb proposed a family of "strange" recursions of metafibonacci type, parametrized by k. Previously we showed that contrary to Golomb's conjecture, for each k there are many increasing solutions, and an explicit construction for multiple solutions was displayed. By reformulating our solution approach using matrix dynamics, we extend these results to a characterization of the asymptotic behaviour of all solutions of the Golomb recursion. This matrix dynamics perspective is also used to construct what we believe is the first example of a "nontrivial" nonincreasing solution, that is, one that is not eventually increasing.

Key Words: metafibonacci recursion; Golomb recursion; matrix dynamics

Tuesday, March 4, 1997
12:10 p.m.

77 **Characterizing Uniquely 4-Colorable Planar Graphs**

Tom Fowler, Georgia Institute of Technology, Atlanta, Georgia 30332

Fiorini and Wilson conjectured in 1977 that a uniquely edge 3-colorable cubic planar graph always contains a triangle. This is equivalent to the statement that every uniquely vertex 4-colorable planar graph has a vertex of degree three and implies that every such graph can be constructed from the complete graph on four vertices by repeatedly adding vertices of degree three. We give a computer-assisted proof of the conjecture. More precisely, using the techniques employed in the proof of the Four-Color Theorem we prove from first principles that every "internally 6-connected" planar triangulation has at least two 4-colorings. The Four-Color Theorem is a corollary. This is joint work with Robin Thomas.

78 **The Domination-Compliance Graph of a Tournament**

Guillermo Jimenez*, J. Richard Lundgren
University of Colorado at Denver, Denver, CO, 80217-3364

Vertices x and y are a dominant pair in a tournament if for all vertices $z \neq x, y$, either x beats z or y beats z. Vertices x and y are a compliant pair in a tournament if for all vertices $z \neq x, y$, either z beats x or z beats y. Let DC(T) be the graph on the same vertex set as T with edges between pairs of vertices that are either a dominant pair or a compliant pair in T. We show that the maximum possible number of edges in DC(T) is $2(n-1)$ and this bound is sharp. In addition we obtain results about the structure of DC(T) such as forbidden subgraphs and the clique number. Since DC(T) is the complement of the Competition/Resource graph of a tournament, complementary results are obtained for this graph.

Key Words: Tournaments, Domination Graph, Competition Graph

79 **Asymptotically Good Graph Colouring**

P. Mark Kayll, The University of Montana, kayll@charlo.math.umt.edu

View the determination of your favourite graph colouring parameter as an integer program. The problem is intractable, right? (Mine is.) This does not mean we should throw up our hands and quit trying to colour graphs! One means of handling the difficulties inherent to graph colouring arises through the consideration of appropriate *linear relaxations*, the LP's associated with the colouring IP's. A colouring parameter (say χ') exhibits *asymptotically good* behaviour in case $\chi'/\chi'^* \to 1$ as $\chi'^* \to \infty$ (χ'^* is the solution of the linear relaxation of the IP defining χ'). This talk will survey some recent developments establishing the asymptotically good behaviour of a variety of graph colouring parameters, including the chromatic index, the list-chromatic index and the total chromatic number. Certain interconnections between these results will be explored.

Key words: (total) graph colouring, integer/linear programming, asymptotics

80 **Searching for Symmetric Venn Diagrams**

Stirling Chow and Frank Ruskey(*), University of Victoria

An n-Venn diagram is a collection of n simple closed curves that partition the plane into 2^n connected regions, one region for each intersection of the interiors of a subset of the curves. The familiar 8 region 3-Venn diagram made from three circles is usually drawn with a 3-fold rotational symmetry. In general, an n-Venn diagram can possess an n-fold rotational symmetry only if n is prime, but no symmetric diagram is known for any $n > 7$. A diagram is simple if no more than two curves meet at any point. For $n = 7$ we've discovered 56 simple symmetric Venn diagrams, all but 8 of which are new, as well as a number of non-simple diagrams. Some interesting connections between Venn diagrams, knots, Hamilton cycles, and Gray codes will be illustrated.

Keywords: Venn diagram, combinatorial search, Gray code, Hamilton cycle.

Tuesday, March 4, 1997
3:20 p.m.

81

Coloring triangle-free graphs with fixed genus

John Gimbel, University of Alaska*
Carsten Thomassen, Technical University of Denmark

We show that if k≥4 and G is a triangle-free graph with fixed genus then we can tell in polynomial time if $\chi(G) \leq k$. Further, the chromatic number of a graph of girth at least six on a fixed orientable surface can be found in polynomial time.

Key Words: genus, triangle-free, algorithm, polynomial time

82

Graphs with Maximum Degree Three and Maximum Domination Number

David C. Fisher and Kathryn Fraughnaugh*, University of Colorado, Denver, CO 80217
Suzanne M. Seager, Mount Saint Vincent University, Halifax, Nova Scotia BM3 2J6

The domination number $\gamma(G)$ of a graph G is the minimum cardinality of a set $S \subseteq V(G)$ such that every node of G belongs to S or has a neighbor in S. We have shown that if the maximum degree of G is at most 3, then $\gamma(G) \leq \frac{1}{4}(3n - e + i)$ where G has n nodes, e edges, and i isolated nodes. Here we characterize those graphs for which this bound is tight. In particular, we show that the only connected cubic graphs with domination ratio exactly 3/8 are the two graphs below.

Keywords: domination number, maximum degree three

83

Pebbling in Diameter Two Graphs and Products of Paths

T. Clarke, Math. Dept., Arizona St. Univ., Tempe, AZ, 85287
R. Hochberg, DIMACS, Rutgers Univ., New Brunswick, NJ, 08903
* G. Hurlbert, Math. Dept., Arizona St. Univ., Tempe, AZ, 85287

Key words: graphs, pebbling number, diameter, connectivity

Suppose p pebbles are distributed to the vertices of a graph G. A pebbling step consists of removing two pebbles from one vertex and then placing one pebble at an adjacent vertex. We say a pebble can be *moved* to a vertex r, the *root* vertex, if we can repeatedly apply pebbling steps so that in the resulting distribution r has one pebble. For a graph G, we define the *pebbling number*, $f(G)$, to be the smallest integer m such that for any distribution of m pebbles to the vertices of G, one pebble can be moved to any specified root vertex r. The concept of graph pebbling was introduced by Lagarias and Saks as a way to prove a number-theoretic conjecture of Erdős and Lemke.

Trivially, $f(G)$ is at least the number of vertices $n(G)$. By characterizing which diameter two graphs G have $f(G) = n(G)$, we prove that every 3-connected diameter two graph G has $f(G) = n(G)$, extending results of Pachter, Snevily and Voxman. We also use this characterization to show the same holds for almost all graphs. We conjecture that there is a function $k(d)$ such that if G is a graph with $diam(G) = d$ and $\kappa(G) \geq k(d)$ then $f(G) = n(G)$. We prove that if k exists then $k(d) \geq 2^d/d$. We believe that $k(d) \leq 2^d$.

If time permits we will discuss other interesting results and conjectures.

84

The Bowtie Algorithm for Steiner Triple Systems

Patricia Boling* and C. C. Lindner
Auburn University

P. Horak and A. Rosa proved that any Steiner triple system with an even number of triples can be partitioned into bowties. The object of this paper is a different algorithm having many applications. In this paper we show how to use the algorithm to give a small embedding for partial bowtie systems.
Key Words: Steiner Triple Systems

85

On the multiplicity of the smallest eigenvalue of a Graph

M. Rosenfeld, Dept. of Computer Science, PLU

To what extend do eigenvalues characterize graphs? In the 60's F. Harary thought that the eigenvalues completely characterize graphs. Later, co-spectral graphs were proven to be abundant. Yet in some cases, graph properties yield strong conditions on the eigenvalues of the underlying graphs. This situation is common in certain extremal graphs. In this talk we prove that if the multiplicity of the smallest eigenvalue of G is $> n/2(n$ number of vertices) then G must containa Triangle.

Related open problems will also be discussed.

86

The connected domatic number

S. T. Hedetniemi*, S. M. Hedetniemi, Jeff Raglin, Clemson University
Doug Rall, Furman University; Bert Hartnell, St. Mary's University

The connected domatic number of a graph G = (V,E) is the maximum order of a partition of V(G) into connected dominating sets. We review the few existing results on this graphical invariant and offer a few new ones concerning 2- and 3-dimensional grid graphs and Kings graphs.

Tightening Bounds for Reconstruction of Bit Strings from Their Substrings

87

Jason Bandlow and Robert Molina*, Alma College

Given a bit string **b** of length n, the k-deck of **b**, denoted $D_k(\mathbf{b})$, is the collection of substrings of **b** of length k. We say that **b** is k-reconstructible if it is determined by its k-deck. It is known that bit strings of length $n > 7$ are k-reconstructible if $k \geq \lfloor \frac{n}{2} \rfloor$, but experimental evidence suggests that this result can be improved. For example, we have shown that bit strings of length 15 are 5-reconstructible. We define the function $f : N \to N$ so that $f(n) = k$ if and only if k is the minimum value such that all bit strings of length n are k reconstructible. We describe an algorithm which has allowed us to compute the values $f(n)$ for $n \leq 25$.

Partitioning the edges of $\lambda K_{c,d}$ into copies of $K_{a,b}$

88

Dean Hoffman and Mark Liatti*
Discrete and Statistical Sciences, Auburn University

We investigate partitioning the edges of the λ-fold complete bipartite graph $\lambda K_{c,d}$ into copies of $K_{a,b}$.

Keywords: complete bipartite graph.

LIST COLORINGS OF GRAPHS OF SMALL MAXIMUM DEGREE

89

Martin Juvan*, Bojan Mohar, Riste Škrekovski
University of Ljubljana, Slovenia

List colorings form a natural generalization of ordinary colorings in which each element of a graph must be colored by a color from its own list of admissible colors. Such colorings were introduced independently by Vizing [1] and Erdős, Rubin and Taylor [2] in the seventies. Recently, they attracted considerable attention when several problems related to list colorings were successfully resolved [3,4]. In the talk, several results relating ordinary colorings of graphs of small maximum degree with their list counterparts will be discussed.

1. V. G. Vizing, Coloring the vertices of a graph in prescribed colors (in Russian), *Metody Diskret. Analiz.* **29** (1976) 3–10.
2. P. Erdős, A. L. Rubin, H. Taylor, Choosability in graphs, *Congr. Numer.* **26** (1980) 125–157.
3. F. Galvin, The list chromatic index of a bipartite multigraph, *J. Combin. Theory Ser. B* **63** (1995) 153–158.
4. C. Thomassen, Every planar graph is 5-choosable, *J. Combin. Theory Ser. B* **62** (1994) 180–181.

90

Partitions and Domination in a graph

B.L. Hartnell*, Saint Mary's University, Halifax, Canada P.D. Vestergaard, Aalborg University, Aalborg, Denmark

We are interested in the following variation of domination. Consider a graph where each vertex is assigned a colour from 1,2,...,k. Besides having a minimum dominating set for the entire graph we also wish to determine, for each colour i, the smallest set of vertices (regardless of colour) which dominate the subset coloured i. We establish an upper bound for the sum of these values and show it to be sharp.

91

Reconstructing Bit Strings from Their Substrings

Jason Bandlow* and Robert Molina, Alma College

Given a bit string **b** of length n, the k-deck of **b**, denoted $D_k(\mathbf{b})$, is the collection of substrings of **b** of length k. We say that **b** is k-reconstructible if it is determined by its k-deck. We show that **b** is k reconstructible if $n > 4$ and $k \geq n/2$ by proving the following stronger result. Let $n > 4$ and $k \geq n/2$. If **b** is a bit string of length n with a ones, and if there are b elements of $D_k(\mathbf{b})$ that start with a one and c elements of $D_k(\mathbf{b})$ that end with a one, then the integers a b and c determine **b**.

92

A recursive construction for cyclic quasiframes and cyclically resolvable cyclic Steiner 2-designs

Miwako Mishima* and Masakazu Jimbo, Dept of Electronics and Computer Engineering
Gifu University, Gifu, 501-11, JAPAN, miwako@info.gifu-u.ac.jp

Let a triple $(V, \mathcal{G}, \mathcal{B})$ be a *group divisible design* (GDD). Note that when $\mathcal{G} = \phi$, the design is a *BIB design* (or *2-design*).

For a subclass $\mathcal{B}' \subseteq \mathcal{B}$ and for a subset $H \subseteq V$, if the union $\mathcal{B}' \cup \{H\}$ is a partition of the *point set* V, then $\mathcal{B}'$ is called a *holey resolution class with respect to* H and H is called a *hole*. Especially when $H = \phi$, $\mathcal{B}'$ is simply called a *resolution class*. Let $\mathcal{H}$ be a collection of h-subsets of V. If $\mathcal{B}$ can be written as a disjoint union $\mathcal{B} = \mathcal{P} \dot{\cup} \mathcal{Q}$ where $\mathcal{P}$ can be partitioned into holey resolution classes with respect to some hole $H \in \mathcal{H}$ and $\mathcal{Q}$ can be partitioned into resolution classes, then we call such a GDD a *quasiframe* and denote it by a quadruple $(V, \mathcal{G}, \mathcal{H}, \mathcal{B})$. It is known that when $\mathcal{H} = \mathcal{G} \neq \phi$ the design is called a *semiframe*, if $\mathcal{P} = \phi$ it is said to be *resolvable* and if $\mathcal{Q} = \phi$ it is called a *frame*.

If $(V, \mathcal{G}, \mathcal{B})$ has an *automorphism* σ of order $|V|$, then the design is said to be *cyclic*. In this case, $\mathcal{B}^\sigma = \{B^\sigma | B \in \mathcal{B}\} = \mathcal{B}$ and $\mathcal{G}^\sigma = \{G^\sigma | G \in \mathcal{G}\} = \mathcal{G}$ hold. For a quasiframe $(V, \mathcal{G}, \mathcal{H}, \mathcal{B})$, assume that $(V, \mathcal{G}, \mathcal{B})$ is a cyclic GDD with respect to σ. Furthermore if σ also preserve holey resolution classes $\mathcal{P}$ and resolution classes $\mathcal{Q}$, that is, $\mathcal{P}^\sigma = \{P^\sigma | P \in \mathcal{P}\} = \mathcal{P}$ and $\mathcal{Q}^\sigma = \{Q^\sigma | Q \in \mathcal{Q}\} = \mathcal{Q}$, then we say that the design is a *cyclic quasiframe with respect to* σ. When $\mathcal{H} = \mathcal{G} = \phi$, the design is called a *cyclically resolvable cyclic BIB design*.

In this talk, we shall give a recursive construction for cyclic quasiframes in the case of $\lambda = 1$ and by using the construction *cyclically resolvable cyclic Steiner 2-designs* can be constructed recursively.

Threshold Role Assignments

93

Fred S. Roberts, DIMACS, Li Sheng* RUTCOR
Rutgers University, New Brunswick, NJ

If G is a graph, a *k-threshold close role assignment* is a function mapping each vertex into a role, a positive integer $1, 2, \ldots, k$, so that if x and y are adjacent, then the *Hausdorff distance* between the sets of roles assigned to their neighbors is at most 1. In 1995, Roberts showed that every graph of at least two vertices is 2-threshold close role assignable and every graph of at least three vertices is 3-threshold close role assignable. We prove in this paper that every graph of at least k vertices is k-threshold close role assignable for $k = 4$ and $k = 5$.

Domination Number of Tournaments and Complete Bipartite Digraphs

94

Tamás Szabó, University of Memphis

In the talk we shall present some results obtained jointly with Béla Bollobás on the domination number of tournaments. We prove that the domination number of almost every random tournament of order n (the edges of K_n are oriented randomly, independently, with equal probabilities) is exactly $\lceil \log_2 n - 2\log_2 \log n + \log_2 \log 2 \rceil$ for almost every n. As a corollary, we obtain a lower bound on the maximum domination number among all orientations of K_n, very close to the trivial upper bound. We also give an explicit construction to obtain a large domination number.

Finally, we consider oriented complete bipartite graphs and show that one of the two classes can always be dominated by a small subset of the other class.

Key words: domination, orientation, random graph.

Threshold Growth Dynamics

95

Tom Bohman MIT

The Threshold Growth Model is a model for crystal growth in the plane. The mode l has two parameters: a neighborhood of the origin $\mathcal{N} \subset \mathbf{Z}^2$ (the neighborhood of x is then $x + \mathcal{N}$) and a threshold θ. T he crystal is a sequence $A_0 \subset A_1 \subset \cdots \subset \mathbf{Z}^2$. Given the initial set, A_0, the rest of the sets are generated in one of two ways. De terministic Threshold Growth Dynamics are generated by the rule:

$$A_{i+1} = A_i \cup \{x \in \mathbf{Z}^2 : |(x + \mathcal{N}) \cap A_i)| \geq eta\}$$

Random Threshold Growth Dynamics are generated by the rule $A_{i+1} = A_i pS$ where S is a set chosen uniformly at random from

$$\{x \in \mathbf{Z}^2 : |(x + \mathcal{N}) \cap A_i)| \geq \theta\}$$

In this talk we discuss some recent results concerning the limiting shape the cr ystal achieves starting from a finite initial set.

