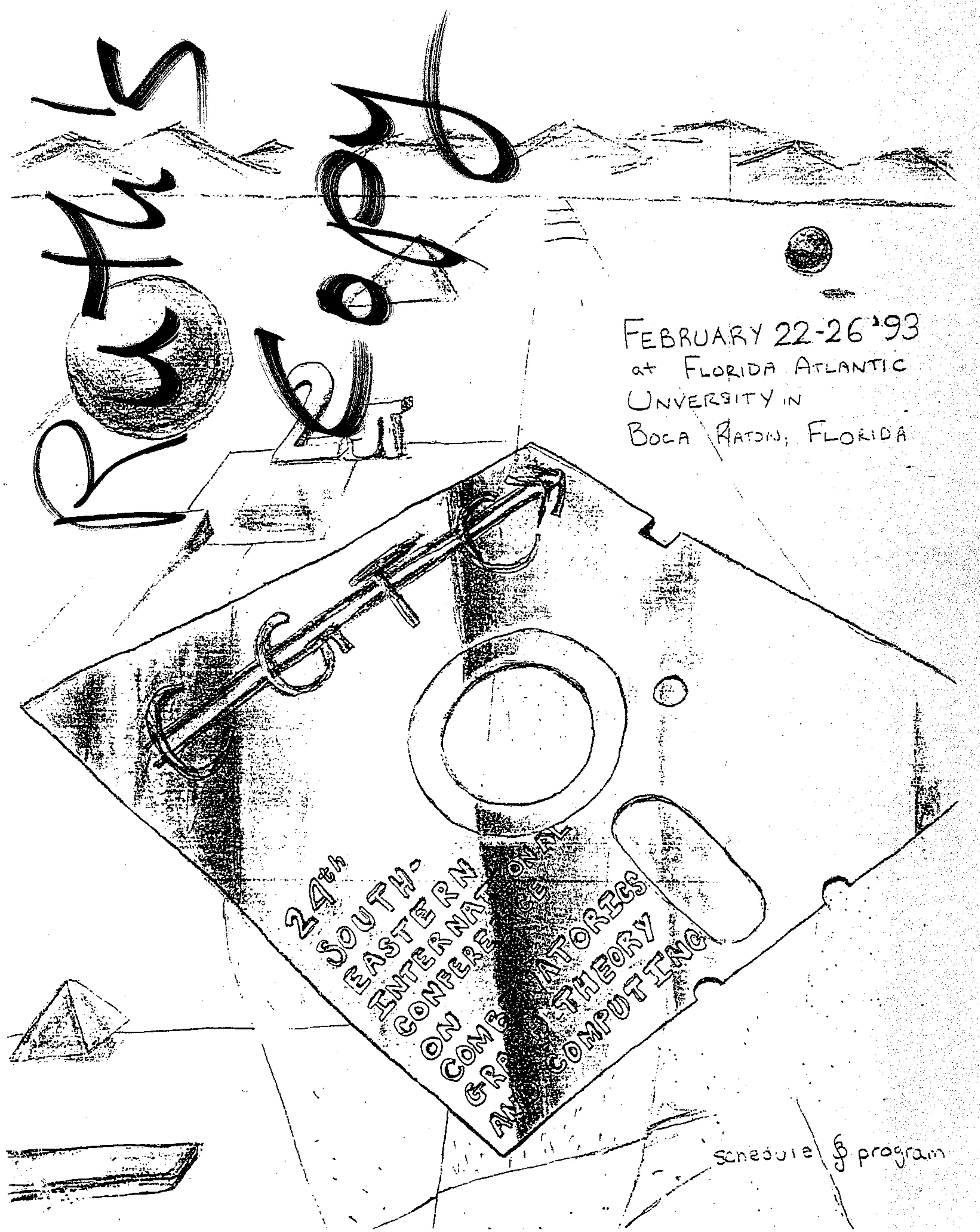
FEBRUARY 22-26 '93
at FLORIDA ATLANTIC
UNVERSITY IN
BOCA RATON, FLORIDA
24th
SOUTH-
EASTERN
INTERNATIONAL
CONFERENCE
ON
COMBINATORICS
GRAPH THEORY
AND COMPUTING
schedule & program

MONDAY, FEBRUARY 22, 1993

REGISTRATION begins at 8:00 A.M. in the downstairs lobby of the University Center, where COFFEE WILL BE SERVED. GCN (left or front) and GCS are the two Halves of the Gold Coast Room. FAU Rooms 202 A and C are reached through the second floor Lounge.

	GCN	GCS	202 A	202 C
9:00 AM	OPENING and WELCOME DEANS CARRAHER AND HOLLAND			
9:30	WALLIS			
10:30	COFFEE			
10:50	1 GRIMALDI	2 TRENK	3 TARJAN	4 SZEKELEY
11:10	5 TANNY	6 LUNDGREN	7 BARKAUSKAS	8 LAWSON
11:30	9 SHIUE	10 McKEE	11 DUNBAR	12 HURD
11:50	13 SULLIVAN	14 MERZ	15 TELLE	16 ZAHID
12:10 PM	17 PRITIKIN	18 CABLE	19 MC KONG	20 HUTCHINSON

12:30 LUNCH (On your own -- Cafeteria open; there are many nearby restaurants - see our lists in your conference package!)

	GCN	GCS	202 A	202 C
2:00	WALLIS			
3:00	COFFEE			
3:20	21 KENNEDY	22 SEAGER	23 WAXMAN	24 MARKUS
3:40	25 MASSEY	26 HARBORTH	27 DEOGUN	28 CHEN
4:00	29 C LIN	30 GOLDWASSER	31 ALTMAN	32 DEAN
4:20	33 FRANCEL	34 ABRHAM	35 CIESLIK	36 CORNEIL
4:40	37 NURMELA	38 ANSTEE	39 WALSH	40 TIMAR
5:00	41 SARVATE	42 GARNICK	43 FOSTER	44 SHREVE
5:20	45 McSORLEY	46 GAREY	47 MAGLIVERAS	48 VARMA

6:15 CONFERENCE RECEPTION in the BOARD of REGENTS ROOM on the THIRD floor of the ADMINISTRATION BUILDING.

There will be Conference transportation back to the motels at 5:50 PM, returning to the reception about 6:30. There will be transportation from the reception back to the motels.

TUESDAY, FEBRUARY 23, 1993

REGISTRATION HOURS (second floor LOBBY, where COFFEE will be served.) 8:15-11:00 A.M. and 1:30-3:30 P.M. GCN (left or front) and GCS are the two halves of the Gold Coast Room. Rooms 202 A and C are reached through the second floor Lounge. There will be book exhibits in Room 232 from 9:00 to 5:00.

	GCN	GCS	202-A	202-C
8:40 AM	49 NAM	50 GRONAU	51 OH	52 WE CLARK
9:00	53 MS ANDERSON	54 KLERLEIN	55 GOVINDARAJU	56 VINCE
9:30	ALSPACH			
10:30	COFFEE			
10:50	57 DANZIGER	58 KLINGSBERG	59 SQUIRE	60 ASHLOCK
11:10	61 KREHER	62 FAUDREE	63WOJCIECHOWSKI	64 GRIGGS
11:30	65 MILLS	66 COX	67 TM-Y WANG	68 PIOTROWSKI
11:50	69 KEY	70 VALDES	71 LUO	72 BEASLEY
12:10 PM	73 PHILLIPS	74 PLUMMER	75 HUANG	76 BAJNOK
12:30	LUNCH BREAK (ON YOUR OWN)			
2:00	ALSPACH			
3:00	COFFEE			
3:20	77 QIU	78 BAGGA	79 WINTERS	80 EO HARE
3:40	81 MORE	82 KUBICKI	83 TIAN	84 HARTNELL
4:00	85 SCHWEIZER	86 QUINTAS	87 PIPPERT	88 VAN WIEREN
4:20	89 FIORINI	90 BALINSKA	91 LINDQUESTER	92 DEJTER
4:40	93 GVOZDJAK	94 STEINER	95 ELMALLAH	96 SLATER
5:00	97 ACKERMAN	98 HOANG	99 CAI	100 LI

6:00 CONFERENCE PARTY at the home of JACK FREEMAN : 741 AZALEA ST, (but park on AURELIA) 395 - 7921.

CONFERENCE TRANSPORTATION will leave for the motels at 5:30. There will be transportation from the UNIVERSITY CENTER to the party at about 5:30, and from the motels at about 6:10. There will be transportation from the party back to the motels. As always, we urge car-pooling, especially with parking spaces scarce near Freeman's. It is a pleasant walk to the Freeman home, should you be adventurous.

WEDNESDAY, FEBRUARY 24, 1993

REGISTRATION HOURS (second floor LOBBY, where COFFEE will be served.) 8:15-11:00 A.M. and 1:30-3:30 P.M. GCN (left or front) and GCS are the two halves of the Gold Coast Room. Rooms 202 A and C are reached through the second floor Lounge. There will be book exhibits in Room 232 from 9:00 to 5:00.

	GCN	GCS	202-A	202-C
8:40 AM	101 TURGEON	102 PENRICE	103 R MARSHALL	104 MANN
9:00	105 HEDETNIEMI	106 GUNTHER	107 McRAE	108 GORDON
9:30	COLBOURN			
10:30	COFFEE			
10:50	109 CHINN	110 McCANNA	111 MEHTA	112 PIAZZA
11:10	113 MACULA	114 HATTINGH	115 HURLBERT	116 HEMMINGER
11:30	117 DC FISHER	118 GIMBEL	119 SJOGREN	120 SCULLY
11:50	121 LEFMANN	122 CARRINGTON	123 RADZISZOWSKI	124 RUSKEY

12:15 PM **CONFERENCE PHOTOGRAPH** at the OUTDOOR STAGE. We will lead you from the lobby, if you can't find it on your own, but PLEASE PARTICIPATE!

12:30 LUNCH BREAK (ON YOUR OWN)

	GCN	GCS	202-A	202-C
2:00	COLBOURN			
3:00	COFFEE			
3:20	125 NASH-WILLIAMS	126 DUTTON	127 PEDERSON	128 LATIFI
3:40	129 C WANG	130 FLETCHER	131 FAN	132 GONZALEZ
4:00	133 ROELANTS	134 S MARSHALL	135 CARSON	136 ORDMAN
4:20	137 KUONG	138 MOON	139 OELLERMANN	140 LEE
4:40	141 HIGGINS	142 J YU	143 FRAUGHNAUGH	144 FIDUCCIA
5:00	145 SIRAN	146 RYAN	147 McKEON	148 STIVAROS
5:20	149 LATKA	150 RODNEY	151 CASEY	152 BERMUDEZ

5:45 An informal tea for women in combinatorics will be held in Room 215 of the Science and Engineering Building from 5:45 to 6:45PM. This event has been organized by Kathy Fraughnaugh and Carolyn Pfeffer Johnston.

The CONFERENCE BANQUET will be held in the Cafeteria Building at 8:00 PM (seating at 7:45). Meet at the Rathskellar in the University Center for a cash bar (beer and wine only) from 6:45 to 7:45 PM. Conference transportation will be available to the motels at 5:45. There will be transportation from the motels to the University at approximately 6:45. There will be transportation back to the motels after the banquet.

THURSDAY, FEBRUARY 25, 1993

REGISTRATION HOURS (second floor LOBBY, where COFFEE will be served.) 8:15-11:00 A.M. and 1:30-3:30 P.M. GCN (left or front) and GCS are the two halves of the Gold Coast Room. Rooms 202 A and C are reached through the second floor Lounge. There will be book exhibits in Room 232 from 9:00 to 5:00.

	GCN	GCS	202-A	202-C
8:40 AM	153 SOMER	154 WILLIAMS	155 NEGRON	156 ASMEROM
9:00	157 BENNETT	158 WATKINS	159 MARCANO	160 GREEN
9:30	ERDOS			
10:30	COFFEE			
10:50	161 GREIG	162 SANKARANARAYANA	163 MAKKI	164 Y-C LIN
11:10	165 HAMM	166 deFIGUEIREDO	167 AMIN	168 KANCHI
11:30	169 FINIZIO	170 LASKAR	171 EDWARDS	172 SOIFER
11:50	173 JEDWAB	174 BORIE	175 NAIR	176 KG FISCHER
12:10 PM	177 DAVIS	178 SCHAAL	179 GEWALI	180 SALZBERG
12:30	LUNCH BREAK (ON YOUR OWN)			
2:00	KOBLITZ			
3:00	COFFEE			
3:20	181 CHOPRA	182 RASMUSSEN	183 ABBAS	184 SAMPATH
3:40	185 DEUTSCH	186 WANTLAND	187 CA ANDERSON	188 HOBBS
4:00	189 LIPKIN	190 BURRIS	191 TESMAN	192 EATON
4:20	193 OSSOWSKI	194 D ZHENG	195 LIU	196 PRUESSE
4:40	197 CAISEDA	198 DOMKE	199 KINNERSLEY	200 CRIBB
5:00	201 ITO	202 GODDYN	203 ULLMAN	284 OPOROWSKI
5:20	205 McNULTY	206 BARR	207 ALAVI	208 ERICKSON

There will be an informal CONFERENCE PARTY 6:15-7:30 in the Cafeteria Patio area--to be moved indoors if weather dictates. There will be Conference transportation back to the motels at 5:45 PM and back to the party at 6:15. There will be transportation back to the motels after the party.