Keywords: cellular automata

Matchings in the leave of equitable partial Steiner triple systems

96

Michael E. Raines* and C. A. Rodger, Auburn University
raineme@dms.auburn.edu

In this talk, a simple proof is given to show that if there exists a partial Steiner triple system on n symbols with exactly t triples that contains a m atching M in its leave, then there exists an equitable partial Steiner triple system on n s ymbols with exactly t triples whose leave also contains M. In particular, w e consider the case when M is a (near) 1-factor.

Keywords: equitable, Steiner triple system, matching

97 **Closest Pair for Two Separated Sets of Points**

R. Litiu(*),Univ. of Michigan D.I. Kountanis, Western Michigan Univ.

Let P and Q two distinct sets of points separated by a straight line on the 2D space or by a plane on the 3D space. Without loss of generality, we assume that the separating line is parallel to one of the coordinate axes for the 2D case and the separating plane is parallel to one of the coordinate planes for the 3D case. A closest pair of points according to their rectilinear distance, one from P and the other from Q, is computed with time complexity $O(n \log^{d-1} n)$, where d is the dimension of the space. The worst and best case distributions of the points in P and Q are determined and a generalization to higher than three dimensions is suggested.

98 **Lights Out!**

Alice A. McRae, Dee A. Parks* Appalachian State University

Lights Out is a Tiger Electronics game played on a 5x5 grid of lights. When the game begins some of the lights are on and others off. The object of the game is to turn off all the lights. Depressing a light toggles it and its neighbors to the north, south, east, and west. The game can be represented as a systemof linear equations which, when solved, tells which lights to depress. By looking at the game as a graph domination problem, we can generalize this game to arbitrary graphs. This talk will describe our generalization to other graphs such as paths, trees, and other grids.

Keywords: domination, Lights Out

99 **A New Algorithm for Finding Vertex-Weight**

Y. Daniel Liang, C S,Indiana Purdue University at Fort Wayne
Fort Wayne, IN 46805 liangy@cvax.ipfw.indiana.edu

We present a simple and efficient $O(m + n \log\log n)$ time algorithm for finding a shortest path in a vertex-weight graph with nonnegative weights. The best-known algorithm takes $O(m + n \log n)$ time using Dijkstra's approach with a Fibonacci heap. Our result immediately improves the time bound of many graph algorithms that rely on reduction to the vertex-weight shortest path problem. Furthermore, the new techniques can be applied to developing efficient algorithms in interval graphs and trapezoid graphs. Specifically, we present an $O(n \log\log n)$ time algorithm for finding a vertex-weight shortest path in trapezoid graphs and an $O(n \log\log n)$ time algorithm for finding a dominating set in interval graphs.

Keywords: Domination, graph algorithm, interval graphs, shortest path, Steiner set, trapezoid graph

On Large Sets of Almost Disjoint STS

100

Jane W. Di Paola

Two Steiner triple systems on v points are almost disjoint if they have at most one triple in common. A set of Steiner triple systems on v points is large if every possible triple occurs at least once. In an invited address at the British Combinatorial Conference in 1995, T.S.Griggs and A. Rosa proposed the problem of finding for which values of x and y do there exist large sets of STS (v) in which each triple occurs with frequency either x or y and suggested x=1, y=2 as a starting point.

After showing that the problem requires v=1(mod 6) the author discusses the proposed construction for v=25 and as a preliminary, finds a needed cyclic BIBD with $\lambda=2$ and v=25. This design appears to be new.

101 **From Hall's matching theorem to optimal routing on hypercubes**

Shuhong Gao*, Beth Novick, Department of Mathematical Sciences, Clemson University,
Ke Qiu, Jodrey School of Computer Science, Acadia University, Nova Scotia, Canada

A well-known theorem of P. Hall says that a family of finite sets has a system of distinct representatives (SDR) if and only if the union of any k sets contains at least k distinct elements. We strengthen this theorem, showing that Hall's "marriage" condition implies certain properties of the permutations of the sets. This result is used to solve a problem in optimal routing on hypercubes. Specifically, we show exactly when there are internally node-disjoint shortest paths from a source node to any given set of s ($s \leq n$) target nodes on an n-dimentional hypercube. We also show that there always exists a collection of internally node-disjoint paths from a source node to any given set of s ($s \leq n$) target nodes such that each path is either shortest or near-shortest (no more than 2 from the shortest). We give an efficient algorithm to contruct these paths.

Key words: Hall's matching Theorem, hypercubes, routings, shortest paths, efficient algorithms.

102 **Independence and Generalized Minimum Degree**

Gayla S. Domke (Georgia State University), Lisa R. Markus* (Furman University)

Let $G = (V, E)$ be a graph on n vertices. Let $\beta_0(G)$ denote the order of a largest independent set of vertices in G. For $1 \leq t \leq \beta_0(G)$, the generalized minimum degree, $\delta_t(G)$ is defined as follows: $\delta_t(G) = min\{|N(S)| : S \text{ is an independent set with } t \text{ vertices}\}$. We show that for any graph G, $\beta_0(G) + \delta_t(G) \leq n$ for all t, $1 \leq t \leq \beta_0(G)$. For an arbitrary graph G we furnish two conditions which are necessary if $\beta_0(G) + \delta_t(G) = n$ and show that for a split graph these conditions are sufficient. Furthermore, we present necessary and sufficient conditions for certain classes of graphs to achieve $\beta_0(G) + \delta_t(G) = n$ for small values of t or $\delta_t(G)$.

Key words: Independence number, generalized minimum degree.

Directed Acyclic Graph Scheduling with Communication Delay on Two Processors

103 Wing Ning Li (wingning@uafsysb.uark.edu), University of Arkansas

Given a set J={ $t_1, t_2, \ldots t_n$ } of tasks, with each t_i having execution time 1, a start-time $s_i \geq 0$ and a deadline $d_i \geq 0$, a communication delay c, and a set of precedence constraints which restrict allowable schedules, the problem of determining whether there exists a schedule on two identical processors that executes each task in the time interval between its start-time and deadline is examined. Without considering the communication delay c, an $O(n^3)$ algorithm exists to solve the problem. We show that with the communication delay the problem is NP-complete.

A construction of a strongly regular graph (64,35,18,20)

W. L. Golightly, W.H. Haynsworth*, D.G. Sarvate Department of
104 Mathematics, University of Charleston, Charleston SC

A symmetric difference method of construction of the Clebsch graph is given by Cameron and Van Lint and by Shirkhande and San. It is shown that this method produces only four strongly regular graphs. In particular, we obtain a s regular graph (64, 35, 18, 20).

Key words: Strongly regular graph

Optimal Packings and Coverings of the Complete Directed Graph with 3-Circuits and with Transitive Triples

Robert B. Gardner, *Dept of Math; East Tennessee State University; Johnson City, TN 37614*

105

Maximal packings and minimal coverings of the complete directed graph with isomorphic copies of the directed graph d are studied in the cases of d being either of the two orientations of a 3−cycle. Necessary conditions are given which are shown to be sufficient through direct constructions.

keywords: packings, coverings, decompositions

Independence, Domination and Generalized Maximum Degree

Gayla S. Domke* (Georgia State University), Jean E. Dunbar (Converse College)
Teresa W. Haynes and Debra J. Knisley (East Tennessee State Universi ty)
Lisa R. Markus (Furman University)

106

Let $G = (V, E)$ be a graph on n vertices. Let $\beta_0(G)$ denote the order of a largest independent set of vertices, $\gamma(G)$ denote the the order of a smallest dominating set of vertices and $i(G)$ denote the order of a smallest independent dominating set of vertices in G. For $1 \leq t \leq \beta_0(G)$, the generalized maximum degree, $\Delta_t(G)$ is defined as follows: $\Delta_t(G) = max\{|N(S)| : S \text{ is an independent set with } t \text{ vertices}\}$. For an arbitrary graph G and for some t, $1 \leq t \leq eta_0(G)$, we furnish conditions which are necessary if $\gamma(G) + \Delta_t(G) = n$ and are sufficient for a graph to achieve $n - t \leq \gamma(G) + \Delta_t(G) \leq n$. We also give some classes of graphs where $\gamma(G) + \Delta_t(G) = n$ or $i(G) + \Delta_t(G) = n$ for some t, $1 \leq t \leq \beta_0(G)$.

Key words: Domination number, independent domination number, gen eralized maximum degree.

A Relationship Between Two Binary Tree Coding Schemes

107 *Rodney O. Rogers, Embry-Riddle Aeronautical University*

A binary tree coding scheme is a bijection between the set of n-node binary trees, $n \geq 1$, and a set of tuples of integers called codewords or encodings. In a coding scheme devised by Zerling, each integer value in the encoding of a tree t reflects the number of edge rotations which must systematically be performed to transform t into a tree where every child node is a left child. Lucas's scheme uses the same codeword sets as Zerling, but implicitly rotates different edges to achieve a tree where every child node is a right child. As a result, her codeword for t is not the same as Zerling's. Intuitively, these two coding schemes seem closely related. Using a known correspondence between n-node binary trees and triangulations of vertex-labelled convex (n+2)-gons, we show that if p is the triangulation corresponding to a binary tree encoded w by Lucas, then w under Zerling's scheme encodes the binary tree corresponding to the triangulation produced when each label of p is shifted clockwise one vertex.

Key words: graph theory; binary trees; edge rotations; polygon triangulations; binary tree coding schemes; encoding and decoding algorithms

Inductive Extensions of some Z-cyclic Whist Tournaments

108 Philip A. Leonard, Arizona State University, Tempe AZ 85287

A method is given for extending Z-cyclic whist tournaments of two types: those in which the number of players is the square of a prime q = 4n+3, and those in which the number of players is q+1 for such a prime. This method, and its connection to some recent work of N.J. Finizio, results in the construction of some new infinite families of Z-cyclic whist tournaments. Techniques from earlier work on starters form the basis for the method.

114

SPACES OF MATRICES OF FIXED RANK OVER GF(2)

LeRoy B. Beasley, Utah State University, Logan UT 84322-3900, USA

We investigate spaces of matrices over the field of two elements. The only known example of a space of m x n matrices of rank k or 0 and of dimension larger than max(m,n) is when the k = 2 and m = n = 3. When k = 2, we show that this is the only exceptional case.

115

On Optimal Orientations of Graphs

Aleksandar Pekeč, BRICS, University of Aarhus

It is well known that a graph G admits a strongly connected orientation $\vec{G}$ if and only if G is 2-edge-connected. There are several efficient algorithms for finding $\vec{G}$. However, in most applications there is a need for an optimal orientation $\vec{G}$ with respect to some measure of optimality. Finding $\vec{G}$ that minimizes the oriented diameter or the oriented radius are known to be NP-hard problems. I will present a simple proof that finding $\vec{G}$ that minimizes $\sum_{u,v \in V(G)} \vec{d}(u,v)$ is also NP-hard. The main part of the talk will be devoted to optimal orientations of certain classes of graphs that are important in practice, such as circular grids, symmetric triangular grids,... There is a common flavor in the constructions of optimal orientations, as well as in the optimality proofs. I will point out these similarities and discuss possible underlying principles in the design of optimal orientations of graphs. Finally, I will state several open problems.

Keywords: Strongly Connected Orientations, Graph Algorithms, Complexity

Optimal Routing in 2-dimensional Meshes with Faulty Blocks

116

Jie Wu, CSE, Florida Atlantic University, jie@cse.fau.edu

In this paper, we propose a sufficient condition for optimal routing in 2-dimensional meshes with faulty blocks. Unlike many traditional models that assume all the nodes know global fault distribution, our approach is based on the concept of limited global fault information. Fault information is distributed on limited number of nodes while it is still sufficient to support optimal routing.

117 **Topology of some graph and hypergraph complexes**

Eric Babson (MSRI), Anders Björner (KTH), Svante Linusson (Stockholm), John Shareshian* (MSRI), Volkmar Welker (Essen)

For positive integers i, k, n, the collection of all k-uniform hypergraphs on a labeled n-set for which the underlying graph can be disconnected by removing $i-1$ vertices forms a simplicial complex on the set $\binom{[n]}{k}$. Questions concerning the topology of these complexes arose in Vassiliev's study of knots in three-space. The case $i = 1$ was previously handled by the second and fifth authors. We examine the case $i = 2$, determining the homotopy type and the Lefschetz character for the natural action of S_n when $k = 2$, and the Euler characteristic when $k > 2$.

Keywords: graph complexes, knot theory, poset topology

118 **Matrix Quadratic Residues mod p^n**

Timothy P. Donovan, Midwestern State University, Wichita Falls, Texas
donovan@abacus.mwsu.edu

In an earlier paper, matrix squares over finite fields were considered. In this paper the matrix congruence $X^2 \equiv A \pmod{p^n}$ for A a 2 x 2 matrix, p an odd prime, and $n \geq 1$. The emphasis is on the matrices A for which there is a solution. With some restrictions a count of the residues is provided. The main approach is through the use of similarity and a simple canonical form.

The Outer Crossing Number of a Graph: A New Parameter with Results for $K_{m,n}$

Deborah Bergstrand* and Alexander Woo, Williams College, Williamstown, MA 01267

119

A drawing of a simple graph G is a mapping of G into the plane such that each vertex is mapped to a different point and each edge is mapped onto a homeomorph of [0,1] with appropriate endpoints. Such a drawing is good if two distinct edges intersect at most once, either at a common end vertex or at a crossing. A drawing is outerfacial if there exists a point in the plane such that a curve can be drawn from it to any vertex in G without intersecting any part of the drawing. We define what we believe to be a new graph parameter, the outer crossing number of G, denoted $ocr(G)$, to be the minimum number of crossings among all good, outerfacial drawings of G. In addition to some basic observations, we present simple formulas for $ocr(K_{n,n})$ and $ocr(K_{qn,n})$ together with optimal drawings. We also present a recursive formula for computing $ocr(K_{m,n})$ in general and a recursive process for creating an optimal drawing. We also extend the parameter to surfaces of genus greater than zero.

Key words: graph drawing, crossing number, outer crossing number

120 **On Distributed Loop Networks**

Weizhen Gu, Southwest Texas State University

Let N be a positive integer and let $A = \{a_1, a_2, \cdots, a_k\}$ be a set of integers with $1 = a_1 < a_2 < \cdots < a_k < N$. A Cayley digraph $G(N, A)$ associated with $Z/(N)$ and A has its vertex set $\{1, 2, \cdots, N\}$ and arcs ij if $j = i + a$ for some $a \in A$. A Cayley digraph can be considered as a model for loop networks. For positive integers d and k, let $N(d, k)$ be the maximum N such that the diameter of $G(N, A)$ is no more than d for some A with k elements. Hsu and Jia established a formula for $N(d, 2)$ and lower bounds for $N(d, k)$ with $k \geq 3$. These lower bounds have been improved by S. Chen and the author in 1992 and further by Jia and Su in 1995. In this paper, a further improvement of these lower bounds is obtained.

121

Reflection Groups, Matroids, and Signed Graphs

Lori Fern, Gary Gordon*, Jason Leasure and Sharon Pronchik, Lafayette College

Let H be a subgroup of an n-dimensional hyperoctahedral group which is generated by reflections. Associated with the normal vectors of the reflecting hyperplanes of H is the dependence matroid M and a signed graph G, both of which encode symmetry information. We explore the connections among these three objects, concentrating on the correspondence between the subgroup H and the automorphism group of the matroid M. Signed graphs are central to our proof methods and are useful for visualizing higher dimensional objects. In particular, we show how certain closure operations on the associated signed graph are induced by closure of the subgroup H and matroid closure in M..

VALUE SETS OF SOME BINOMIALS OVER FINITE FIELDS

Alberto Cáceres, Math Dept, Univ. of Puerto Rico–Humacao, Humacao PR 00791

122

Any function defined on a finite field into itself is representable as a polynomial. A polynomial over a finite field F which, upon evaluation is a one–to–one function $F \rightarrow F$ is called a *Permutation Polynomial* (PP). These objects are of interest in information theory, cryptography and general algebra.

The Dickson-Hermite's (1920) is the best known criterion to decide if a polynomial is a PP, however due to the high powers that this criterion involves, it is not workable for even small fields. Research then has turned its attention to the study of the value sets of polynomials, i.e., the range, and its size, of the induced function. General bounds on the size of value sets are already known and the size of some value sets of monomials and some particular polynomials have been already calculated. In this work we evaluate the size of the value sets of binomials of the form $aX^r + bX^{q-1-r} \in F[X]$ when r and $q-1$ are coprime. Base field is either a prime or an extension field. These computations lead us to improve some of the known bounds on value sets and should clear the way for the search of new PPs.

Key words: permutation polynomials, value sets.

123

A Construction Which Yields Strongly Well-Covered Graphs

Mike Pinter, Belmont University, Nashville, TN

A graph is well-covered if every maximal independent set is a maximum independent set. If a well-covered graph G has the additional property that G-e is also well-covered for every edge e in G, then we say the graph is strongly well-covered. A construction which produces connected strongly well-covered graphs with arbitrarily large (even) independence number will be exhibited. The construction is in terms of a graph product given in a paper by Topp and Volkmann several years ago.

KEYWORDS: well-covered, independent set, independence number, edge-critical

124

Constructive Characterization Of Delta-Wye Graphs Having Splits But No Good Splits

L. Leslie Gardner, University of Indianapolis

This paper extends the decomposition theory of 3-connected graphs by presenting a constructive characterization of the delta-wye graphs having splits but no good splits under the Coullard-Gardner-Wagner definition of 3-decomposition. Graphs with splits but no good splits are interesting because they are the well-struct ured members of decompositions. Bonds and polygons are the well-structured members of the Tutte decomposition of 2-connected graphs, and wheels and twirls are the well-structured members of the Coullard-Gardner-Wagner decomposition. The delta-wye graphs having splits but no good splits are a rich and complex class with a unifying structural property that allows them to be characterized completely. This class of graphs illustrates many of the issues that arise in extending decomposition results to graphs of higher connectivity.

Key Words: graph decomposition, delta-wye graphs, constructive characterization

Finite convex sets and the beta invariant. Preliminary report 125

Christina Ahrens* (Oberlin College), **Gary Gordon** (Lafayette College), **Elizabeth McMahon** (Lafayette College)

A generalization of Crapo's β invariant is applied to finite point sets with convex subsets. Specific results are presented, and the following theorem is outlined: For a finite subset C of the plane, $\beta(C) = |int(C)|$, where $|int(C)|$ is the number of interior points of C, i.e., the number of points of C which are not on the boundary of the convex hull of C.

keywords: convex set, β invariant

SOME METHODS OF COMMUTATIVE ALGEBRA ON CODING THEORY

126

C. Rentería (Escuela Superior de Física y Matemáticas, IPN-MEXICO, e-mail: `renteri@esfm.ipn.mx`)
H. Tapia-Recillas* (Departamento de Matemáticas, U. A. Metropolitana–I, MEXICO, e-mail: `htr@xanum.uam.mx`)

Since V.D. Goppa introduced the use of algebraic geometry in the study of error detecting-correcting linear codes, particularly algebraic curves over finite fields, many interesting and outstanding results have been obtained in this area of research. Lately some concepts and results from commutative algebra have been used in the theory of linear codes, such as the case of Gröbner Bases for decoding. The purpose of this talk is to describe how some other concepts from commutative algebra such as the Hilbert function, the a-invariant of an ideal, projective resolutions, etc. can be used to describe properties of linear codes. As an example the case of Reed-Muller codes will be presented.

127 Listing Nearly Planar Graphs

Hiryoung Kim* and Alan P. Sprague, UAB, Birmingham, AL
kim1@cis.uab.edu

Let G be a graph and $v_1, v_2, \cdots, v_n$ be the vertices of G. Let G_i be a vertex deleted graph obtained from G by deleting a vertex v_i and its incident edges. If G is nonplanar and for every v_i, G_i is planar, then G is a nearly planar graph (NPG). Wagner (1967) and O'Neil (1973) study NPGs, in particular, the latter in regard to the graph reconstruction problem. NPGs are an infinite class of graphs, however, Wagner shows that all NPGs are necessarily finite and can be determined explicitly. Though Wagner provides well-defined classes of NPGs, they are not completely explicit. Thus our objective of the paper is to provide more explicit description of NPGs as a finitely describable class as well as characteristics of NPGs. Our study is partly experimental by implementing a simple algorithm to generate NPGs. Also, by listing NPGs, we further hope to take a close look at the NPG reconstruction problem.

Keywords: Planar Graphs, Graph Reconstruction.

Fast Backtracking Principles Applied to Find New Cages

128

Brendan McKay, Australian National Univ.;
Wendy Myrvold* and Jacqueline Nadon, Univ. of Victoria

We describe rules of thumb based on many years of practical experience for designing fast exponential backtracks. These ideas were successfully applied to the problem of characterizing $(3, g)$-cages, the minimum order 3-regular graphs of girth g. It took just 5 days of cpu time (compared to 259 days for previous authors) to verify the $(3, 9)$-cages, and we were able to confirm that $(3, 11)$-cages have order 112 for the first time ever. The lower bound for a $(3, 13)$-cage is improved from 196 to 202 using the same approach. Also, we determined that a $(3, 14)$-cage has order at least 258.

Keywords: Graph theory, girth, cage, computer techniques, backtracking.

129

Matroid Ramsey Numbers

J.E. Bonin - George Washington; J. McNulty, Univ.of Montana; T.J. Reid*, Univ. of Mississippi

A tight upper bound on the number of elements in a connected matroid with fixed rank and largest cocircuit size is given. This upper bound is used to show that a connected matroid with at least thirteen elements contains either a circuit or a cocircuit with at least six elements. In the language of matroid Ramsey numbers, $n(6,6) = 13$; this is the largest currently known matroid Ramsey number.

Shifting on finite vector spaces

Éva Czabarka, University of South Carolina, czabarka@math.sc.edu

130

Erdős, Ko and Rado have proved that for $2k < n$, the maximum cardinality of a system of pairwise intersecting of k-subsets of an n-element set is at most $\binom{n-1}{k-1}$, and that in an intersecting system system with cardinality $\binom{n-1}{k-1}$ every k-set contains a fixed element of the base-set. In this proof they introduced a shifting technique on the sets: for a pair of integers i, j such that $i < j$ we replace j with i in every set of the system satisfying certain conditions. This technique has proved very useful. A system of k-dimensional subspaces of an n-dimensional space is pairwise intersecting, if the intersection of any two subspaces in the system contains a nonzero vector. The analogue of the Erdős-Ko-Rado theorem for finite vector spaces states that if V is an n-dimensional space over the q-element field, then a pairwise intersecting system of k-dimensional subspaces of V contains at most $\begin{bmatrix} n-1 \\ k-1 \end{bmatrix}_q$ elements, provided $n > 2k$. This was proved by Hsieh using computations.

In this work we define a shifting technique for finite vector spaces and use it to prove Hsieh's theorem. Using Greene and Kleitman's proof for the case $n = 2k$, the shifting will give an induction proof for Hsieh's theorem.

131

A STRENGTHENING OF THE KURATOWSKI PLANARITY CRITERION FOR QUASI 4–CONNNECTED GRAPHS

Alexander Kelmans, Rutgers University and University of Puerto Rico

The Kuratowski planarity criterion for graphs is well known:

(1) *A graph is non–planar if and only if it contains a subdivision of either K_5 or $K_{3,3}$.*

In 1981 we proved the following strengthening of the Kuratowski theorem.

(2) *A 3–connected graph distinct from K_5 is non–planar if and only if it contains a subdivision of $K_{3,3}$ in which three edges (forming a matching) are not subdivided.* We also described a construction providing infinitely many 3–connected non–planar graphs that do not contain subdivisions of $K_{3,3}$ with more than three non–subdivided edges. Therefore the above theorem is tight. In 1982 we showed that the planarity problem for 3–connected graphs can be easily reduced to the problem for so called *quasi 4–connected graphs* (i.e. 3–connected graphs having no essential vertex 3–cuts and no triangles), and we proved the following analogue of the Kuratowski theorem for quasi 4–connected graphs.

(3) *A quasi 4–connected graph is non–planar if and only if it contains a subdivision of either Q or W where Q is the graph–skeleton of the 3–dimensional cube with one main diagonal and W is the graph obtained from the 8–cycle by adding all four main diagonals.*

Our new result is the following strengthening of the Kuratowski theorem:

(4) *A quasi 4–connected graph is non–planar if and only if it contains a subdivision of $K_{3,3}$ with at least five non–subdivided edge s.*

We also show that this theorem is tight.

Keywords: graph, planarity, subdivision, 3–connectedness, quasi 4–connectedness.

132

Toroidal Cayley Graphs with Hexagonal Faces

Linda Valdes, San Jose State University

All groups are found that have alternative Cayley graphs with hexagonal embeddings on the torus. The groups are distinguished by the symmetry of their embeddings; that is, strong, weak symmetry or nonsymmetry.

On Dowling Lattice Homology and Lie Superalgebras

133

Eric Gottlieb* and Michelle Wachs, University of Miami

There is a remarkable relationship between the representation of the symmetric group on the homology of the partition lattice and the free Lie algebra. This relationship has been studied from various view points by Joyal, Barcelo, Hanlon, and Wachs. Bergeron extended this relationship to the action of the hyperoctahedral group on the homology of the signed partition lattice. In this paper we further extend the relationship to general Dowling lattices. A wreath product group acts naturally as an automorphism group of the Dowling lattice, generalizing the action of the symmetric group S_n on the partition lattice Π_{n+1} and the action of the hyperoctahedral group on the signed partition lattice. The action of the wreath product group on the homology of the Dowling lattice is shown to be isomorphic (up to sign) to its action on a multilinear component of the enveloping algebra of a certain Lie (super)algebra. For the symmetric group and the partition lattice this reduces to a result of Stanley that the representation of S_n on Π_{n+1} is the regular representation. For the hyperoctahedral group and the signed partition lattice, this generalizes the results of Bergeron.

On Primitive Optimal Normal Bases in Finite Fields

134

Nelson A. Carella
City University of New York

Let the integer n≡1,2,3 mod4, 2n+1 be a prime, and suppose the order of 2 in F_{2n+1} is ord(2)=n or 2n. Further, assume ω is a (2n+1)th primitive root of unity in the (2n+1)th cyclotomic field extension $F_2(\omega)$ of the field of two elements F_2. In the main result of this paper we will provide some theoretical foundation for the computation of the order or index of the element $\omega+\omega^{-1}$, called a period of degree n, in F_q over F_2, $q=2^n$. This result proves an expanded version of a conjecture concerning the primitivity of the element $\omega+\omega^{-1}$ stated in *S. Gao and S. Vanstone, On Orders of Optimal Normal Basis Generators, Mathematics of Computations, Vol. 64, No. 211, July 1995, p. 1227-1233.* We then combine various results into a single result, which we refer to as the *Primitive Optimal Normal Basis Theorem*.

Keywords: Primitive roots; Optimal Normal Basis; Finite Fields.

Line Greatest Common Subgraphs

135

* Kathleen Cherry, U.S. Government   Robert Brigham, UCF

(Keywords: line graph, greatest common subgraph)

The concept of line greatest common subgraphs (linegcs) evolved in an attempt to answer the following question-- when does $L(gcs(G_1,G_2))$ = gcs $(L(G_1),L(G_2))$ for given graphs G_1 and G_2? This question does not evoke a simple answer. Difficulty results because both sides of the equation are sets of graphs and many times the cardinality of the two sets differ, which means that set equality is doomed to fail in these cases. So, the more interesting question evolved to-- do the sets have any common members, that is, is there a nontrivial graph member of $L(gcs(G_1,G_2)) \cap gcs(L(G_1),L(G_2))$? If such a graph member exists, then G_1 and G_2 are said to have a line greatest common subgraph, that is, linegcs(G_1,G_2) = G, for which corresponding $L(G) \in L(gcs(G_1,G_2)) \cap gcs(L(G_1),L(G_2))$.

Introductory material will include examples of graph pairs G_1 and G_2 that do have a linegcs member and a demonstration of linegcs existence. Two demonstration/proofs will characterize the relationship between graph pairs G_1 and G_2 and their greatest common subgraph member G that prevents G from being a member of linegcs(G_1,G_2).

The Planar Finite Fields of Prime Order

136

Dawn M. Jones, Western Michigan University

A Cayley graph is a way to model groups by directed graphs. We extend this idea to model finite fields. Using this we define the genus of a finite field. A characterization of the planar finite fields of prime order is given.

Key Words: finite fields, Cayley graphs, genus

137

Path Vectors and Optimal Trees

Alfred J. Boals, Western Michigan University
Gary Chartrand, Western Michigan University
Jamal Nouh, Beirzeit University
Donald W. VanderJagt*, Grand Valley State University
Julie Yates, The Nulty Agency, Kalamazoo, MI

The path vector of a digraph D of order n is the $(n-1)$-vector $(d_1(D), d_2(D), \ldots, d_{n-1}(D))$, where $d_i(D), 1 \leq i \leq n-1$, is the number of (directed) paths of length i in D. Here we restrict our attention to directed trees. For directed trees T_1 and T_2 of order n, T_1 is said to dominate T_2 if

$$\sum_{i=1}^{j} d_i(T_1) \geq \sum_{i=1}^{j} d_i(T_2)$$

for all j $(1 \leq j \leq n-1)$. Let $\mathcal{T}$ be the set of directed trees of a fixed order. A directed tree $T \in \mathcal{T}$ is dominant if T dominates every tree in $\mathcal{T}$. Several results dealing with these concepts are presented. A tree T is optimal if there exists an orientation T' of T that dominates all orientations of T. Several classes of optimal trees are described.

Key Words: digraph, path vector, optimal tree.

138

The Transitive Sylow p-subgroups of S_{p^2} and the Cayley Isomorphism Problem

Edward Dobson,Louisiana State University

Let p be prime. We determine the transitive Sylow p-su bgroups of S_{p^2}. This allows us to determine necessary and sufficient conditions f or two Cayley objects of $\mathbf{Z}_p^2$ or $\mathbf{Z}_{p^2}$ in any class of combinatorial objects to be isomorphic.

139

Parallel Knockouts in Graphs

Peter J. Slater, University of Alabama in Huntsville

Keywords: domination, independence, irredundance, knockouts.

The independence/domination/irredundance sequence of parameters, ir(G) $\leq \gamma(G) \leq i(G) \leq \beta(G) \leq \Gamma(G) \leq IR(G)$, arises because every maximal independent vertex set dominates, and every minimal dominating set is irredundant. The sequence has been extended outward by considering maximal irredundant sets. To extend the sequence inward and have a pair of parameters between i and β, one seeks a property such that subsets minimal with respect to this property are independent. Motivated by this, the knockout concept arose. Joint work with Douglas Lampert will be described, including a successful insertion $i(G) \leq nk(G) \leq NK(G) \leq \beta(G)$.

The principal focus of this paper will be new results and open questions concerning situations in which knockouts are performed in parallel.

Additional Results on Fairness in Spouse-Avoiding Mixed Doubles Round-Robin Tournaments

140

David R. Berman, Sandra C. McLaurin*, Douglas D. Smith
University of North Carolina at Wilmington

Brayton, Coppersmith, and Hoffman[1974] represent a spouse-avoiding mixed doubles round-robin tournament for n couples (SAMDRR(n)) as an idempotent self-orthogonal Latin square (ISOLS(n)) of order n. Berman, McLaurin, and Smith [1996] consider "fairness" for SAMDRR's in which both men and women are ranked from strongest to weakest. We extend the results of that paper. We show that every cyclic ISOLS(n) has fair rankings. We consider products of tournaments and techniques for finding tournaments that are fair for certain rankings.

Key words: Latin square, self-orthogonal, tournament.

141

Domination Graphs of Tournaments

David Guichard, Whitman College

The domination graph of a digraph has the same vertex set as the digraph, and an edge between vertices u and v if and only if there are arcs (w, u) and (w, v) for some vertex w. At the 1996 Southeast Conference, Richard Lundgren presented some results on domination graphs of tournaments. Left undecided was the question: Can a disjoint union of five caterpillars be the domination graph of a tournament? Even in the case of five paths this was unknown. We present a complete characterization of the forests of non-trivial caterpillars that are the domination graphs of tournaments. The only new case is five caterpillars, but we provide a single proof that covers all cases.

Keywords: domination graph, tournament, caterpillar

142

Elementary proofs that Z_p^2 and Z_p^3 are CI-groups

Brian Alspach* and Lewis Nowitz
Simon Fraser University and New York City

Let G be a finite group. If every Cayley graph X on G has the property that whenever Y is isomorphic to X, there is an isomorphism of X to Y that is also a group automorphism of G, then G is said to be a CI-group. Using O'Nan's Theorem, C. Godsil proved that Z_p^2 is a CI-group. Using the classification of finite simple groups, M. -Y. Xu proved that Z_p^3 is a CI-group. E. Dobson proved the latter result without using the classification. We provide elementary proofs of these two results.

143

Analyzing Graph Theory

Gary Chartrand*, Western Michigan University
Linda Eroh, Western Michigan University
Michelle Schultz, University of Nevada, Las Vegas
Ping Zhang, Western Michigan University

We look at graph theory in terms of several metric spaces associated with graphs and in terms of several parameters defined on graphs.