FRIDAY, FEBRUARY 26, 1993

REGISTRATION HOURS (second floor LOBBY, where COFFEE will be served.) 8:15-11:30 A.M. GCN (left or front) and GCS are the two halves of the Gold Coast Room. Rooms 202 A and C are reached through the second floor Lounge. There will be book exhibits in Room 232 from 9:00 to 11:30.

	GCN	GCS	202-A	202-C
8:40 AM	209 L WANG	210 O'REILLY	211 DR HARE	212 MAJEWSKI
9:00	213 TAPIA	214 PITTEL	215 GARGANO	216 SINNAMON
9:30	KOBLITZ			
10:30	COFFEE			
10:50	217 CHAVEY	218 RAYBURN	219 ADHAR	220 SPRAGUE
11:10	221 MARTIN	222 KILLGROVE	223 RANDIC	224 LEW
11:30	225 TERWILLIGER	226 JURISIC	227	228 SHAHROKHI
11:50	229 WHITE	230 NOSTRAND	231 BALKRISHNAN	232 BOURJOLLY
12:10 PM	233 REES	234 DIPAOLA	235 Y ZHAO	236 H-J LAI
12:30	LUNCH (ON YOUR OWN)			
2:00	237 TONCHEV	238 McGUINNESS	239 KLASA	240 GIUDICI
2:20	241 IHRIG	242 KIAER	243 PETERSON	244 T LU
2:40	245 B ANDERSON(IHRIG)	246 COSTA	247 BRAND	248 LEWIS
3:00	249 PERVIN	250 MACGILLIVRAY	251 HAIN	252 BRAWLEY
3:20	253 RADAS	254 BREWSTER	255 McCAULEY	256 JOHNSON
3:40	257 GRIFFIN	258 GOLDBERG	259 LAUE	260 HOCHSTATTLER

There will be transportation back to the motels following the last talks.

THANKS FOR COMING!!

There will be an informal after-dinner SURVIVORS PARTY, at the home of Aaron Meyerowitz and Andrea Schuver, 454 NE Third Street, beginning about 8PM. Tell us if you need transportation.

WE'LL SEE YOU **HERE** FOR THE TWENTY-FIFTH SOUTHEASTERN INTERNATIONAL CONFERENCE ON COMBINATORICS, GRAPH THEORY AND COMPUTING,

***** FEBRUARY ??-??, 1994 *****

INVITED INSTRUCTIONAL LECTURES

MONDAY, FEBRUARY 22, Professor Walter D. Wallis of Southern Illinois University will speak on *Hadamard Matrices, 1893-1993*, at 9:30AM and 2:00PM.

TUESDAY, FEBRUARY 23, Professor Brian R. Alspach of Simon Fraser University will speak on *Graph Decompositions I:Cycle Covers and Decompositions*, at 9:30AM and on *Graph Decompositions II: Orthogonal Factorizations*, at 2:00PM.

WEDNESDAY, FEBRUARY 24, Professor Charles J. Colbourn of the University of Waterloo will speak on *Reliability Polynomials*, at 9:30AM and on *Edge-Partitioning Multigraphs into Triangles*, at 2:00PM.

THURSDAY, FEBRUARY 25, Professor Paul Erdos of the Hungarian Academy of Sciences will speak on *Some Surprises*, at 9:30AM.

THURSDAY, FEBRUARY 25, at 2:00PM and FRIDAY, FEBRUARY 26, at 9:30AM,Professor Neal I. Koblitz of the University of Washington will speak on *Kid-Krypto: Combinatorially Based Cryptography for Kids (and Adults)*.

Hadamard Matrices 1893-1993
Walter D. Wallis, Southern Illinois University

This will be a historical survey of definitions and existence and uniqueness questions in the theory of Hadamard matrices over the past hundred years. Applications and relations to other structures will be discussed.

Graph decompositions I and II

Brian Alspach Simon Fraser University alspach@cs.sfu.ca

Graph decompositions I deals with three main threads concerning cycle decompositions of graphs. The first thread starts with Walecki's decompositions of complete graphs into Hamilton cycles. This leads to a variety of Hamilton decomposition questions. The second thread starts with Steiner triple systems and leads to questions about decomposing complete graphs into cycles. The third thread starts with Euler's solution of the Kõnigsberg bridges problem. This leads to the present situation of considerable interest in decomposing weighted graphs into cycles. The topics will be surveyed with an emphasis on unsolved problems.

Graph decompositions II deals with orthogonal and suborthogonal factorizations of graphs. Suppose two factorizations $\mathcal{F} = \{F_1, F_2, \ldots, F_r\}$ and $\mathcal{H} = \{H_1, H_2, \ldots, H_s\}$ of a graph G are given. We say that $\mathcal{F}$ and $\mathcal{H}$ are *suborthogonal* if and only if $|E(F_i) \cap E(H_j)| \leq 1$ for all $1 \leq i \leq r$ and $1 \leq j \leq s$. We say they are *orthogonal* when equality always holds. Many classical combinatorial topics can be discussed in this context.

For example, a pair of orthogonal $n \times n$ Latin squares can be interpreted as two orthogonal 1-factorizations of the complete bipartite graph $K_{n,n}$. A Room square of order $2n$ can be interpreted as two suborthogonal 1-factorizations of the complete graph K_{2n}.

There are many unsolved problems dealing with suborthogonal factorizations. These will be the focus of the discussion on this topic.

Reliability Polynomials
Charles J. Colbourn, University of Waterloo

The probability that a network is connected when each link in the network operates with the same probability p can be succinctly represented as a polynomial in p. Various forms of this polynomial reveal combinatorial structure inherent in its coefficients; this combinatorial structure can be employed to obtain bounds on the coefficients, and (surprisingly) also on the value, of the reliability polynomial. Theorems relating combinatorial structure of the network to its reliability polynomial are outlined, and some open questions discussed.

Edge-Partitioning Multigraphs into Triangles
Charles J. Colbourn, University of Waterloo

A number of questions concerning designs with block size three, such as existence, completion, and embedding, reduce to the problem of determining when a multigraph has an edge-partition into triangles. We examine necessary conditions for a multigraph to have such a partition, and then concentrate on partitions of complete multipartite graphs (= group-divisible designs). We mention a number of open questions.

Combinatorially Based Cryptography for Children (and Adults)
Neal Koblitz

In these two talks I describe how certain notions of modern cryptography can be presented to youngsters using combinatorial constructions. Among the topics discussed are the use of Boolean circuits for bit commitment protocols and hash functions, and the construction of a public key message transmission system using perfect codes in a graph. I also discuss how this project relates to math education reform. In particular, experience suggests that ''crayon-technology'' topics in discrete math and cryptography might have a special value for female and minority children in the U.S. and for schoolchildren in the Third World.

INDEX OF ABSTRACTS

This index contains a list of all authors of abstracts, not just presenters. The list is complete as of 4:00 p.m. on Thursday, February 18, 1993.