Key Words: graph, metric space, graphical parameter.

144

Critical sets for orthogonal latin squares: first steps.

A. D. Keedwell, University of Surrey, Guildford, England.

A *critical set* C for a single latin square of order n is a subset of its n^2 cell entries which together determine the square uniquely (that is, these cell entries are a subset of only one latin square) but which has the additional property that, if any member of C is deleted to leave a subset C', then C' no longer has unique completion.

We extend this idea to the case of t pairwise orthogonal latin squares of order n. There are now n^2t cell entries. We obtain a lower bound for the size of a minimal critical set of these cell entries and show that this lower bound can be attained for orders less than 7. In the case of order 7, we give an upper bound for the size of a critical set.

Key words: latin square, orthogonal latin squares, critical set, uniquely completable set.

Smallest Nonmeshy Trees in Triangular and Hexagonal Lattices

145 Christian Thurmann, Technische University at Braunschweig

The smallest numbers of vetices are determined for trees with maximum degree d such that a tree is not isomorphic to a subgraph of the triangular or hexagonal lattice. (Common work with Heiko Harborth.)

146

Vertex Disjoint Cycles on the Cayley Color Graph

A. Gregory Starling, University of Arkansas

In 1996, Starling and Klerlein studied vertex disjoint cycles on a subgraph of $C_n X_2 C_m$ with m even. The subgraph is a Cayley color graph on $(Z_m, +)$ and a set of generators $\{1, 2, m-2\}$. That paper gave an algorithm for the coverings of the directed graph by vertex disjoint cycles represented by lengths of the cycles in partitions of the integer m. In this paper, the algorithm is extended for the case when m is odd.

Keywords: vertex disjoint cycles, Cayley color graph, partitions of integers

147

On uniquely intersectable graphs

N.V.R. Mahadev and Tao-Ming Wang(*), Northeastern University

P. Erdos, A. Goodman, and L. Posa first introduced the topic *intersection graphs* in 1966. For a nonempty set S, let α be a family of subsets of S, We say a graph G is an *intersection graph with respect to* (S, α) if α is in one-to-one correspondence with the vertex set of G and two vertices in G are adjacent if and only if the corresponding subsets in α have nonempty intersection. (S, α) is called a set representation of G, and (S, α) is minimal if $|S|$ is the least among all set representations. It is well known that every finite simple graph is an intersection graph with respect to (S, α) for some S. In 1977, R. Alter and C.C. Wang introduced the concept of *unique intersectable(ui) graphs*, i.e. those graphs with unique minimal set representation up to isomorphisms. They showed that triangle-free is a sufficient condition for a graph to be ui. In 1990, M. Tsuchiya studied the concept of unique intersectability with respect to antichains(uia) and showed that triangle-free is also a sufficient condition for a graph to be uia. In this paper we generalize the above results by proving that if a graph is diamond-free and twins-free, then it is ui and if a graph is diamond-free and non-pendant brothers-free, then it is uia. Also we characterize diamond-free graphs that are ui and the line graphs of triangle-free graphs that are ui. We also consider the concept of unique intersectability with respect to multifamilies(uim) and obtain a necessary and sufficient condition of such graphs. Most recently, we have obtained a characterization of hereditary uim graphs, namely NP_4-free and NC_4-free, which also generalized some of previous results.

Keywords: intersection graphs, uniquely intersectable graphs

Transversals in Cyclic Latin Squares

148

Dean Clark and James T. Lewis*
University of Rhode Island

The left cyclic Latin square L of order n has first row $0, 1, \ldots, n-1$ and subsequent rows obtained by cyclic shifts of one position to the left. A transversal of a Latin square of order n is a set of n positions, no two in the same row or column, containing each of the n symbols. We investigate the number of transversals of L and the number of double transversals; that is, simultaneous transversals of both L and L', the right cyclic Latin square. Use of transversals in some games will be mentioned.

Key words: Latin square, transversal

149

Stable Graphs for Inclusive Connectivity

D.W. Cribb, U.S. Air Force Academy, cribbdw.dfms@usafa.af.mil

The inclusive edge (vertex, mixed) connectivity of a vertex v is the minimum number of edges (vertices, graph elements) whose removal yields a subgraph in which v is a cutvertex. A λ_i- (κ_i-, μ_i-) neutral edge is an edge whose removal does not change the respective inclusive edge (vertex, mixed) connectivity value of any vertex. Also, a graph is λ_i, κ_i, or μ_i-stable if the sum of all the respective λ_i , κ_i, or μ_i- values remains the same no matter what edge is deleted from the graph. An investigation into a special case of neutral edges for inclusive connectivity provides us with the first example of a type of stable graph. Various combinations of stability among the three parameters is explored.

Keywords: Inclusive connectivity, neutral edges

150

1-factors and Connectivity in Vertex-transitive Graphs

Yusheng Qin, Dept. Math. & Stats., Simon Fraser University, CANADA

Bill Jackson raised the following problem: Let X be a vertex-transitive graph of even order. Does X contain a 1-factor F such that $X - F$ is connected? In this paper we obtain some partial results on this problem.

Key words: vertex-transitive graphs, Cayley graphs, 1-factors, connectivity

151

On Bridgland's Tree Degree Problem

Robert E. Jamison, Mathematical Sciences, Clemson University,Clemson, SC 29634-1907
off: (803) 656-5219fax: (803) 656-5230 rejam@clemson.edu

The following intriguing problem was posed by Michael Bridgland in connection with the closure of isomorphism types of trees:

For which integers $0 < d < n$ does there exist a graph of order n and maximum degree d which contains (up to isomorphism) every tree of order n and maximum degree at most d?To simplify the discussion, a graph graph yielding a solution to Bridgland's problem for $d < n$ will be called [d,n]-arboreal. This talk will focus on computer searches for [d,n]-arboreal graphs for small n ($<=12$) and present a couple of existence results for more general situations.

KEYWORDS: Tree, isomorphism, maximum degree

152

TUPLES IN INFINITE LATIN SQUARES

Michael E. Mays, West Virginia University

We adapt a construction for countably infinite sequenceable groups to construct "quarter plane" and "full plane" infinite latin squares whose entries are integers in which every m-tuple of integers occurs exactly once as adjacent entries in some row. We give some specific examples of sets of finite latin squares that contain special sets of m-tuples. Other variations are possible, including the use of more than one occurrence of each m-tuple or an embedding in more than one latin square.

153

TOUGHNESS AND TOUGH COMPONENTS

M.D. Plummer, Vanderbilt University,Nashville, Tennessee 37240, USA

Let $t(G)$ denote the toughness of graph G. If $S \subseteq V(G)$ is a cutset of G and $\omega(G-S)$ is the number of components of $G-S$, then if $|S|/\omega(G-S) = t(G)$, set S is called a *tough set* in G and any component C of $G-S$ is called a *tough component* in G.

Theorem: If C is a tough component of graph G and $t(G) = c/d$ where c and d are positive integers with $c \leq |S'|$ and $d \leq \omega(G-S')$, where S' is any largest tough set in C, then:

(i) $t(C) \geq ((d-1)/d)t(G)$ and hence if $d \geq 2$, $t(C) \geq (1/2)t(G)$.

(ii) Moreover, if in addition, $t(G) \geq 1$, then $t(C) \geq (1/2)\lceil t(G) \rceil$.

Sharpness of the bounds in this Theorem are discussed. Finally, a corollary of the Prime Number Theorem is used to investigate forbidden values for the toughness of tough components.

keywords: *toughness, tough component*

Circulant Graphs which are Cayley on a Second Group

154

Joy Morris, Simon Fraser University; morris@cs.sfu.ca

This paper extends some work by Anne Joseph, by determining two other equivalent conditions for when a circulant graph on p^n vertices (p an odd prime) can be described as a Cayley graph on some other group, and on which groups of order p^n such a graph is a Cayley graph.

Structure and Recognition of n-Tuple Vertex Graphs

155

David Weinreich, University of Memphis

The n-tuple vertex graph of a graph is the graph whose vertex set consists of all n-tuples of V(G) and whose edge set consists of all pairs of n-tuples (u,w) such that the symmetric difference between u and w is an element of E(G). We discuss the uniqueness of this mapping and some of its structural characteristics, particularly its relationship with the line graph L(G).

KEYWORDS: Graph theory, line graph, graph-valued function

156

Amalgamating Infinite Latin Squares

Anthony J. W. Hilton, Dept. of Math., University of Reading, UK
Jerzy Wojciechowski*, Dept. of Math., West Virginia University

A latin square is an n by n matrix whose elements are in $\{1, \ldots, n\}$ and no element is repeated in any row or column. Given equivalence relations on the set of rows, the set of columns, and the set of symbols, respectively, we can use these relations to identify equivalent rows, columns and symbols, and obtain an amalgamated latin square. There is a set of natural equations that are neccessary to be satisfies by an amalgamated latin square. Using these equations we can define the notion of an outline latin square and it becomes an obvious conclusion that an amalgamated latin square is an outline latin square. In 1980, Anthony Hilton proved that the opposite implication holds as well, that is, that every outline latin square is an amalgamated latin square. In this paper, we present an infinite generalization of that result.

157

THE CROSSING NUMBER OF $C_7 \times C_7$

Mark S. Anderson*, Rollins College,
R. Bruce Richter, Carleton University,
Peter Rodney, Carleton University

Using the curve systems approach developed by Richter and Thomassen (*Disc. Comp. Geom. 13 (1995)*), we prove that the crossing number of $C_7 \times C_7$ is 35. This is in line with the general conjecture that the crossing number of the Cartesian product $C_m \times C_n$ of two cycles is $(m-2)n$, for $m \leq n$.

158

Hamiltonian Decompositions of Cayley Graphs on Abelian Groups of Even Order

Jiuqiang Liu, Eastern Michigan University

In the paper "Hamiltonian decompositions of Cayley graphs on abelian groups of odd order, Journal of Combinatorial Theory, Series B Vol. 66(1996), 75-86", we proved that the Cayley graph $cay(A,S)$ has a hamiltonian decomposition if $S = \{s_1, s_2, \cdots, s_k\}$ is a minimal generating set of an abelian group A of odd order. Here we prove that if A is an abelian group of even order and $S = \{s_1, s_2, \cdots, s_k\}$ is a strongly minimal generating set (i.e., $2s_i \notin < S - \{s_i\} >$ for each $1 \leq i \leq k$) of A, then $cay(A,S)$ can be decomposed into hamiltonian cycles.

159

A Bijection for Partitions With All Ranks Positive

Carla D. Savage(*)
Radhika Venkatraman
North Carolina State University

For an integer partition π, the i-th *rank*, r_i, of π is the difference between the i-th part of π and the i-th part of the conjugate of π. The sequence of *successive ranks of* π is the sequence $r_1, r_2, \ldots, r_d$ where d is the size of the Durfee square of π.

It follows from work of Andrews and Bressoud that the number of partitions of n with all successive ranks positive is equal to the number of partitions of n with no part of size '1'. However, no simple bijective proof of this identity has appeared in the literature.

In this talk we give a simple bijection, based on a result of Cheema and Gordon for 2-rowed plane partitions.

Keywords: integer partitions, plane partitions, Atkins ranks

160

Hamiltonian Latin Squares

A.J.W. Hilton* (Reading University, England)
C.A. Rodger (Auburn University)

We introduce the concept of Hamiltonian latin squares and of pairs of orthogonal Hamiltonian latin squares and we give a construction method for them. We also discuss symmetric Hamiltonian latin squares, and relate these to Hamiltonian decompositions of complete graphs. We give examples of pairs of symmetric orthogonal Hamiltonian latin squares, and raise the general problem of their construction.

We show that a partial Hamiltonian latin square of order n can be embedded in a Hamiltonian latin square of order *2n*, and that a partial symmetric Hamiltonian latin square of order *2n*. Both these results can be proved by a direct argument, or by an argument involving matroids.

Key words: Hamiltonian, Latin squares, orthogonality, matroids

ON UNIQUELY REALIZABLE DEGREE SEQUENCES

161

by
*J.T.B.BEARD,JR.
TENNESSEE TECHNOLOGICAL UNIVERSITY
and
ANN D. DORRIS
SCIENCE APPLICATIONS INTERNATIONAL CORP.

The relationship between integer partitions with positive summands and degree sequences for undirected graphs with no loops or multiple edges continues (from Euler, 1736) to be a topic of major interest (e.g., Erdos and Richmond, 1993) . In 1976, the "artistic" portion of our 1973 results on this topic were published. Here, we present the technically difficult portions of the latter, which evidently remain unknown even as folklore, and seek to characterize those graphical partitions which are uniquely realizable by a connected graph. Results in three areas contribute to our primary results: complementation of partitions, graphical partitions uniquely realizable by a disconnected graph, and graphical partitions uniquely realizable by a bipartite graph. The major theorem gives twelve sufficient conditions for any two connected realizations of a degree sequence to be isomorphoric; a partial converse is also established.

Keywords: Degree sequences, graphical partitions, threshold graphs.

Group connectivity of 3-edge-connected chordal graphs

162

Hong-Jian Lai, Department of Mathematics
West Virginia University, Morgantown, WV 26506

Let A be a finite abelian group and G be a digraph. The *boundary* of a function $f : E(G) \mapsto A$ is a function $\partial f : V(G) \mapsto A$ given by $\partial f(v) = \sum_{e \text{ leaving } v} f(e) - \sum_{e \text{ entering } v} f(e)$. The graph G is A-connected if for every $b : V(G) \mapsto A$ with $\sum_{v \in V(G)} b(v) = 0$, there is a function $f : E(G) \mapsto A - \{0\}$ such that $\partial f = b$. In [J. Combinatorial Theory, Ser. B 56 (1992) 165-182], Jaeger *et al* showed that every 3-edge-connected graph is A-connected, for every abelian group A with $|A| \geq 6$. It is conjectured that every 3-edge-connected graph is A-connected, for every abelian group A with $|A| \geq 5$; and that every 5-edge-connected graph is A-connected, for every abelian group A with $|A| \geq 3$.

In this note, we investigate the group connectivity of 3-edge-connected chordal graphs and characterize 3-edge-connected chordal graphs that are A-connected for every finite abelian group A with $|A| \geq 3$.

On the problem of a matching orthogonal to a 2-factorization

163

Mateja Šajna, Simon Fraser University

We present a partial answer to a problem posed by B. Alspach: for a fixed 2-factorization $\mathcal{F}$ of a $2d$-regular graph G, does G contain a matching orthogonal to $\mathcal{F}$? We show that there is
always an orthogonal subgraph consisting of disjoint 1- and 2-paths. We also give an upper bound for the number of 2-paths in the optimal case.

Independent knights on hexagonal honeycombs

164

Heiko Harborth, Technische Universit"at Braunschweig, Germany
e-mail: h.harborth@tu-bs.de

For triangular parts of the hexagonal tesselation of the plane the independence number of knights is discussed.

Generalized Common Factor Graphs

173

Irene Gassko, irina@cs.bu.edu, Boston University, CS Dept.

The common factor graph $G(S)$ is a graph that has integers on a set S as vertices, and edges between those pairs which have a common factor larger than 1. R.B. Eggleton introduced common factor graphs in his paper "Common Factors of Integers: a Graphic View", and obtained a number of interesting results about these graphs. We generalize this notion to $G(S,c)$, where edges exist only between pairs of integers with a prime common divisor greater than c (Eggleton's graphs being a special case of $c = 1$). We show that some of the Eggleton's theorems are also true for generalized common factor graphs. Consider $S = S_N$, where S_N is a sequence of N consecutive integers, every element of which has a common prime divisor with another element of the sequence. Eggleton proved that $G(S_N)$ is always connected. This is not always true for $G(S_N, c)$. We prove that a connected $G(S_N, c)$ can always be found for each sufficiently large N. It is also shown that for any $c, d \in \mathbb{N}$ there is $N(c,d)$ such that for each $N > N(c,d)$ one can construct $G(S_N, c)$ with the minimal degree greater than d. More results are obtained concerning constellations - special graphs having common factor graphs as vertices.

Keywords: graphs, factors, prime, connected.

ERNST GABOR STRAUS: EINSTEIN'S CO-AUTHOR AND PAUL ERDOS' COLLABORATOR AND GREAT FRIEND

174

Judith L. Arkin* and David C. Arney
United States Military Academy,
West Point, New York 10996.

Recently in the February, 1997 edition of the NOTICES (American Mathematical Society) there was a major article about Albert Leon Whiteman pp.217-219 in which there was a request for literature about the late Professor Whiteman. When I read this article I was reminded of an old (1983) tape where Professor Whiteman had recorded a eulogy of Professor Straus. I immediately contacted Mrs. Louise Straus, the wife of Professor E.G. Straus. She wrote to me (Mrs. Judith L. Arkin).

In this paper, Professor David C. Arney and I have included the entire letter and tape of Mrs. Straus and Professor Whiteman.

ON THE STRUCTURE OF EXTREMAL GRAPHS WITH A LOWER BOUND ON GIRTH

175

Felix Lazebnik*, Dept. of Mathematical Sciences,
University of Delaware, Newark, DE 19716; fellaz@math.udel.edu
Ping Wang, Dept. of Mathematics, Computing and Information Systems,
St. Francis Xavier University, Antigonish, Nova Scotia,Canada;
pwang@juliet.stfx.ca

Let $n \geq 3$ be a positive integer, and let G be a simple graph of order v containing no cycles of length smaller than $n+1$ and having the greatest possible number of edges (an extremal graph). Does G contain an $(n+1)$ – cycle? In this talk we present some properties of extremal graphs and several new results where the question is answered affirmativly. For example, this is always the case for (i) $v \geq 8$ and $n = 5$, or (ii) when v is large compared to n: $v \geq 2^{a^2+a}n^a$, where $a = n - 3 - \lfloor \frac{n-2}{4} \rfloor$, $n \geq 12$.

On the other hand we show that the answer to the question can be negative if v is small compared to n, e.g., $v = 2n + 2 \geq 24$.

Keywords: girth, forbidden cycles, structure of extremal graphs.

176 The Gallai-Edmonds algebra of graphs

M. Bartha Dept of CS, Memorial U of Newfoundland St. John's, NF, Canada, A1B 3X5 (Author on leave from József Attila University, Szeged, Hungary)bartha@cs.mun.ca

Molecular switching devices are promising components of future computers. Graphs with labeled external vertices, called open graphs, are introduced to model the switching behavior of molecules. Open graphs are given the structure of a simple algebra $\mathbf{Gph}_\Sigma$ sorted by the set of all finite subsets of the alphabet Σ used for labeling the external vertices. The constants of this algebra are the star graphs, and the only operation is composition, which practically merges two graphs along their external vertices wearing the same label.

The same algebraic structure is introduced on graphs having a perfect internal matching. A perfect internal matching (p.i.m.) of an undirected graph G is a matching that covers all the vertices of G with degree at least 2. Such vertices are called internal, whereas vertices with degree one are called external in G. Since composition is introduced as a different operation on open graphs having a p.i.m., the resulting algebra is not a subalgebra of $\mathbf{Gph}_\Sigma$. Rather, it becomes a homomorphic image of that algebra under a suitable homomorphism C. For any open graph G, $C(G)$ is the subgraph of G characterized by the well-known Gallai-Edmonds structure theorem as the one covered by all maximum (internal) matchings in G. Hence the name Gallai-Edmonds for the algebra of open graphs having a p.i.m.

177

A Common Subgraph of Two Graphs

Heather Gavlas

For a subgraph H of a graph G, we say that H is distance-preserving if $d_H(u,v) = d_G(u,v)$ for every pair u,v of vertices of H. A greatest common distance- preserving subgraph of two connected graphs G_1 and G_2 is a graph G of maximum size such that G is a distance-preserving subgraph of G_1 and G_2. This idea of common subgraphs under distance constraints arises naturally in drug reception/interaction designs. It turns out that a distance- preserving subgraph is an induced subgraph, and that the size of a greatest common distance- preserving subgraph can be arbitrarily smaller than the size of a greatest common induced subgraph. Also, for many classes of graphs G, there exist graphs of arbitrarily large size such that G is the unique greatest common distance-preserving subgraph of the graphs under consideration. These concepts are further investigated for trees.

keywords: graph, subgraph, distance

178

IN MEMORY OF PAUL ERDÖS: Our Joint Problems in Combinatorial Geometry

by Alexander Soifer
University of Colorado at Colorado Springs, P.O.Box 7150, Colorado Springs, CO 80933
asoifer@mail.uccs.edu http://www.uccs.edu/~asoifer

About a decade ago I was influenced by my first meeting with Paul Erdős to switch from Abelian group theory to geometry of combinatorial flavor. During the period of time that followed I was blessed with an intense communication and work with Paul. This talk will include reminiscences of collaboration interwoven with open problems created by one of us as a result of our interaction, as well as joint problems. Here are a couple of examples:

1. I proved that the set of all integers n such that every triangle can be cut into n triangles congruent to each other, is precisely the set of perfect squares. This prompted Paul Erdős to pose the following two $25 problems [1]:

   Problem 1. Find all triangles that can only be cut into n^2 congruent triangles for any positive integer n. *Problem 2.* Find all positive integers n such that at least one triangle can be cut into n triangles congruent to each other.

2. Let S be a finite point set in the plane. The symbol $\min\Delta(S)$ will denote the minimum area of a triangle with its vertices in distinct points of S. A convex n-gon P is the set of its n vertices; the interior of P is understood as the interior of its convex hull. The following is one of our joint problems [2]:

   Problem 3. Over all convex n-gons P_n of area 1 and all k-element sets S_k in the interior of P_n find $\Delta(n,k) = \max(\min\Delta(P_n \bigcup S_k)$. For example, $\Delta(4,1) = 0.5(\sqrt{2}-1)$.

Over the past few years we have worked together on the book *Problems of pgom Erdős* which may be out in 1998. The historical essay on how this project came about is a part of the January 1997 issue [3] of *"Geombinatorics"* that is entirely dedicated to Paul Erdős.

Bibliography: [1] Soifer, A. *How does one cut a triangle?* Center for Excellence in Mathematical Education, Colorado Springs, 1990. [2] Erdős, P. and Soifer, A., Triangles in convex polygons, *Geombinatorics* II(4), 19993, 72-74. [3] Soifer, A. "And you don't even have to believe in G'd but you have to believe that the Book exists," *Geombinatorics* VI(3), 1997.

179

Construction of Indecomposable Heronian Triangles

Paul Yiu, Florida Atlantic University.

We give a simple characterization in terms of the tangents of the half - angles of a primitive Heronian triangle for the triangle to be decomposable into two Pythagorean triangles. This characterization leads to easy constructions of Heronian triangles not so decomposable.

180

Application of Fuzzy Node Network Flow

Chang-Shyh Peng, Creighton University, Omaha, NE

In this paper, Fuzzy Node Network Flow is introduced. Efficient dynamic algorithms are presented to solve three major issues with this new model. The first issue is the maximum fuzzy network flow where the flow constraint is on the vertices instead of the arcs, vertices are divided into distinct subsets, nodes in the same subsets are not connected, and arcs connect nodes in consecutive subsets. The second issue is to identify the maximum path flow which satisfies a minimum path membership grade requirement m while the maxflow is intact. The solution is based on the maxflow derived in the first model. The third issue is the maximum path membership grade with minimum path flow requirement. Given a minimum path flow requirement, we try to find a path which satisfies this flow requirement and possesses the maximum possible path membership grade while the maximum network flow is achieved.

Key Words: Fuzzy, Network Flow.

Thursday, March 6, 1997
12:10 p.m.

181

Line Graphs and Forbidden Induced Subgraphs

Hong-Jian Lai and Lubomir Soltes*, Department of Mathematics,
West Virginia University, Morgantown, WV26506-6310

Beineke characterized line graphs in terms of nine forbidden induced subgraphs. we show that the number of forbidden induced subgraphs can be decreased by imposing minor connectivity and minimum degree conditions. In particular, a 3-connected graph with minimum degree at least seven is a line graph of a simple graph if and only if it does not contain any of certain three graphs as an induced subgraph. Moreover, the number of forbidden induced subgraphs cannot be decreased further even if the connectivity is increased. As a corollary we obtain a new sufficient condition for a graph to be hamiltonian-connected.

Keywords: Line Graphs, Forbidden Subgraphs

182

Distance in Convex Geometries

John Pfaltz
Univ. of Virginia

We explore a metric that is defined in terms of the closed/convex sets of the space, instead of arcs, paths, or some underlying Euclidean distance. Specifically, we let

$$d(X, Y) = \text{number of closed sets containing } X + \text{number containing } Y - 2 \cdot (\text{number containing both})$$

With this metric, the distance between sets of points is NOT the distance between the closest points, but rather involves the entire sets. For example, we can have $X \subset Y$ with $d(X, Y) > 0$.

183

ON THE BIPARTITION NUMBERS OF RANDOM TREES

A. Meir, York University
J.W. Moon*, University

If T_n is a rooted tree with n nodes, let $p(T_n)$ and $q(T_n)$ denote the number of nodes of T_n at even and odd distance from the root, respectively. Our object is to determine the limiting distribution of $D(T_n) = |p(T_n) - q(T_n)|$ over trees T_n in certain simply generated families of trees $\mathcal{F}$ whose generating function $y(x)$ satisfies a relation of the form $y = x\Phi(y)$. In paticular, we show that the expected value of $D(T_n)$ is asymptotic to $(An/2\pi)^{1/2}$ where A is a constant whose value depends on $\mathcal{F}$.

Key words: *simply generated trees, distances.*

184

Error-Correcting Nonadaptive Group Testing

Anthony J. Macula, SUNY Geneseo

d-disjunct matrices constitute a basis for nonadaptive group testing algorithms and binary d-superimposed codes. The rows of a d-disjunct matrix represent the tests in a NGT algorithm which identifies at most d defects (positives) in a population A d-disjunct matrix M is called d(e)-disjunct if given any d + 1 columns of M with one designated, there are e + 1 rows with a 1 in the designated column and a 0 in each of the other d columns. d(e)-disjunct matrices form a basis for e error-correcting NGT algorithms. In this paper, we construct d(e)-disjunct matrices. These nonadaptive group testing algorithms have can impact the Human Genome Project.

Thursday, March 6, 1997
2:00 p.m.

Automaticity of rational functions

F. von Haeseler, University Bremen and FAU, Boca Raton

Let $\mathcal{R}$ be a finite, commutative ring. A rational function $R(X,Y)$ with coefficients in $\mathcal{R}$ is the quotient of two polynomials $P(X,Y)$, $Q(X,Y) \in \mathcal{R}[X,Y]$, i.e., $R(X,Y) = P(X,Y)/Q(X,Y)$, and we always suppose that $Q(0,0)$ is an element of the multiplicative group $\mathcal{R}^*$ of $\mathcal{R}$. A rational function with coefficients in the ring $\mathcal{R}$ defines a double sequence $\{s(n,t)\}_{n,t\geq 0}$ which is defined as

$$F(X,Y) = \frac{P(X,Y)}{Q(X,Y)} = \sum_{n,t=0}^{\infty} s(n,t)X^nY^t.$$

We discuss automaticity properties of double sequences defined by rational functions.

We start with a characterization of double sequences defined by rational functions which are definable in the structure $\langle \mathbb{N},+\rangle$. A sequence definable in the structure $\langle \mathbb{N},+\rangle$ is k-automatic for all $k \geq 2$. It turns out that the double sequence $s(n,t)$ is definable in $\langle \mathbb{N},+\rangle$ if and only if $Q(X,Y)$ satisfies a purely algebraic condition, the monomial factor property. A polynomial $Q(X,Y) \in \mathcal{R}[X,Y]$ has the monomial factor property if there exists a polynomial $S(X,Y)$ such that $S(X,Y)Q(X,Y)$ is a finite product of factors of the form $1 - r_iX^{\alpha_1}Y^{\beta_i}$.

As a next step, we study rational functions with coefficients in the ring $\mathbb{Z}/(m) = \mathbb{Z}/m\mathbb{Z}$, i.e., the integers modulo m, where $m \in \mathbb{N}$ and $m \geq 2$. We present a complete characterization of the automaticity properties of rational functions with coefficients in $\mathbb{Z}/(m)$. In order to state the result we introduce the set

$$M_m(Q) = \left\{ p \;\middle|\; \begin{array}{l} p \text{ is a prime divisor of } m \text{ and } Q(X,Y) \bmod p \\ \text{does not have the monomial factor property} \end{array} \right\}$$

Then we have: Let $m \in \mathbb{N}$, $m \geq 2$ and let $F(X,Y) = P(X,Y)/Q(X,Y)$ a rational function with coefficients in $\mathbb{Z}/(m)$, then

1. If $\mathrm{Card}(M_m(Q)) \geq 2$, then $F(X,Y)$ is not k-automatic for all $k \geq 2$.
2. If $\mathrm{Card}(M_m(Q)) = 1$, then $F(X,Y)$ is p^α-automatic, where $p \in M_m(Q)$, and for all $\alpha \in \mathbb{N}$. If k is not of the form p^α The $F(X,Y)$ is not k-automatic.
3. If $\mathrm{Card}(M_m(Q)) = 0$, then $F(X,Y)$ is definable in $\langle \mathbb{N},+\rangle$.

Finally, we present a sufficient criterion for a polynomial $Q(X,Y) \in \mathbb{Z}/(p)[X,Y]$, where p is a prime number, not having the polynomial factor property. This criterion is based on the Hausdorff dimension of a geometric object which is associated with the rational function $1/Q(X,Y)$.

Keywords: Automatic sequences, finite automata, logic, fractal geometry

Thursday, March 6, 1997
3:20 p.m.

197

Pacing a Software Project - A Mathematical Model

Michael L. Gargano*, Frank LoSacco, Stuart Varden
Pace University

How should a software project be paced? That is how should resources be managed to finish the job as soon as possible while satisfying various constraints. First we consider a purely mathematical model, solve it analytically, and discuss its limitations. A genetic algorithm, satisfying more general conditions, will then be explored.

Keywords: project management, mathematical model, genetic algorithm

On Permanents of Adjacency Matrices of Iterated Line Digraphs

Joe Klerlein, Scott Sportsman, Charles Wallis* (cwallis@wpoff.wcu.edu)
Western Carolina University

198

A graph-theoretic interpretation of matrix permanents and properties of line graphs will be used to determine the values of permanents of adjacency matrices of iterated line digraphs of certain classes of directed graphs.

keywords: permanent, adjacency matrix, line digraph

CAT Generation of Subsets with a Given Parity Subsequence

199

Dominique Roelants van Baronaigien, University of Victoria

An *alternating subset* of the set $\mathbf{N} = \{1, 2, \ldots, n\}$ is a subset $\{s_1, s_2, \ldots s_k\}$ where $s_i < s_{i+1}$ and exactly one of s_i and s_{i+1} is odd for all i, $1 \leq i < k$. A generalization of the idea of alternating subsets is to specify a parity sequence $\mathbf{B} = b_1, b_2, \ldots b_k$, and then consider only those subsets $\mathbf{S} = \{s_1, s_2, \ldots, s_j\}$ of $\mathbf{N}$ that have the property that $j \leq k$ and $\exists f$, $\ni 0 \leq f \leq n - j$, and $s_i \bmod 2 = b_{i+f}$ for all i, $1 \leq i \leq j$. A set with this property is said to have *parity subsequence* $\mathbf{B}$

In this paper a Constant Amortized Time (CAT) algorithm is given that for any given parity sequence $\mathbf{B}$, it will list all subsets of $\mathbf{N}$ that have parity subsequence $\mathbf{B}$.

On Asymptotic Properties of Radial Basis Regression Function Estimates and Classification Rules

200

A. Krzyzak* and S. Klasa, Concordia University, Montreal, Canada

In the paper we will investigate asymptotic properties of nonparametric regression estimates based on radial basis networks. The network will be trained by a sequence of independent, identically distributed random variables and and its performance will be measured by the mean absolute integrated error. We will show how to choose radial functions, centers, and the size of hidden layer in order to obtain consistent estimates. The rates of convergence in large classes of functions will also be considered.

201

Graph Semantics vs. Tree Semantics for Common Knowledge in Distributed Systems

Marek A. Suchenek, Department of Computer Science
Cal State U at Dominguez Hills, Carson, CA 90747
Suchenek@dhvx60.csudh.edu

In this talk, we investigate a nonmonotonic multi-modal variant of logic *S5*, called multi-epistemic (*ME*) logic. *ME* logic allows for expressing statements of the form "*X* knows that *Y* knows that *Z* knows that ...", as well as "It is commonly known that ..." (common knowledge sentences) in distributed systems. *ME* logic is deductively complete with respect to a Halpern-Moses-style maximal-graph semantics which may be visualized in a form of graph with colored edges. Semantics of modal operator of common knowledge is provided by the connected components of that graph. Consequently, evaluation of common knowledge sentences involves construction of a spanning tree of a connected component of that graph. If system's common knowledge is a result of a sequence of broadcasts then the graph may be reduced to one with some connected components being isolated vertices. In such a case, evaluation of common knowledge sentences may be done without construction of spanning tree. In this talk we will present a method of graph reduction which achieves the mentioned above effect, and discuss its computational advantages.

Keywords: *distributed knowledge, maximal-graph semantics, multi-modal logic S5, nonmonotonic logic.*

202

A GRAPH STRUCTURE FOR INCLUSION-EXCLUSION INEQUALITIES

Terry McKee, Wright State University (Dayton, Ohio)

Probabilists use the Inclusion-Exclusion Principle to get bounds on the cardinality $|\cup_i A_i|$ of the union of a family of sets from the cardinalities of various intersections of the A_i's. Such "Bonferroni Inequalities" led to "Hunter-Worsley Inequalities," a nice application of maximum spanning trees of the intersection graph of the A_i's. Recent extensions of these inequalities have used *k*-trees, simplicial complexes, and the like.