INDEX (continued)

Monday, February 22, 1993
10:50 a.m.

1 Alternating Subsets and the Fibonacci Numbers

Ralph P. Grimaldi
Rose-Hulman Institute of Technology

For $n \in \mathrm{N}$, the nth Fibonacci number F_n is defined recursively by (1) $F_0 = 0$, $F_1 = 1$; and, (2) $F_n = F_{n-1}+F_{n-2}$, for all $n \geq 2$. For $n \in \mathrm{Z}^+$ let $[n] = \{1, 2, 3, \ldots, n\}$. A nonempty subset A of $[n]$ is called *alternating* if $A = \{a_1, a_2, \ldots, a_k\}$ where $1 \leq k \leq n$, $a_1 < a_2 < \ldots < a_k$, and the parity of a_i and that of a_{i+1} are different for all $1 \leq i \leq k-1$. If a_1 is odd, the number of such subsets A is $F_{n+2} - 1$ — as first observed by Olry Terquem (1782-1862). In this paper we examine properties of the numbers (i) $F(n, m)$, the number of alternating subsets of $[n]$ of cardinality m, where $1 \leq m \leq n$; and, (ii) $F(n) = \sum_{m=1}^{n} F(n, m)$.

We also consider the numbers: (i) $N(n, k)$ which counts the number of alternating subsets of $[n]$ that contain k, for $1 \leq k \leq n$; (ii) $T(n)$ which counts the total number of entries in all the alternating subsets of $[n]$; and , (iii) $S(n)$ which is the sum of all the entries in all the alternating subsets of $[n]$.

2 Bipartite Tolerance Orders

Kenneth P. Bogart, Dartmouth College
Ann N. Trenk*, Wellesley College

Tolerance orders and tolerance graphs arise as a generalization of interval orders and interval graphs in which some overlap of intervals is tolerated. The classes of bounded, proper and unit tolerance orders and graphs have been studied by other authors. Here we introduce two-sided versions of these classes. We show that *all* the classes are identical for bipartite ordered sets and give some examples to show that the classes differ in the nonbipartite case. We also give an algorithm for determining whether a bipartite ordered set is in these classes, and if so finding a representation of it.

Key words: tolerance orders and graphs, interval orders and graphs

3 Dominating Sets in Triangulated Graphs

Lesley R. Matheson and Robert E. Tarjan
NEC Research Institute and Princeton University

We consider the problem of asymptotically minimizing the size of dominating sets in planar graphs. Specifically, we wish to find the smallest ϵ such that, for n sufficiently large, every n-vertex planar graph contains a dominating set of size at most ϵn. We prove that $1/4 \leq \epsilon \leq 1/3$, and we conjecture that $\epsilon = 1/4$. For triangulated discs we obtaina tight bound of $\epsilon = 1/3$. The upper bound proof yields a linear-time algorithm for finding a satisfactory dominating set.

4 MIN-MAX THEOREM FOR A MULTIWAY CUT PROBLEM

by L. A. Szekely
Eotvos University and University of New Mexico

The multiway cut problem is as follows: given a graph G and a subset N of its vertex set, delete the smallest number of edges to separate any pair of vertices of N. By the work of Dahlhaus et al., the multiway cut problem is NP-hard already for $|N|=3$.
In this paper we show a min-max theorem and a polynomial time algorithm, if every cycle of G contains at least one element of N. The results generalizes for edge weighted graphs.
This is a joint work with P. L. Erdos and A. Frank.

Monday, February 22, 1993
11:10 a.m.

5

More Well-Behaved Meta-Fibonacci Sequences

J. Higham & S. Tanny* University of Toronto

A seemingly close relative of the Hofstadter sequence, the meta-Fibonacci sequence $T(n)$ given by $T(n) = T(n-1-T(n-1)) + T(n-2-T(n-2))$, $n > 2$ with $T(0) = T(1) = T(2) = 1$ is known to behave in a completely predictable way. Varying the initial values for the recursion produces a range of results, including new well-behaved but much more complex sequences. Alternatively, the sequences $t_k(n)$ satisfying the same recursion, with $k \geq 4$ initial conditions defined by $t_k(i) = \lfloor \frac{i}{2} \rfloor$, $0 \leq i < k$, are also well-behaved meta-Fibonacci sequences, so long as $k \not\equiv 3 \pmod 4$). Even further, these sequences do not "drift" very far from the original $T(n)$ sequence, in a sense that can be made precise. Finally, we extend the original recursion for $T(n)$ in the natural way to allow for k terms on the right hand side, and note that this produces another well-behaved meta-Fibonacci sequence with properties analogous to those of $T(n)$.

Key Words: meta-Fibonacci sequence; recursion.

6

INVERTING COMPETITION GRAPHS

J.RICHARD LUNDGREN* AND SARAH K. MERZ, U. OF COLORADO AT DENVER

Competition graphs have appeared in a variety of applications from communication networks to food webs to energy models. The competition graph, $C(D)$ of a digraph D has the same vertex set as D and (x, y) is an edge in $C(D)$ if and only if there is a vertex z such that (x, z) and (y, z) are arcs in D. For a given graph G, there may be several digraphs D such that $G = C(D)$. We examine this class of digraphs $\mathcal{D}$, finding a canonical representative that reflects the structure of G. For example, if G is chordal or interval the structure of $\mathcal{D}$ is of particular interest. This approach may shed new light on the open question of determining acyclic digraphs with interval competition graphs. For strongly connected digraphs representing communication networks, we investigate the problem of maximizing the number of arcs in a digraph without changing the competition graph.

7

COUNTING EFFICIENT DOMINATING SETS IN ORIENTED TREES

David W. Bange[1], Anthony E. Barkauskas*[1], Lane H. Clark[2], Linda H. Host[1]
[1]University of Wisconsin – La Crosse, [2]Southern Illinois University at Carbondale

A digraph D is said to have an efficient dominating set S if S is a set of vertices of D such that for each vertex v of D, either v is in S and has no neighbors in S, or v is not in S but is adjacent from exactly one member of S. For a labeled graph G, an *efficiency* of G is an ordered pair $(\vec{G}, S)$ where $\vec{G}$ is an orientation of G and S is an efficient dominating set of $\vec{G}$. The number of efficiencies of G is denoted by $\eta(G)$. If T is a labeled tree, $\eta(T)$ is equal to the number of orientations of T which can be efficiently dominated. Two open questions are discussed. "What is the structure of trees T_q^* having q edges and the maximum number of efficiencies?" "What is the asymptotic behavior of the sequence $(\eta(T_q^*))^{1/q}$?"

8

Node and edge clique cover numbers of E-graphs

Linda M. Lawson* and Teresa W. Haynes
East Tennessee State University

E-graphs are constructed by replacing each edge in a core graph, G, with a copy of a graph, H. We determine node and edge clique cover numbers of E-graphs based on invariant values of G and H.

Monday, February 22, 1993
11:30 a.m.

10 F-*free Interval Graphs*

Terry McKee, Dept. of Mathematics, Wright State University, Dayton OH 45435

Kruskal's "greedy tree algorithm" can be viewed as underlying much of chordal graph theory and its applications. A naive "greedy path algorithm" leads naturally to a special sort of interval graphs called "F-*free interval graphs*." These include Roberts's *indifference graphs* (a.k.a. *unit* (or *proper*) *interval graphs*) and Wolk's P_4C_4*-free graphs* (i.e., *nested interval graphs*), and so also Chvatal & Hammar's *threshold graphs*, now characterizable in greedy, but Prim terms.

11 **Nearly Perfect Sets in Graphs**

* Jean Dunbar, Converse College, Spartanburg, S.C.
Sandra Hedetniemi, Stephen Hedetniemi, Renu Laskar,
Alice McRae, Clemson University, Clemson, S.C.

In a graph G = (V,E) a set of vertices S is nearly perfect if every vertex in V - S is adjacent to at most one vertex in S. Nearly perfect sets are closely related to 2-packings of graphs or strongly stable sets, dominating sets and efficient dominating sets. We say a nearly perfect set S is 1-minimal if for every vertex u in S, the set S - {u} is not nearly perfect. Similarly, a nearly perfect set S is 1-maximal if for every vertex u in V - S, the union of S with {u} is not a nearly perfect set. Lastly, we define n sub-p (G) to be the minimum cardinality of a 1-maximal nearly perfect set, and N sub-p(G) to equal the maximum cardinality of a 1-minimal nearly perfect set. In this paper we look at various bounds on these parameters and we characterize their values for some classes of graphs. We show that finding n sub-p(G) is an NP-complete problem and we discuss a linear algorithm for determining n sub-p(T) for a tree T, and a polynomial algorithm for determining N sub-p (G) for any graph G.

key words: domination, efficient domination, perfect domination

12 **A Discrete Version of a Stochastic Process: The One-Third Game**

by Spencer P. Hurd, The Citadel, Charleston, SC, 29409
and J.S. McCranie, 1505 East Park Ave, V-11, Valdosta, Ga, 31602.

We imagine a card game where a set of players sitting around a table complete a "play" by each passing 1/3 of his cards to the player on his left and then the other 2/3 to the same designated "end" player. Fractions of a card are rounded to the nearest integer number of cards. The card distributions are vertices of a digraph and paths follow the play of the game. We determine the cycle structure for all the digraphs with 3 players and N cards. When M is a fixed number of players, we show that the cycle structure for the 1/M Game and N cards is periodic and easily calculable.

Monday, February 22, 1993
11:50 a.m.

13 Some Comments on Binary Codes of Steiner Triple Systems

F. E. Sullivan, Clemson University

A *Steiner triple system* $\mathcal{D} = (\mathcal{P}, \mathcal{B})$ is an incidence structure of points $\mathcal{P}$ and blocks $\mathcal{B}$ such that every block is incident with precisely three points and any two distinct points are together incident with exactly one block, i.e a 2-(v,3,1) design. The binary *code* $C_2(\mathcal{D})$ of $\mathcal{D}$ (over the finite field F_2) is the subspace that is spanned by the incidence vectors of the blocks. If d is an integer such that $2^d - 1 \leq v < 2^{d+1} - 1$, then the binary code of the design contains a subcode that can be shortened to the binary Hamming code $\mathcal{H}_d$ of length $2^d - 1$ (joint work with J. D. Key). In the case v=15, examples for each dimension from 11 to 15 are given. Doyen, Hubaut and Vandensavel(Math. Z., 163: 251-259, 1978) noted that the 2-rank of a 2-(15,3,1) is 15,14,13,12 or 11 according as the system contains 0,1,3,7, or 15 2-(7,3,1) "sub-systems." We give a straightforward way of finding these subsystems in the binary case.

14 A CHARACTERIZATION OF GRAPHS WITH INTERVAL SQUARES

J. RICHARD LUNDGREN AND SARAH K. MERZ*, U. OF COLORADO AT DENVER
CRAIG W. RASMUSSEN, NAVAL POSTGRADUATE SCHOOL

The *square* of a graph $G = (V, E)$, denoted G^2, is a graph on the same vertex set V such that two vertices x and y are adjacent in the square if there is a path of length one or two between x and y in G. Squares of graphs are useful in the study of radio communication networks.

G $\quad G^2 \quad$ G $\quad G^2$

We say a graph is an *interval graph* if it is the intersection graph of some family of intervals on the real number line. One nice quality of interval graphs is that the chromatic number can be easily calculated. In the interest of efficiently assigning radio frequencies in a communication network, we would like to determine which graphs have interval squares. Short of construction, can we determine whether or not the square of a given graph is interval? Ideas and results pertaining to this question will be discussed.

15 **Domination parameters and partial k-trees**

Jan Arne Telle - CIS Department, University of Oregon.

Many graph parameters optimize an objective function over selected subsets of vertices with some constraint on how many selected neighbors a vertex can have. Classic examples are minimum dominating set and maximum independent set, and the list of papers related to such parameters is steadily increasing. We give a formal description of some of these graph parameters which on the one hand serves to unify their definitions and on the other hand is instrumental in giving low-constant linear time algorithms for computing them on partial k-trees, for fixed values of k. This formalism also eases the introduction of new parameters and we do just that with a few problems that have not previously been studied, showing some NP-completeness results.

16 **Tie-Breaking Rules for 4 x n Warnsdorff's Tours**

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The Citadel
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It has been shown earlier that a fixed tie-breaking rule can produce a Warnsdorff's tour for all 3 x n boards, n >= 12 (results to appear in the Journal of Recreational Mathematics). This paper addresses the correctness of Warnsdorff's rule for 4 x n chess boards.

The following questions will be the focus of this paper.
1. What is the minimal set of start positions, if any, that will yield a Warnsdorff's tour for all 4 x n boards, by the Leftmost Tie-breaking Rule (defined below)?
2. Is there a fixed tie-breaking rule for all 4 x n boards which yields a Warnsdorff's tour from start position (1,1)?
Definition: (Leftmost Tie-breaking Rule) If there is a tie for the next move of the knight by the Warnsdorff's rule, select the first from the list of ties obtained clockwise from 1 o'clock to 11 o'clock.

Monday, February 22, 1993
12:10 p.m.

17 **Graph Embeddings from Hamming Bases**

Dan Pritikin, Miami University, Oxford, Ohio

An algorithm due to Aiello and Leighton dynamically embeds arbitrary binary trees into hypercubes, with small dilation and congestion and (with high probability) small load. The operation and performance level of the algorithm both depend upon first specifying a convenient choice of basis for each binary Hamming code. Making use of particular labellings of double-rooted complete binary trees, in this report we answer an open problem (posed in Leighton's text Introduction to Parallel Algorithms and Architectures) by improving upon the previously best known choices of bases, and thereby reduce the upper bounds for the dilation and congestion associated with the embeddings produced by the algorithm.