I extend the intersection graph approach by introducing a graph-theoretic structure in terms of which versions of the Inclusion-Exclusion Principle and Inequalities can be formulated that avoid unnecessary cancellations of terms.

203

Z-cyclic triplewhist tournaments - new results

Norman J. Finizio **University of Rhode Island**

Let p denote a prime that is congruent to 1 modulo 4. Z-cyclic triplewhist tournaments for $v = 35p^n + 1$ players will be discussed.

Key Words: BIBD, whist tournaments.

204

On Graphs in Which Every Minimal Total Dominating Set is Minimum

Bert Hartnell, Saint Mary's University and Douglas Rall*, Furman University

In some classes of graphs a greedy algorithm is guaranteed to find an optimal subset of vertices with a specified property. For example, in the class of well-covered graphs a greedy approach will find a maximum independent set of vertices since all maximal independent sets have the same cardinality. If a graph *G* has no isolated vertices, then for any ordering of $V(G)$ the obvious greedy algorithm will produce a minimal total dominating set. In this paper we initiate a study of those graphs in which the resulting minimal total dominating set is always minimum.

key word: total dominating set

A Network Flow Model for Binary Tomography on Lattices. 205

Pablo M. Salzberg and Ariel Rodríguez, Department of Mathematics and Computer Sciences, University of Puerto Rico, P.O. Box 23355, Río Piedras Campus, Puerto Rico 00931. e-mail: psalzbe@upracd.upr.clu.edu

A Binary Tomography problem deals with the reconstruction of a black and white image from the knowledge of parallel projections (sums) along a minimal set of directions. Recently, an important application was proposed by Peter Schwander *et al.* (1994), in connection with the study of the atomic structure of crystal lattices. Using high resolution electron microscopy, Schwander developed a technique that counts the number of atoms in each atomic column along different zone axes of a crystal. In case that there is only one type of atom, this could be thought as a three dimensional array consisting only of 0's and 1's, of which it is possible to measure projections in two to four directions. The goal is to reconstruct the atomic configuration of the crystal. In this contribution the author presents an algorithm to find an approximate solution, based on network flows techniques, a type of model generally used in Operations Research. This problem leads to an flow optimization problem on several networks simultaneously.

This research has been supported by MBRS-NIH Grant S06GM08102 and by the AT&T Foundation.

Additive Permutation: Recent Results and Open Problems

206

Jean M. Turgeon, University of Montreal

The notion of additive permutations arose in connection with the problem of constructing large perfect systems of difference sets, which are in turn related to a problem of spacing antennas in radioastronomy. A vector of integers $X = (x_1, x_2, ..., x_n)$, with $x_1 < x_2 < ... < x_n$, is called a basis of additive permutations if there exists a permutation $Y = (y_1, y_2, ..., y_n)$ of X such that $X + Y = (x_1 + y_1, x_2 + y_2, ..., x_n + y_n)$ is again a permutation of X. We summarize the basic properties of additive permutations, some recent results and open problems.

207

Zigzag Graphs

Michael L.Gargano and John W. Kennedy*
Pace University

The idea of number sequences is generalized to collections of terms that are organized as the vertices of an underlying graph. If the graph is a path, we recover the usual notion of a sequence. In particular, we consider zigzag(or up-down) sequences which leads naturally to the notion of zigzag graphs. After characterizing these graphs, we consider combinatorial questions related to the number of generalized zigzag sequences that have a given underlying structure.
Keywords: zigzag graph, sequences, counting

On k- edge-magic wheels. 208

Sin-Min Lee *(San Jose State University), Chee-Lin Kwok (Changta National University of Education, Taiwan)

Key words: edge-graceful graphs, edge-magic, edge-magic spectrum of graph, wheel, labeling algorithm.

A (p,q)-graph G =(V,E) is said to be k-edge-graceful for any integer k, if there exists a bijection f: E —> {k,k+1,...,k+q-1} such that the induced map f^+: V —> {0,1, 2,...,p-1} defined by $f^+(u) = \sum \{ f(u,v) : (u,v) \in E \} \pmod p$ is also a bijection map. G is called k-edge-magic if f^+ is a costant map.

We completely determine for each number n >2 the set of integers k such that the wheel $C_n + K_1$ is k-edge-magic. Some k-edge-magic algorithms for the wheels are also given.

209

Reliable Communication Schemes for Hyper-Rings

Tom Altman, Dept. of Computer Science, University of Colorado at Denver, Denver, Colorado 80217

A circulant graph $G = (V, E)$ is called a hyper-ring with N nodes if $V = \{0, \cdots, N-1\}$ and $E = \{\{u, v\} | v - u$ modulo N is a power of 2$\}$. Hyper-rings (HRs), are a multi-machine organization that is, in a sense, a generalization of the standard hypercube architecture. The number of processors in an HR, however, does not have to be a power of two.
In this talk, we discuss the connectivity and reliable communication protocols for HR architectures. We show that the edge-connectivity of an N-node HR is equal to the degree of any node. We then present a reliable communication protocol for HRs that maximizes the number of independent paths from any source node to any destination node.

Keywords: Hyper-rings, connectivity, distributed/parallel architecures.

210

Wiener Numbers of Some Pericondensed Benzenoid Molecular Systems

Wai Chee Shiu* and Peter Che Bor Lam, Hong Kong Baptist University

The Wiener number (W) of a molecular graph, or more generally of a connected graph, is equal to the sum of distances between all pairs of its vertices. Harold Wiener and other researchers reported the existence of correlation between W and a variety of physico-chemical propertie s of alkanes and of benzenoid hydrocarbons. Despite numerous results obtained in the theory of the Wiener number, some basic problems remains open. For example, no recursive method is known for the calculation of W if a general (molecular) graph, especially of polycyclic graphs. This is particularly frustrating in chemical applications, where the majority of molecular graphs are polycyclic. Recently, the authors of this paper made a significant breakthrough with regard to this problem by designing a method for finding W for a homologous series of compact pericondensed benzenoid molecular systems. In a few subsequent papers, the authors, in joint efforts with other researchers, apply similar method to obtain W for other series. In this paper, we shall summarize results obtained in this respect, and apply the method to find the Wiener number of another series - the non-symmetric hexagonal bitrapeziums.

Key words and phrases: Wiener numbers, polycyclic molecular graphs.

211

Lattice Paths with Weighted Left Turns Above a Parallel to the Diagonal

Christian Krattenthaler, Universität Wien, and
Heinrich Niederhausen*, Florida Atlantic University, Boca Raton

A lattice path starts at the origin and takes unit steps in the North and East directions. Counting paths to (n, m) that stay above a given line is basically the same problem as counting those which stay below a corresponding line; however, this 180° "rotational invariance" is lost if we ask how many paths reach (n, m) staying above the line and make exactly l left turns. The problem can be combinatorially approached via two-rowed plane partitions. Seen as an initial value problem, we can find the solution through Umbral Calculus applying symmetric Sheffer polynomials.

212

A Splitter Theorem for Internally 4-Connected Graphs

Thor Johnson - Georgia Institute of Technology

A consequence of Seymour's Splitter Theorem is that, given two simple, 3-connected graphs H and G with H a minor of G and H not a wheel, G can be obtained from H by repeatedly applying the operations of vertex uncontraction and edge addition such that every intermediate graph is simple and 3-connected. Excluded minor theorems concerning simple, 3-connected graphs can be proven using this result. For some excluded minor theorems, however, it would be useful to have an analogue of Seymour's Theorem for internally 4-connected graphs. (A graph G is internally 4-connected if it is simple, 3-connected, and for every partition (E_1, E_2) of $E(G)$ where $|E_1|, |E_2| \geq 4$, there are at least four vertices incident to both edges in E_1 and E_2.) We prove such a theorem. More precisely, we show that any internally 4-connected graph, G, can be constructed from any internally 4-connected minor H by repeatedly performing one of a set of operations such that each graph in the resulting sequence from H to G has its predecessor as a minor and is "nearly" internally 4-connected in a sense that we make precise. We apply this result to excluded minor theorems. This is joint work with Robin Thomas.

Thursday, March 6, 1997
4:40 p.m.

213 Operating Component Reliability - Properties and Synthesis Results

F. Boesch, Stevens Institute of Technology; D. Gross*, Seton Hall University; C. Suffel,Stevens Institute of Technology

Applications such as multiprocessor networks, where the edges are perfectly reliable but the nodes operate with known probabilities, have been recently modeled by a new reliability function called the operating component reliability. Unlike the classical node reliability model, the new model handles the case of node failures without suffering the pathological property of non-coherence. We show that the model, while coherent is not shellable. We also determine some of its properties and present several synthesis results which involve properties of graphs which optimize this new reliability measure.

Key Words: Node Reliability, Coherence, Shellability

214 On the Weakness of an Ordered Set

Ann N. Trenk, Wellesley College

We extend the notion of a ranking of elements in a weak order to a ranking of elements in general ordered sets. The *weakness* of an ordered set $P = (X, \prec)$ (denoted $wk(P)$) is the minimum integer k for which there exists an integer-valued function $lev : X \to \mathbb{Z}$ satisfying (i) if $x \prec y$ then $lev(x) < lev(y)$, and (ii) if $x \sim y$ then $|lev(x) - lev(y)| \leq k$ (where "$\sim$" denotes incomparability). A *forcing loop* L in P is a sequence of elements $L : x = v_0, v_1, \ldots, v_m = x$ of P so that for each $i \in \{0, 1, \ldots, m-1\}$ either $v_i \prec v_{i+1}$ or $v_i \sim v_{i+1}$.

Our main result relates these two concepts; we prove $wk(P) = \max_L \left\lceil \frac{up(L)}{over(L)} \right\rceil$ where $up(L) = \#\{i : v_i \prec v_{i+1}\}$, $over(L) = \#\{i : v_i \sim v_{i+1}\}$ and the maximum is taken over all forcing loops L in P. We also give an algorithm to compute $wk(P)$ to produce an appropriate ranking function.

Key words: ordered sets, weak orders

215 Triangular Blocks of Zeros in (0,1)-Matrices With Small Permanents

John Goldwasser, West Virginia University

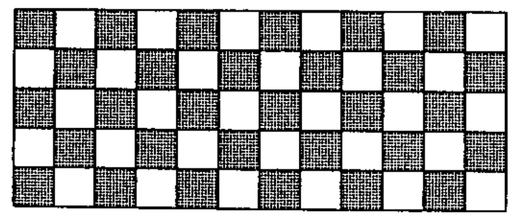
Let A be a square matrix and t a positive integer. We say A is t-triangular if there exist permutation matrices P and Q such that PAQ = B = [b(i,j)] has b(i,j)=0 whenever j ¿ i+t-1. (So 1-triangular means the rows and columns can be permuted to get a lower triangular matrix.) We ask for which positive integers t is this statement true: STATEMENT: If A is any square matrix with nonnegative integral entries such that 0 ¡ per A ¡ (t+1)! then A is t-triangular. If t=1 the statement reduces to a theorem of Brualdi. We prove the statement is true for t=2 and t=3, but false for t=6. Of course the problem could be stated in terms of the number of perfect matchings in a bipartite graph.

Key words: permanent

216 Domination Problems on Rectangular Chessboards

*David C. Fisher (*dfisher@orphan.cudenver.edu*) – Univ. of Colorado at Denver*

A $k \times n$ chessboard is *dominated* if each square either has a piece on it or attacking it. *Can you dominate a* 5×12 *board with 8 Kings (easy)? 4 Queens (hard)? 10 Bishops (hard)? 10 Knights (very hard)? Or 5 Rooks (very easy)?* The $k \times n$ King (*resp.*, Queen, Bishop, Knight, or Rook) domination number is the least number of Kings (*resp.*, Queens, Bishops, Knights, or Rooks) which can dominate a $k \times n$ board.



- We show the $k \times n$ King domination number is $\lceil k/3 \rceil \lceil n/3 \rceil$.
- We find the $k \times n$ Queen domination number for $k \leq 10$ and for all n with a sophisticated exhaustive search *(with Aesoo Chung and Melissa Wilt)*.
- We find the $k \times n$ Bishop domination number for $k \leq 8$ and for all n using Min-Plus Algebra and searching for periodicities.
- We find the $k \times n$ Knight domination number for $k \leq 7$ and for all n with an algorithm of Eleanor Hare using Min-Plus Algebra and searching for periodicities *(with Anne Spalding)*.
- We show the $k \times n$ Rook domination number is $\min(k, n)$.

Keywords: Domination, Min-plus Algebra, Chessboards.

Thursday, March 6, 1997
5:00 p.m.

Component Size Connectivity - A Graph Invariant Related to Operating Component Reliability

217

F. Boesch, Stevens Institute of Technology; D. Gross, Seton Hall University; C. Suffel*, Stevens Institute of Technology

Applications such as multiprocessor networks, where the edges are perfectly reliable but the nodes operate with known probabilities, have been recently modeled by a new reliability function called the operating component reliability. Unlike the classical node reliability model, the new model handles the case of node failures without suffering the pathological property of non-coherence. This reliability measure is associated with the following definition of a new graph invariant. The component size connectivity, denoted by $\kappa_c^{(k)}$, is the minimum number of nodes whose removal produces a graph having no components having k or more nodes. We show how this parameter is related to reliability, and we present several of its properties. We also study several realizability problems relating $\kappa_c^{(k)}$ and κ - the classical connectivity.

Key Words: Node Reliability, Connectivity

Groups generated by unimodal cycles

218

J. Aleš and T.D. Rogers

Groups generated by transitive unimodal permutations are studied. Transitive unimodal permutations on n elements, denoted by Δ_n, generate S_n (A_n) whenever $n \geq 3$ is even (odd), respectively. Presentations of S_n and A_n are given in terms of transitive unimodal generators.

We construct a group on Δ_n for a given binary operation $*$. The group Δ_n is proven to be isomorphic to the cyclic group $C_{|\Delta_n|}$ for a positive integer n. A group isomorphism between Δ_n and the cyclic group $C_{|\Delta_n|}$ is derived. An example for $n = 5$ is given.

Enumerating One-factors of the Cube

L. Clark, J. George*, T. Porter—Southern Illinois University at Carbondale

219

The authors find upper and lower bounds on the number of one-factors in the n-dimensional cube. Previous results counted the number directly using computer search. The permanent of the appropriate matrix together with estimates of the size of permanents of sparse matrices yield very tight bounds for arbitrary n. *Keywords*: Cube, one-factor, permanents.

220 Bounds on Periodicity in the Min-Plus Algebra

*Anne Spalding * and David C. Fisher – University of Colorado at Denver*
email: aespaldi@tiger.cudenver.edu

Many graph theory problems such as domination, coloring and independence number can be solved using the min-plus algebra (*e.g.*, Fisher's talk). In the min-plus algebra, matrix multiplication is computed by replacing multiplication with addition and addition with minimization. Let

$$A = \begin{bmatrix} 7 & 1 & 8 \\ 6 & 9 & 2 \\ 3 & 5 & 6 \end{bmatrix} \quad \text{and} \quad x_0 = \begin{bmatrix} 6 \\ 8 \\ 3 \end{bmatrix}$$

For $k > 0$, define $x_{k+1} = Ax_k$ using min-plus algebra. Then:

$$x_1 = \begin{bmatrix} 9 \\ 5 \\ 9 \end{bmatrix} x_2 = \begin{bmatrix} 6 \\ 11 \\ 10 \end{bmatrix} x_3 = \begin{bmatrix} 12 \\ 12 \\ 9 \end{bmatrix} x_4 = \begin{bmatrix} 13 \\ 11 \\ 15 \end{bmatrix} x_5 = \begin{bmatrix} 12 \\ 17 \\ 16 \end{bmatrix} x_6 = \begin{bmatrix} 18 \\ 18 \\ 15 \end{bmatrix}$$

Note that $x_6 = 6 + x_3$. Thus, $x_k = 6 + x_{k-3}$ for all $k \geq 6$.

How many iterations does it take before this periodicity occurs?

It will be shown that, under certian conditions, the number of iterations is at most $n^2 - n + 2$ where n is the size of the matrix.

Keywords: Domination, Min-Plus Algebra

Finding the connection classes of an unknown subgraph

Weiping Shi (University of North Texas) and Douglas B. West* (University of Illinois)

221

Given a graph G, we wish to find the connection classes (the vertex sets of the components) of an unknown subgraph H. We obtain information by submitting a vertex subset S to an oracle. The oracle returns the set of vertices reachable from the set S by paths in H. The *query number* of G is the smallest worst-case number of queries needed to find the connection classes of an unknown subgraph of G. We determine the query number for several classes of graphs by presenting optimal algorithms. We also discuss the nonadaptive analogue, which corresponds to submitting tests in parallel. The problem is the graph-theoretic model for interconnect diagnosis of wiring networks: the vertices of G represent wires, and the edges represent short faults that may possibly occur.