From a coding theory point of view, for each binary Hamming code we construct a basis comprised of words of weight 3 with the property that for each coordinate there are at most 5 basis vectors with a 1 in that coordinate position.

18 **NICHE NUMBER OF UNIT INTERVAL GRAPHS**

Steve Bowser and Charles Cable, Allegheny College

In this paper we present some sufficient conditions on the maximal cliques of a unit interval graph which will insure that the niche number is zero. An algorithm is presented for finding an appropriate niche digraph which has no isolated vertices.
We also show that the niche number of k-partite graphs of sufficiently large size is infinity.

19 **Computing Domatic Numbers on Permutation Graphs**

M.C. Kong, University of Kansas

Given a simple graph G = (V,E). A set of vertices D of V is a dominating set of G iff every vertex in V-D is adjacent to a vertex in D. The domatic number d(G) of G is the maximum number k such that V can be partitioned into k disjoint dominating sets, and the domatic number problem (DNP) is to decide for a given graph G and an integer k > 0 if d(G) >= k. DNP is known to belong to the class of NP-complete problems even when restricted to the classes of circular-arc graphs, split graphs, and bipartite graphs. We prove in this paper that DNP belongs to P for permutation graphs by presenting an efficient polynomial algorithm for computing d(G).

Keywords: Algorithm, computational complexity, dominating set, domatic
number, permutation graphs.

20 **When three people shake the same number of hands: triples in degree sequences.**

Joan P. Hutchinson, Macalester College

It is well known that the degree sequence of a simple graph has a repeated degree and that if there is only one repeated degree on n vertices, then the repeated degree lies between (n-2)/2 and n/2. We prove an analogous result for a triply-repeated degree.
Theorem: There is a graph on $n \geq 4$ vertices with all degrees distinct except for one degree, j, that is repeated precisely three times if and only if $(n-3)/4 \leq j \leq (3n-1)/4$.

Key words: degree sequence

Monday, February 22, 1993
3:20 p.m.

21 **MAXIMUM PACKINGS OF Kn WITH HEXAGONS**

Janie Ailor Kennedy, Discrete and Statistical Sciences
Auburn University email: jailor@ducvax.auburn.edu

A packing of Kn with hexagons is a pair (S, P) where P is an edge disjoint collection of hexagons. The set of uncovered edges is called the leave. If (S, P) is a packing and the cardinality of P is as large as possible then (S, P) is called a maximum packing. We give a complete solution of the maximum packing of Kn with hexagons problem, covering all possible leaves.

22 **Relaxations of Niche Graphs**

Suzanne Seager, Mt. St. Vincent Univ. Halifax NS, B3M 2J6

Niche graphs were introduced as a generalization of competition graphs. Determining even which caterpillars are niche graphs appears to be difficult. We examine the effect of relaxing the definition of niche graph to allow cycles in the underlying digraph, and apply the results to triangle-free graphs, and specifically to caterpillars.

Keywords: niche graph, caterpillar

23 **Experimental Analysis of the Complexity of Level Graph Algorithms**

Jacob Shapiro, Baruch College, Jerry Waxman, Queens College

A level graph, denoted by G = (V,E,k,f), is a structure consisting of a set of vertices V, a set of edges E, a positive integer k, and an onto function f: E { 1,2,3, ..., k }. A level graph is called rank constrained, if for any two edges (u,v), (u',v), abs(f(u,v)-f(u',v)) $ 1.
Two algorithms, LGS and ALGS, previously introduced by the authors, exhibited asymptotic optimality with respect to quasi-shortest path generation in level graphs. This paper examines the time complexity/path optimality of Dijkstra, LGS and ALGS on rank constrained level graphs.
Level graphs are useful in modeling situations where the edges of the graph may be assigned a level in addition to an edge cost; a highway system is an example. Important applications include such areas as path generation in maps, network design and routing problems.

Keywords: level graph, rank constrained level graph, approximate, run-time complexity shortest path algorithms.

24 **Hamiltonian cycles in $K_{1,r}$-free graphs**

L. R. Markus, Furman University

Let G be a graph and let δ denote the minimum degree of G. One of the oldest results about Hamilton cycles is due to Dirac: If $\delta \geq \frac{p}{2}$ then G is Hamiltonian. A graph is $K_{1,r}$-free if it does not contain $K_{1,r}$ as an induced subgraph. It is claw-free if it does not contain $K_{1,3}$ as an induced subgraph. Matthews and Sumner proved that if G is a 2-connected, claw-free graph with $\delta \geq \frac{p-2}{3}$, then G is Hamiltonian. In this talk we investigate Hamilton cycles in $K_{1,r}$-free graphs with respect to a minimum degree condition.

Keywords: $K_{1,r}$-free, minimum degree, Hamilton cycle.

25 CYCLE PACKINGS IN GRAPHS AND DIGRAPHS

JENNIFER J. Q. MASSEY, UNIVERSITY OF WISCONSIN–MADISON

A cycle packing in a (directed) multigraph is a vertex disjoint collection of (directed) elementary cycles. If D is an Eulerian multidigraph we show that the arcs of D can be partitioned into Δ_{in} cycle packings - where Δ_{in} is the maximum indegree of a vertex in D. This result is used to show that the maximum length cycle packings in any digraph D contain a common vertex. Similarly, if G is an Eulerian multigraph of maximum degree Δ, we show that the edges of G can be partitioned into $\Delta/2$ cycle packings. We *conjecture* that the maximum length cycle packings in any graph contain a common vertex. This conjecture can be verified for outerplanar multigraphs, vertex transitive multigraphs, regular multigraphs of even degree, and cubic multigraphs.

keywords: cycle packing decomposition, cycle packing intersection

26

Ramsey Colorings for Diagonals of Convex Polygons

Heiko Harborth, Techn. Univ. Braunschweig, Germany

As in the case of Ramsey numbers it is asked for the smallest number $r=r_c(G)$ such that every two-coloring of the diagonals (included sides) of a convex polygon with r vertexpoints contains a monochromatic configuration G, for example, a convex cycle C_4. - First results are given for small G. - This is common work with A. Bialostocki.

27 Computational Complexity of Weighted Edge-Integrity - A Measure of Network Vulnerability

Sibabrata Ray, Jitender S. Deogun*, University of Nebraska

In this paper we introduce the concept of weighted integrity. The concept of weighted integrity is motivated by the fact that utility and cost of destruction of various elements of a network could be different. A set of sufficient conditions for the weighted edge integrity problem to be NP-complete are presented. These conditions can be used to prove that weighted edge-integrity problem is NP-complete on many classes of graphs on which the unweighted problem is known to be polynomial or is of unknown complexity. A similar result for weighted vertex integrity can be easily proved. A polynomial time algorithm for weighted integrity problem on Interval Graphs is developed.