Keywords: graph searching, component, subgraph, wiring network, algorithm

Contributions to the Existence of Some Binary Combinatorial Arrays

222 D.V. Chopra, Wichita State University

An array T with m constraints, N runs (treatment-combination) and with s levels is merely a matrix of size $(m \times N)$ with s symbols (say; $0, 1, 2, \ldots s-1$). We restrict ourselves to arrays with $s = 2$ (i.e. elements 0 and 1). These arrays with some combinatorial structure find great use in statistical design of experiments, and are of interest in combinatorics. We consider the following combinatorial arrays. T is called a balanced array (B-array) of strength $t(0 < t \leq m)$ if in every $(t \times N)$ submatrix T_{ij} of T, every t-vector with i 1's $(0 \leq i \leq t)$ in it appears a fixed number μ_i(say) times. The vector $\underline{\mu}^1 = (\mu_0, \mu_1, \ldots \mu_t)$ is called the index set of the array T. We discuss the existence of some such arrays T.

NORMAL MATRICES, PSEUDO-CUBES AND PSEUDO-PRODUCTS

223

Fabrizio Luccio and Linda Pagli*.,Università di Pisa

We work in B^n and consider sets of 2^m points, $m \leq n$, represented in binary *balanced* matrices, whose columns contain half 0's and half 1's, and this property repeats recursively in proper submatrices. We introduce the concept of *pseudo-cube* of order m, that is a subset of 2^m points of B^n whose matrix is balanced. A subcube $B^m \subseteq B^n$ is a special case of pseudo-cube and shares most of its properties. For a given pseudo-cube P we define the class $\mathcal{P}(P)$ of the pseudo-cubes obtained from P by complementing any subset of variables, and show that the elements of $\mathcal{P}(P)$ are disjoint and tessellate B^n. Furthermore, the union of any two pseudo-cubes of the same class $\mathcal{P}$ is a pseudo-cube, and the intersection of two arbitrary pseudo-cubes is either empty or is a pseudo-cube. We then introduce the *pseudo-product* as the characteristic function of a pseudo-cube P. This function inherits all the properties of P, and has a compact expression EXP(P). Given two pseudo-products P_1, P_2 belonging to the same class $\mathcal{P}$, we give an algorithm to construct in linear time the expression of the union EXP($P_1 \cup P_2$) from EXP(P_1) and EXP(P_2). Finally we show how a standard procedure to generate a minimal algebraic form of a Boolean function f can be extended to generate a minimal "Union-of-EXP" form, which is generally much shorter than the former.

keywords: pseudo-cube, pseudo-product, binary matrix, Boolean function

Domination Matrices of Kneser Graphs

224

Boris Shekhtman, Univ of South Florida

We investigate the domination number of graphs whose vertices are suspaces of a finite dimensional vector space and whose edges are are pairs of subspaces with a preassigned dimension of the intersection.

$O(1)$ Query Time Algorithm for APSP on an Interval Graph

233 Alan P. Sprague*, University of Alabama at Birmingham and
Tadao Takaoka, Ibaraki University

We present an approach to the All Pairs Shortest Path (APSP) problem on an interval graph on n vertices which sidesteps the obvious $\Omega(n^2)$ lower bound. After $O(n)$ preprocessing time, the algorithm can deliver a response to a distance query in $O(1)$ time. It is assumed that an interval model for the graph is given, and left and right ends of intervals are already sorted by coordinate. The algorithm is based on a recoordinatization of an underlying proper interval graph. The algorithm may be extended to circular arc graphs.

KNIGHT'S TOURS ON TRIANGULAR HONEYCOMBS

234 John J. Watkins, Colorado College

The *Knight's Tour Problem* — can a knight visit each square of a chessboard exactly once, by a sequence of knight's moves, and finish on the same square as it began — has a history that goes back to Euler and De Moivre. Recent interest in chessboard problems involving the knight can be traced to one of Martin Gardner's columns in *Scientific American*. In particular, in 1991, A.J. Schwenk answered completely the question of which rectangular chessboards have a knight's tour. Knight's tours have also been considered on other surfaces such as the torus. Chessboards in the shape of a triangular honeycomb have also been introduced with moves adapted for the standard chess pieces. For example, the knight moves two hexagons in a single direction and then one hexagon either left or right 60 degrees. We will show that a knight's tour exists for any triangular honeycomb of order at least 8.

Pushing Vertices in Tournament-Like Digraphs

235 William Klostermeyer, Christina Myers* and Lubomir Soltes
West Virginia University, Morgantown, WV
e-mail: {wfk, myers}@cs.wvu.edu, soltes@math.wvu.edu

The effect of the push operation (when a vertex is pushed, the direction of each of its incident edges is reversed) on Hamiltonicity is studied for tournaments and k-partite tournaments (orientations of complete k-partite graphs). It is shown that all sufficiently large k-partite tournaments, $k \geq 3$, can be made to have a Hamiltonian cycle, where each partition has an equal number of vertices. It is also shown that almost all bipartite tournaments can be made to have a Hamiltonian cycle. Sufficient conditions are given for k-partite tournaments to be pushable to a graph with a directed Hamiltonian cycle, when the partitions are not necessarily of equal size. We also prove that using pushes, any sufficiently large tournament can be made to have an exponential number of Hamiltonian cycles, considerably strengthening a previous result.

Keywords: tournaments, k-partite tournaments, Hamiltonian cycles

237

Optimal Sequential Assignment of Random Objects into Cells Subject to Order Restriction

Dieter Reetz, Department of Mathematics and Computer Science, University of Puerto Rico, Rio Piedras, PR 00931, USA

At discrete times $t = 1, ..., m$ objects appear and a decision concerning their placement into one of the cells $i = 1, ..., m$ must immediately be made. The state of an object at time t is a random variable X_t taking on values $k = 1, ..., n$ with given probabilities $p_k = P(X_t = k)$. Let y_i be the value of an object assigned to cell i. Placement of objects continues as l ong as the values of the objects assigned to cells are nondecreasing ($y_i \leq y_j$ for $i \leq j$). The process stops if an object cannot be placed subject to this order restriction. We calculate a strategy maximizing the expected number of objects that can be assigned to cells.

Keywords: Dynamic programming, sequential assignment, sequential sorting.

238

A Game Inspired by Covering Rectangles with 1 x 1 and 1 by 2 Rectangles

Phyllis Z. Chinn* and Dale R. Oliver
Humboldt State University, Arcata, CA
pzc1@axe.humboldt.edu
dro1@axe.humboldt.edu

Cuisenaire rods ("c-rods") are a set of rectangular solids with cross-section of 1 cm by 1 cm squares, color-coded by length, and varying from 1 cm long white rods and 2 cm long red rods to 10 cm long orange rods. A variety of number-theoretic and combinatorial geometry problems can be modeled using the c-rods. In this presentation we explore a game involving the tiling of rectangles by 1 x 1 and 1 x 2 rectangles. When the rectangle being tiled is a 1 by m rectangle, the game is equivalent to the game of Kayles, as described in "Winning Ways" by Berlekamp, Conway and Guy.

Key words: Tiling problems, Kayles, Nim-heap games, using manipulatives to motivate mathematical discoveries

On consecutive edge-coloring of bipartite graphs

239

K. Giaro, M. Kubale*

Technical University of Gdańsk, Foundations of Informatics Dept., 80952 Gdańsk
kubale@pg.gda.pl

A proper edge-coloring c of G with natural numbers is *consecutive* if the colors of edges incident with each vertex form an interval. The *span* of c is the number of colors used in the consecutive coloring and $\chi_c'(G)$ stands for the minimum span among all consecutive colorings of G. If G does not have such a coloring then the *deficiency* $d(G)$ of G is the minimum number of pendant edges whose attachment makes G consecutively colorable. In the talk we investigate these concepts in the case of bipartite graphs. Namely, we show that the problem of deciding if G is consecutively Δ-colorable is NP-complete for $\Delta \geq 5$. Then we review special cases of bipartite graphs that have such a coloring e.g. trees, cacti, complete bipartite, $(2,\Delta)$-regular, grid graphs and show that for some of them $\lim_{n\to\infty} \chi_c'(G)/\Delta = \infty$. Finally, we investigate bipartite graphs which are not consecutively colorable and show that for some of them $\lim_{n\to\infty} d(G)/n = 1$.

An upper bound for the number of spanning trees of a graph that is sharp for complete multipartite graphs

240

J. Rodriguez*, Long Island University, Brooklyn, NY 11201.
e-mail: rodiguez@comsci.liunet.edu

L. Petingi, Universidad de la Republica, Montevideo, Uruguay.
e-mail: petingi@fing.edu.uy

Let G be a simple graph with degree sequence $(d_1, d_2, \ldots, d_n)$. We est ablish the following upper bound for the number of spanning trees of G:

$$t(G) \leq n^{n-2} \prod_{j=1}^{n} (d_j/n)^{(n-1-d_j)/(n-d_j)}.$$

with equality if and only if G is a complete multipartite graph. Earlier bounds by Grimmett, Grone and Merris, Nosal, and Kelmans were sharp for complete graphs only. The bound above is used to solve a problem in optimal design that arises in connection with Network Reliability.
Keywords: Spanning Trees, Laplacian Matrix.

241

Using Continuous Nonlinear Relaxations to Solve Constrained Maximum-Entropy Sampling Problems

Kurt M. Anstreicher, University of Iowa; Marcia Fampa, Federal University of Rio de Janeiro; Jon Lee, University of Kentucky; Joy Williams*, University of Kentucky

We consider a new nonlinear relaxation for the Constrained Maximum - Entropy Sampling Problem - the problem of choosing the $s \times s$ principal submatrix with maximal determinant from a given $n \times n$ positive definite matrix, subject to linear constraints. We implement a branch-and-bound algorithm for the problem, using the new relaxation. The performance on test problems is far superior to a previous implementation using an eigenvalue-based relaxation. A parallel implementation of the algorithm exhibits approximately linear speed-up for up to eight processors, and has successfully solved problem instances which were heretofore intractable.

Key words: branch-and-bound, positive definite matrix

242

Forcing Numbers of Graphs

Lior Pachter*, MIT and Peter Kim, NCSSM

Let G be a graph that admits a perfect matching. The forcing number of a perfect matching M of G is defined as the smallest number of edges in a subset S of M, such that S is in no other perfect matching. We show that for the 2n x 2n square grid, the forcing number of any perfect matching is bounded below by (2/3)n and above by n^2. The upper bound is sharp. We also establish a connection between the forcing problem and the minimum feedback set problem. Finally, we present some conjectures about forcing numbers in other graphs.

Keywords: Matching, Forcing Number, Minimum Feedback Set

THE INDUCED PATH NUMBER OF THE HYPERCUBE

243

Salar Y. Alsardary
Department of Mathematics, Physics, & Computer Science
Philadelphia College of Pharmacy & Science
Philadelphia, PA 19104-4495

The induced path number $\rho(G)$ of a graph G is the minimum number of subsets in a partition of V(G) so that each subset induces a path. Chartrand and others proved that $\rho(k_2^d) \leq 2^{d-5}$ and conjectured that $\rho(k_2^d) \leq d$. We prove that $\rho(k_2^d) \leq 16$.

New Proofs of the Uniqueness of Maximal Noneven Digraphs with the maximum number $M(n)$ of arcs and $M(n)-1$ arcs

244

Chjan C. Lim
Department of Mathematical Sciences,
Rensselaer Polytechnic Institute

A digraph is noneven if there is a weighting of its arcs by ± 1 such that the corresponding weighted adjacency matrix is sign-nonsingular, that is, each term in its determinant expansion has the same sign. This problem has its origins in mathematical biology, economics and statistical mechanics where it is related to the Kastelyn method for enumerating the number of perfect matchings. It is also related to a theorem on the classical Polya problem on the conversion of the permanent of $0,1$ matrix, which states that the entries of a given $0,1$ matrix H can be signed so that the resulting $0,\pm 1$ matrix H' has determinant $\det H' = \pm per\ H$ if and only if H is the zero pattern of a sign-nonsingular matrix. Gibson gave a proof of the uniqueness of the sign-pattern that is sign-nonsingular and contains the least number $m(n) = \frac{1}{2}(n-1)(n-2)$ zeroes. In this paper we give a graph-theoretic proof of Gibson's result which has several interesting properties, one of which is its easy extension to a proof of the uniqueness of the maximal sign-nonsingular pattern with $m(n)+1$ zeores, or the maximal noneven digraph with $M(n)-1$ arcs.

Keywords: Noneven digraphs, extremal properties, sign-nonsingular patterns, Polya's permanent problem.

245

Solving Graph Problems by DNA

Natasha Jonoska*, Stephen A. Karl, Masahico Saito
University of South Florida, Tampa FL, 33620

The breakthrough work of Adleman in 1994 demonstrating that a well known NP-complete problem, the Hamiltonian Path Problem (HPP), can be solved by molecular experiments, opened the field of DNA-based computing. The solution of HPP in the Adleman's work required polynomial number of steps relative to the size of the graph. In our work we explore the three dimensional structure of DNA. On a simple example we show how graphs can be constucted by DNA molecules. With our construction, the HPP can be solved in constant number of steps, i.e. one step of hybridization and ligation, one step of restriction endonuclease digestion and one step of gel electrophoresis. We also show that without additional reaction constraints, any graph construction by predesigned oligos yield topologically a covering space of the graph.

KEYWORDS: DNA-computing, Hamiltonian Path Problem, knots.

246

Rectangle Breaking in Grids

(or, How Do You Make A Grid More Hip?)

Paul Dreyer* and Therese Biedl, Rutgers University

Given an $n \times n$ square grid, there are the outlines of n^2 1×1 squares, $(n-1)^2$ 2×2 squares, and so on. What is the fewest edges that can be removed from the grid such that there are no complete outlines of squares remaining? Using techniques from tiling problems and other graph theoretic methods, we solve the problem for all n and also prove similar results for rectangular grids for which the side lengths differ by no more than six. We introduce another version of the problem where we ask for the fewest number of edges to "break" all of the rectangles in a grid.

Keywords: Tiling, Graph Theory, Packing

247

(D;n)-cages: Some new results

Margaret Francel, The Citadel; Nirmala Limaye, University of Bombay
Dinesh G. Sarvate*, University of Charleston

We present the study of two questions which arise naturally in the study of (D;n)-cages. A known and sharp upper bound for f(D,4) is obtained by constructing a bipartite graph. Professor Roger Eggleton asked whether a cage with girth 4 must be bipartite. We show that there is at least one family where the cage must be non-bipartite. Also there are families where both bipartite and non-bipartite cages exists. In the process we construct all cages for D= {r,s,t} and girth 4 for $2^2 r<s<t^2 10$. The second question deals with whether a cage must be connected when $|D| > 1$.

248

The Hall Index of Complete and Complete Bipartite Graphs

Matthew M. Cropper*, West Virginia University, cropper@math.wvu.edu
A.J.W. Hilton, The University of Reading, A.J.W.Hilton@reading.ac.uk

Let L be a vertex list assignment of a finite simple graph, G, and $\alpha(\sigma,L,G)$ denote the size of the largest independent set of vertices of G whose lists contain the symbol σ. Clearly then G being L-list colorable implies that $\sum_{\sigma \in \cup L(v)} \alpha(\sigma,L,H) \geq |V(H)|$ for every induced subgraph H of G, where the union is taken over all $v \in V(H)$. If it is true that G and L satisfy $\sum_{\sigma \in \cup L(v)} \alpha(\sigma,L,H) \geq |V(H)|$ for every induced subgraph H of G then we say that G and L satisfy Hall's condition.

The Hall Number of G, h(G), is the smallest positive integer h such that there is a proper list coloring of G whenever the lists are of length at least h and Hall's Condition is satisfied. For $k \geq \chi(G)$, $h_k(G)$ is defined similarly except that the lists are to be formed from a stock of k symbols.

The Hall Index of G is the edge analogue of Hall Number and is denoted by h'(G) and $h'_k(G)$ respectivley. By examining the Hall Number of the line graph of $K_{m,n}$ we show that $h'(K_{m,n}) \in \{n-1,n\}$ for $2 \leq m \leq n$ and $h'_n(K_{m,n}) = n-1$ for $2 \leq m \leq \lceil \frac{n}{3} \rceil$. We also show directly that $h'(K_{2n+1}) \in \{2n,2n+1\}$ and so $h'(K_{2n+2}) \in \{2n,2n+1\}$.

249

Problem-Specific Heuristics in a Branch-and-Bound Timetabling Procedure

Lynn Kiaer, Rose-Hulman Institute of Technology

The use of heuristic solution algorithms in conjunction with a branch-and-bound procedure has proven useful in obtaining feasible solutions, which in turn permi ts pruning of the problem tree, and also allows the branch-and-bound procedure to f ocus on moving the relaxation bound as rapidly as possible. In this paper, a particu lar version of the timetabling problem is considered: given n events, each requiri ng certain resources, t timeslots, r_{sk} resources of type s available in ti meslot k, costs c_{ik} of scheduling event i in timeslot k, and interaction cos ts w_{ikjl} of scheduling event i in timeslot k while also scheduling event j in timeslot l, find a minimum-cost assignment of events to timeslots. The impact of using problem-specific heuristic criteria to determine branching choices within the branch-and-bound procedure is discussed, and the effect of various criteria are compared and contrasted.

Determinant Algorithms for Random Planar Structures

250

David Bruce Wilson

Colbourn, Myrvold, and Neufeld developed a fast algorithm for generating random arborescences of a graph, using the fact that the determinant of a certain matrix enumerates these arborescenes. There are a variety of other combinatorial structures that can be enumerated by evaluating a determinant, structures of interest in both the physics and mathematical communities. Randomly generating such objects has been a useful tool in studying their properties, and has guided mathematicians by suggesting theorems that might be true. We show here how to adapt and extend the techniques used by Colbourn *et al.* to efficiently randomly generate such objects. These new algorithms offer significant improvements over previous algorithms in both their generality and their speed. Specifically, we show how to generate a random perfect matching of an n-vertex planar graph in $O(n^{3/2})$ arithmetic operations, and a random set of nonintersecting paths from p sources to p sinks in a planar directed graph in $O(p^{1.688}n)$ arithmetic operations. Using exact arithmetic, the random perfect matching algorithm uses $O(n^{5/2}\log^2 n \log\log n)$ bit operations.

Key words: perfect matching, random generation, determinant, linear algebra, Kasteleyn matrix

251

Thin Polygonal Graphs Revisited and Anew

Manley Perkel, Wright State University, Dayton, OH, U.S.A.

(This talk is based on joint research with Cheryl Praeger (University of Western Australia) and Richard Weiss (Tufts University).)

A connected graph Γ of girth $m \geq 3$ is called a polygonal graph (or m-gon graph) if it contains a set Π of m-gons (i.e. simple cycles of length m) such that every path of length two of Γ is contained in a unique element of Π. The meaning of "thin", which relates to the nature of the action of a group of automorphisms, will be explained in the talk.

Here we confine ourselves to investigating thin 6-gon graphs which are highly symmetric (i.e. having automorphism groups transitive on vertices, with a vertex stabilizer (at least) 5/2-transitive on its neighbors). Previous characterizations of these have been intensively group-theoretical. Now, using the computer system MAGMA, we are able to provide shorter proofs as well as find new examples.

252

Some Results Concerning 1 - Well-covered Graphs

Arthur S. Finbow*, B.L. Hartnell, Saint Mary's University, Halifax, Canada

A graph G is said to be well-covered if every maximal independent set of vertices has the same cardinality. If the subgraph of the well-covered graph G induced by G - {v} is also well-covered for each vertex v in G then G is said to be 1 - well-covered. This class was first studied by Staples (1975) and more recently by Pinter(1991-1997). We present some new results concerning the structure of such graphs, the relationship of this class to other classes of well-covered graphs, and some constructions for obtaining such graphs.

Asymptotic Hypercube Embeddings of Dynamic k-ary Trees

253

Ömer Eğecioğlu* and Maximilian Ibel
Department of Computer Science, University of California Santa Barbara, CA 93106

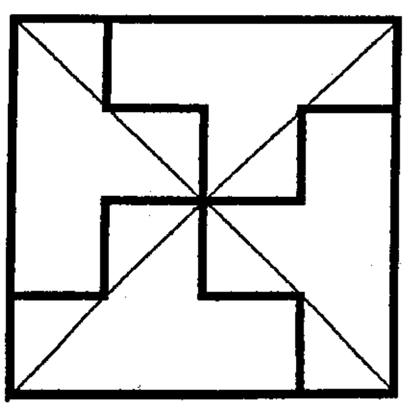
Several algorithms are known for embedding dynamically growing trees onto hypercubes. In a dynamic k-ary tree each leaf node may spawn k new children at any given time. The embedding process must not reassign any tree node to another host node in the hypercube once it has been placed. Desirable properties of the embedding are low dilation and optimal load-balance.

The existing algorithms are mainly directed toward optimizing the load balance for trees that are comparable in size to the hosting hypercube. It has been observed that in this case the naive approach of assigning newly spawned leaves to random neighbors in the hypercube host yields suboptimal results. We consider the asymptotic behaviour of this naive placement algorithm. For symmetry reasons it is to be expected that the resulting process should lead to an asymptotically balanced load for dynamic k-ary trees. We show that a formal proof of this is a consequence of the Matrix-Tree theorem for graphs.

254 TILINGS BY TRIANGLES AND BY POLYOMINOES

Aaron Meyerowitz, Florida Atlantic U.

Given a tile, a basic problem is to determine which regions can be tiled by copies of the tile and to classify and/or enumerate all such tilings. For certain triangular tiles, such as an isosceles right triangle, this is quite easy. Take such a tiling and "approximate" it by placing it on a square grid and replacing each tile by a polyomino consisting of the union of all the cells it covers and some of the partially covered cells. The result is a tiling by polyominoes and potentially, by congruent polyominoes. When do all tilings by the resulting polyomino arise in this fashion?



255 Weakly Triangulated Comparability Graphs

Elaine Eschen, Fisk University, Ryan B. Hayward, University of Lethbridge, Jeremy Spinrad, Vanderbilt University, R. Sritharan*, Indiana State University

The class of weakly triangulated comparability graphs and their complements are generalizations of interval graphs and chordal comparability graphs. We show that problems on these classes of graphs can be solved efficiently by transforming them into problems on chordal bipartite graphs. We show that recognition and computing maximum independent set on weakly triangulated comparability graphs can be solved in $O(n^2)$ time in this manner, and that the number of weakly triangulated comparability graphs is $2^{\Theta(n \log^2 n)}$. We also give algorithms to compute transitive closure and transitive reduction in $O(n^2 \log\log n)$ time, if the underlying graph of the transitive closure is a weakly triangulated comparability graph.

Key words. weakly triangulated graph, comparability graph, algorithms.

257 **Shortest Path Heuristics Under Heavy Traffic Conditions**

Robert Goldberg, Jacob Shapiro, Jerry Waxman* City University of New York

Shortest path heuristics have played an important role in routing schemes for networks and associated topologies. However, if multiple requests for routing are all provided with the identical shortest path, then the traffic approaches the capacity of the best path; the result is that none of these paths can accommodate the traffic. In this case, shortest path algorithms become unstable and often oscillate between a set of paths. In these "heavy load" conditions, such heuristics can cause the network to overload and even to collapse. The current research investigates quasi-shortest path routing using level graphs to model network topology and implements the ALGS* algorithm as the basis for a "quasi-dynamic" shortest path routing protocol. Given the algorithm's efficiency and ability to adapt to "dynamic" network situations, ALGS* can alleviate many of the difficulties experienced with shortest path routing under demanding load conditions.

Parallel Tetrangle-Inequality Bound-Smoothing on a Cluster of Workstations

258 Kumar Rajan*, Narsingh Deo, and Nishit Kumar, UCF

Given an incomplete, weighted, undirected graph $G = (V, E, W)$, with the node set V representing points in 3-D space, the set of weights W representing (imprecise) measured distances between node pairs belonging to the edge set E; one often needs to compute tight bounds on distances between pairs of nodes not belonging to the edge set E. Imprecision is handled by using a pair of numbers for upper and lower bounds on the measured distance. This problem known as *bound smoothing* arises, for example, in determining 3-D molecular structures, in which nodes are atoms and edges are atom pairs for which imprecise interatomic distances are available.

A simple method for bound smoothing is to repeatedly apply limits imposed by *triangle inequality*. The distance bounds obtained can be further tightened by applying *tetrangle inequalities* — the limits imposed by sets of four nodes (instead of three for triangle inequalites) and given by Cayley-Menger determinants. For tetrangle-inequality bound-smoothing only iterative algorithms for approximate solutions are known. For every possible quadruple of nodes, these iterative procedures find upper and lower limits on each of the six distances in a quadruple and then use them to tighten the distance bounds. Thus, each iteration requires solving a constant number of inequalities for all quadruples in a graph on n nodes.

Here, we study the parallelization of one such iterative algorithm on a cluster of DECstation 5000/25 working under the PVM (Parallel Virtual Machine) environment, in which an important step is scheduling a set of edge-disjoint quadruples on the workstations (to avoid write conflicts). Our experience along with extensive empirical results are discussed with special emphasis on the scheduling problem which is closely related to the well-known

Augmenting a tree so that every three vertices lie on a cycle

259 W.D. Goddard, P. Dankelmann, H.C. Swart, University of Natal
Ortrud R. Oellermann*, The University of Winnipeg.

Let P be a property possessed by the complete graph. The augmentation number of a graph with respect to property P, is the smallest number of edges that need to be added to G to produce a graph with property P. The process of adding a minimum number of edges to G to produce a graph with property P is referred to as augmenting G with respect to P. A graph has the nVC property if every n vertices of G lie on a common cycle. The augmentation number of a tree with respect to the 3VC property is determined and an efficient procedure for augmenting a tree with respect to the 3VC property is outlined.

Key Words: Augmenting, trees, cycle properties

A new upper bound for $R_3(K_4{-}e)$

260 Konrad Piwakowski

Technical University of Gdańsk, Foundations of Informatics Dept., 80952 Gdańsk
coni@pg.gda.pl

The Ramsey number $R_m(G)$ is defined to be the least integer n such that every assignment of m colors to edges of K_n results in a monochromatic subgraph isomorphic to G.

Among three-color Ramsey numbers $R_3(G)$ where G contains at most five edges and no isolated vertices only $R_3(K_4{-}e)$ remains unknown.

With the help of computer algorithms we improve by two the upper bound for $R_3(K_4{-}e)$ given by G. Exoo in 1991 and thus we show that $28 \le R_3(K_4{-}e) \le 30$.

Keywords: Ramsey numbers, edge-coloring, exhaustive search.

261

Different Measures for Evaluating Quality of a Partition

Robert Goldberg*, Jacob Shapiro, Jerry Waxman
City University of New York

This paper addresses a well known NP-Hard problem of partitioning a set of n elements into k subsets (termed a k-partition) whose sum of weights are as equal as possible. Many papers in the literature considered this problem with defining the notion of equality using one specific metric. We will survey the various metrics possible and show the relationship of these metrics to each other

The most common metric used in the literature is that of Graham. This metric gauges performance based on the size of the largest subset in the resultant partition which was useful in scheduling problems. Coffman and Luecker consider a metric that involves more than one subset, the difference between the weights of the maximum and minimum subset. Goldberg, Shapiro and Waxman have considered the variance of all of the subsets within the partition.

These metrics are used in analyzing different heuristics of approximation algorithms in obtaining a k-partition. The heuristics analyzed in this paper are local approximations to the optimal solution. These include best fit and first fit heuristics which were first introduced in connection with the bin packing problem. A greedy variant of best fit is also considered and all of these are compared to the basic method of randomly assigning elements to the k subsets.

262

Parallel Computation of a Diameter-Constrained MST and Related Problems

A. Abdalla, N. Deo, N. Kumar*, and T. Terry, UCF

Computing spanning trees with specified properties and constraints constitutes a large class of discrete optimization problems, which arise in a variety of practical situations. Here, 35 such problems culled from various sources in the literature are discussed. One such problem *Diameter-Constrained Minimum Spanning Tree* is the following: given an undirected, edge-weighted graph G and a positive integer k, find a spanning tree with the smallest weight among all spanning trees of G which contain no path with more than k edges. This problem is known to be NP-complete.

Since most of these constrained MST problems are NP-hard, good approximate algorithms are required. It appears that algorithms for this class of problems can be classified into two broad methods. The first one, *iterative refinement*, in each iteration computes an MST of the graph with current weights and then penalizes (increases the weight of) a subset of tree edges so that they are discouraged from appearing in the MST in the next iteration. The subset must contain edges whose removal from the spanning tree (and replacement by other edges) is likely to reduce (and eventually eliminate) the constraint violation. The second method, on the contrary, constructs a spanning tree once and for all in a modified greedy fashion, employing problem-specific heuristics in the selection of the next edge to be added during the tree construction. Both these methods are simple, fast, and lend themselves to efficient parallel implementation, specially on massively-parallel SIMD machines.

A game of cops and robber on isometric subgraphs of strong products of pa ths.

263

*Shannon L. Fitzpatrick, Richard J. Nowakowski
Dalhousie University

We investigate playing the game of "cops and robber" on graphs which can be isometrically embedded in the strong product of paths. In general, if G requires k cops and H requires l cops then $G \boxtimes H$ requires $\max\{k, l\}$ cops. Consequently, only one cop is required on the strong product of paths. The situation is more complicated on subgraphs of products of paths, although some of the structure carries over.

Key words: isometric, embedding, strong product, injective hull, paths, distance, metric.

The Ramsey number for a quadrilateral vs. a complete graph on six vertices.

264

Chula J. Jayawardene* , Cecil C. Rousseau

February 16, 1997

Let $r(C_4, K_n)$ be the smallest integer N such that if a graph on N vertices contains no C_4, then its independence number is at least n.It has been shown by G. Exoo using computer techniques that $r(C_4, K_6) \geq 18$ (Congressus Numerantium, 59 (1987) 31-36).First we will show that there exists exactly one graph on 13 vertices containing no C_4 and having independence number 4. Next using this result, it will be shown that $r(C_4, K_6) \leq 18$. Finally we will give six graphs of order 17 containing no C_4 and having independence number 5, all of which would give the same lower bound obtained by Exoo.

Key Words: graph theory, Ramsey numbers.

Friday, March 7, 1997
4:00 p.m.

265 **The H-polytope of a graph**

G. Sampath, University of Massachusetts at Dartmouth

The set of undirected simple graphs of n vertices is mapped one-to-one to a set of convex polytopes in R^{n^2}. A convex potential function is defined in the interior of the polytope, and the *center of the graph* defined as the point in the interior where the convex function reaches its minimum. This center is computable in polynomial time. It bears a simple relationship to the vertex degrees which is easily derived from a quadratic approximation of the potential function. A heuristic to detect the presence of Hamiltonian paths or cycles in the graph which takes the form of an interior-point algorithm is described.

Keywords: Graph polytopes; Hamiltonian cycles; interior-point algorithms

266 **A Comparison of Two Parallel Algorithms for the Degree-Constrained Minimum Spanning Tree Problem**

Li-Jen Mao* Narsingh Deo Nishit Kumar Sheau-Dong Lang
University of Central Florida, Orlando, FL 32816

The minimum-weight spanning tree (MST) problem with an added constraint that no node in the spanning tree has the degree more than a specified integer number, d, is known as the degree-constrained minimum-weight spanning tree problem (d-MST). The need to compute degree constrained MST arises in many applications such as network designs and back-plane wiring. Since d-MST is an NP-Hard problem, several approximation algorithms have been proposed in the literature. We developed a parallel algorithm on the MasPar MP-I machine using the Weight-Adjust heuristic; the experimental results showed very good time-efficient performance. In this paper we present a new approximation algorithm for the d-MST. The algorithm uses a greedy heuristic and is based on Sollin's MST algorithm adapted to the parallel environment. We implemented this algorithm on the MasPar, and tested it with random graphs and with the TSPlib benchmark. The test results show that while the Weight-Adjust heuristic has a better speed performance, the new approach consistently produced solutions with better quality. Also this new approach has a more robust performance on the lower degree-constrained graphs.

267 **On the Girth of Digraphs**

Jian Shen, Department of Mathematics and Statistics
Queen's University at Kingston K7L 3N6 Canada

It was conjectured that the girth of every digraph with n vertices and minimum outdegree k is at most $\lceil n/k \rceil$. This conjecture was proved for $k = 2$ by Caccetta and Häggkvist. In this note, we generalize Caccetta and Häggkvist's result by showing that the girth of every digraph with n vertices and minimum outdegree 1 is at most $\lceil n/2 \rceil + \lfloor t/2 \rfloor$, where t is the number of vertices having outdegree exactly 1.

268 **Majorizing Real Valued Functions on a Poset**

Hosien S. Moghadam, University of Wisconsin Oshkosh

Let f and g be two order preserving real valued functions on the partially ordered set P. We say f is majorized by g if

$\sum_{x\in P} f(x) \le \sum_{x\in P} g(x)$ for any filter F of P and

$\sum_{x\in P} f(x) = \sum_{x\in P} g(x)$. We will consider some special cases for P

and discuss when f is majorized by g along with some well known results.

Key words: Partially Ordered Sets