Key Words: Computational Complexity, Graph Integrity, Network Vulnerability.

28

Hamiltonian Graphs with Large Neighborhood Unions

Guantao Chen, North Dakota State University

In this paper we prove the following. Let G be a 4(k-1) connected graph of order n. If $|N(S)| > (n-1)/2$ for every independent vertex set S of k vertices, then G is hamiltonian.

The key words are: Cycles, Hamiltonian Cycles, Neighborhood Unions, and Degrees.

Monday, February 22, 1993
4:00 p.m.

29 **Hadamard matrices of order 32**

C.Lin*, University of Nevada, Las Vegas
W.D. Wallis, Southern Illinois University
Zhu Lie, Suzhou University, P.R. China

We use several methods to construct Hadamard matrices of order 32 and find 4-profiles of these Hadamard matrices to give lower bounds on the number of equivalence classes of Hadamard matrices of order 32.

30

Maximum subsets of (0,1) with no solutions to x+y = kz

F. Chung, Bellcore and J. Goldwasser*, West Virginia Univ.

If k is a positive integer we say S is a k-sum-free set if there are no solutions to x + y = kz with x, y, z in S. For k greater than 3, we find the maximum size of a measurable k-sum-free subset of (0,1). The essentially unique maximum 4-sum-free subset of (0,110) is the union of [1,2), [7,14) and [55,110). Remarks will also be made about maximum k-sum-free subsets of {1, 2, ..., n} and maximum densities of k-sum-free subsets of the positive real numbers and the positive integers.

31

Algebraic Algorithms for the Graph Isomorphism Problem

Tom Altman, University of Colorado at Denver, Denver, CO 80217

We present two algorithms for the graph isomorphism problem. The first algorithm is based on the Principal axis theorem and a specific marking of the diagonal elements of the respective adjacency matrices. It is a backtracking algorithm, hence, in the worst case its running time is not bounded by a polynomial. The class of graphs that forces it to run in exponential time will be identified.
In the second algorithm, the backtracking phase is replaced by a heuristic that uses a sequence of at most n steepest descent projections to perform the required marking. Since each projection involves a solution of $O(n)$ assignment problems, the running time of the second algorithm is $O(n^5)$.

Categories and Subject Descriptors: F2.2 Analysis of Algorithms and Problem Complexity; G.2.2 [Discrete Mathematics]: Graph Theory-graph algorithms

32 The Complexity of Hamiltonian-connected Graphs

Alice M. Dean, Skidmore College

We determine the computational complexity of several problems related to hamiltonian-connected graphs. In particular, we show that it is NP-complete to determine whether a graph G is hamiltonian-connected. We also show that it is NP-complete to determine whether G is hamiltonian-connected from a distinguished vertex v. Lastly, we consider the complexity of multiple-solution and unique-solution variations.

KEY WORDS: Computational complexity, hamiltonian-connected, graphs, NP-complete

NOTE: Please schedule this talk Monday-Wednesday.

Monday, February 22, 1993
4:20 p.m.

33 **Certain relationships and nonexistence results for affine μ-resolvable balanced ternary designs**

Margaret Francel* (The Citadel) and Dinesh G. Sarvate (College of Charleston)

Some new parametric relationships of affine μ-resolvable balanced ternary designs are presented including the fact that a balanced ternary design, BTD(V, B= βt, R, K, Λ), is affine μ-resolvable with t classes if and only if B-V = t-1. These relationships are then used to give some nonexistence results and to prove that an affine μ-resolvable balanced ternary design is column regular.
1

34 **EXISTANCE THEOREMS FOR CERTAIN TYPES OF GRACEFUL VALUATIONS OF SNAKES**

Jaromir Abrham, Department of Industrial Engineering
University of Toronto, Toronto, Ontario, Canada M5S 1A4

A graceful valuation of a graph G with m vertices and n edges is a one-to-one mapping ψ of the vertex set V(G) into the set {0,1,...,n} with the following property: If we define, for any edge e ∈ E(G) with the end vertices u,v ∈ V(G), the value $\bar{\psi}(e)$ of e by $\bar{\psi}(e) = |\psi(u) - \psi(v)|$ then $\bar{\psi}$ is a one-to-one mapping of E(G) onto the set {1,2,...,n}. An α-valuation of G is a graceful valuation of G for which there exists a number γ $(0 \le \gamma < |E(G)|)$ such that, for any edge e ∈ E(G) with the end vertices u,v, it is $\min\{\psi(u), \psi(v)\} \le \gamma < \max\{\psi(u), \psi(v)\}$. An α_k-valuation of P_n (the snake with n edges) is an α-valuation of P_n in which k is the smaller of the values of the end vertices. For each integer $k \ge 0$, the paper presents all values n for which P_n has an α_k-valuation.

Key words: graceful graphs, snakes.

35 **Shortest Trees in Banach-Minkowski Spaces**

(by Dietmar Cieslik, University of Greifswald, Germany)

For a finite set of points in a space a Steiner Minimal Tree (SMT) is a shortest tree which interconnects these points. We also consider some relativs of this problem: 1. The restricted Steiner Problem where we only allow k additional points in the tree (k-SMT) and 2. The Fermat Problem, where a point has to be found such that the sum of the distances to the given points is as small as possible. Solutions depend essentially on the way to determine the distances in the space. We consider the Steiner Problem and its relativs in arbitrary Banach-Minkowski spaces.

A 0-SMT can be found easily by the well-known Minimal Spanning Tree algorithm. There is an algorithm solving the Fermat Problem by solving a linear programming problem, at least approximatly. Furthermore, it is shown that the degrees of the vertices in a SMT or k-SMT are bounded by numbers which only depend on the space. We give a solution for the 1-SMT problem. This method needs polynomial bounded time only. In the Euclidean plane and the plane with rectilinear norm we construct SMT and k-SMT for all $k \geq 0$.

Since the Steiner Problem is at least NP-hard, but it is possible to compute a 0- or 1-SMT in polynomial bounded time, we are interested in bounds for the error which arises by replacing a SMT by a 0- or 1-SMT. These errors are called Steiner Ratios. We give bounds for these numbers of the spaces.

36 **THE LINEAR STRUCTURE OF GRAPHS: AN UPDATE**

Derek Corneil*, Dept. of Computer Science, University of Toronto,
Stephan Olariu, Dept. of Computer Science, Old Dominion University,
Lorna Stewart, Dept. of Computing Science, University of Alberta.

Many families of perfect graphs such as interval graphs, permutation graphs, and cocomparability graphs demonstrate a type of linear ordering of their vertex sets. These graphs are all asteroidal triple free (an independent triple {x,y,z} where between every pair there is a path that avoids the neighbourhood of the third). In this talk we present new results on asteroidal triple free graphs that support the notion that it is this property that is enforcing the linear structure. This talk is an update of the talk that we presented at SE #22.

Key Words: linear graphs, asteroidal triple free graphs, perfect graphs, cocomparability graphs.

Monday, February 22, 1993
4:40 p.m.

37 **Upper Bounds for Covering Designs by Simulated Annealing**

Kari J. Nurmela* & Patric R. J. Östergård, Helsinki University of Technology

A $t-(v,m,k,\lambda)$ *covering design* is a pair $(X,\mathcal{A})$, where X is a set of v elements (called *points*) and $\mathcal{A}$ is a multiset of k-subsets of X (called *blocks*), such that every m-subset of X intersects at least λ members of $\mathcal{A}$ in at least t points. It is required that $v \geq k \geq t$ and $v \geq m \geq t$. Such a covering design gives an upper bound on $C_\lambda(v,m,k,t)$: the number of blocks in a minimal $t-(v,m,k,\lambda)$ covering design. In this work it is shown how simulated annealing can be used in searches for general covering designs. The method is used to calculate upper bounds on $C_1(v,t,k,t)$ for $k \leq 9$.

Keywords: Combinatorial optimization, covering design, simulated annealing.

38 General Graph Factors

R. P. Anstee * and Yunsun Nam
Mathematics Dept., University of British Columbia,
Vancouver, B.C., V6T 1Z2, Canada

We consider the following degree consrained subgraph problem solved by Cornuéjols. Let $G=(V,E)$ be a (multi)graph and for each $v \in V$ we have a set B_v. We look for a subgraph H of G with $d_H(v) \in B_v$. The problem is NP-complete if we allow gaps in B_v of size 2 or more. Cornuéjols solved the case for gaps of size 1. We extend his results by providing a direct algorithm that uses augmenting paths (which need not be alternating!) and is strongly polynomial.

39 **Theoretical and Experimental Comparison of Four Binary Tree Generation Algorithms**

P. Auger and T.R. Walsh*, University of Quebec in Montreal,

Four algorithms for generating codes for binary trees are compared for efficiency using both theoretical average-case analysis and measured CPU time. These are the lexicographic generation of parenthesis systems, of the positions of the left parentheses in these systems, of the Ruskey-Hu level-sequences, and the Van Baronaigien-Ruskey generation of binary trees in A-order. Some minor errors in the latter two algorithms are noted and corrected.

Key Words: Binary tree generation, average-case time complexity,

40 **SOME PROGRESS ON THE LOVASZ CONJECTURE**

C. Cary Timar, Vanderbilt University.

Dirac proved that, given k vertices in a k-connected graph, there is a cycle passing through all of them. Lovasz conjectured that the same is true for k independent edges, unless they form an odd edge-cut. Haggkvist and Thomassen proved this for up to k-1 independent edges. I show that given k independent edges, at least k-1 of them lie on an admissible cycle.

Keywords: long cycles, admissible cycles

Monday, February 22, 1993
5:00 p.m.

41

On ternary designs with a specified number of blocks with repeated elements.

Thomas Kunkle and Dinesh G. Sarvate* (College of Charleston)

In the Combinatorics and Graph theory special session of the Dayton meeting of the American Mathematical Society, Oct. 1992, Professor Elizabeth Billington gave a talk on Ternary designs. In the discussions which followed the talk Professor Preece suggested the study of designs with a specified number of blocks in which treatments are repeated. We refer such designs as Balanced Part Ternary Design. A balanced part ternary design is an arrangement of V distinct objects into $B=b_1+b_2$ blocks such that each of the b_1 blocks contains exactly K distinct objects, each of the b_2 blocks contains K not-all-distinct objects, each object occurs exactly R times, and every pair of distinct objects occurs exactly Λ times.
We will present some elementary results and constructions for such designs. It is easy to see that the necessary conditions are sufficient for the existence of all balanced part ternary designs with block size three.

42

Enumerating the one-factorizations of K12

Jeffrey H. Dinitz, U. Vermont, *David K. Garnick, Bowdoin College

We will describe progress on the enumeration of all nonisomorphic one-factorizations of K_{12}. It is known that there is a unique one-factorization for each of K_2, K_4, and K_6, there are 6 one-factorizations of K_8 and 396 of K_{10}. We have found 14 million one-factorizations of K_12 so far. The computation is currently running on 12 workstations, and we anticipate completing (or nearly completing) the enumeration by the conference. We will describe the algorithm we are using, and techniques we have used to verify partial results.
Keywords: cmplete graphs, enumeration, one-factorizations

43

Representing Tree Structures as Binary Strings

Paul W. Oman, Karen Van Houten, James A. Foster

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A binary string is *self delimiting* when it contains exactly one more "one" than it has "zeros", and every prefix has at least as many "ones" as "zeros". Such strings can be recognized by scanning them from left to right, counting zeros and ones. Hence, they to not need to be explicitly delimited, which makes them an inherently compact data structure.

We demonstrate a bijective mapping between self delimiting binary strings and tree structures. This makes it possible to represent tree structures with an arbitrary number of nodes with these strings.

Based on these proofs we create an algorithm for generating representative binary strings directly from commonly used graph data structures. This technique effectively compresses the structure of a tree with n nodes into $2n-1$ bits plus the original information contained within the tree (which can be compressed using existing compression algorithms). The compressed structure and information can then be used for data transmission, transformation, and archival purposes.

We have shown elsewhere that this particular representation of tree structures is useful for arbitrary data compression, randomly generating circuits, verifying join tree properties for relational database query optimization, and for analyzing random walks.

44

CYCLES OF LENGTH 2 MOD k

G.Chen, No Dak St Univ; N.Dean, Bellcore; W.Shreve*, No Dak St Univ.

Graphs with a cycle of length 2 mod 4 are considered. Theorem: If G is a 2-connected graph with at least 6 vertices and minimum degree at least 4, then G contains a cycle of length 2 modulo 4. Theorem: If G is a graph with minimum degree at least (k+5)/2 then G contains a cycle of length 2 mod k or a cycle of length 6 mod k.
Key words: cycle, path, minimum degree, modulo k.

Monday, February 22, 1993
5:20 p.m.

45

SOME FURTHER RESULTS ON SEQUENTIAL COVERING DESIGNS

John P. McSorley, Dept. Mathematics, Southern Illinois University, Carbondale. IL 62901.

A Sequential Covering Design SD(v,k,t) is a sequence of length t based on a v-element set, such that every pair of elements of the set occur in a block of size k somewhere in the sequence. It is desirable to minimize t for a fixed v and k. Let g(v,k) denote this minimum value of t. We present some exact values of g(v,k); some upper and lower bounds; a greedy construction; and other miscellaneous results and conjectures.

46 **MULTIDERIVATIVE METHODS FOR SECOND ORDER FREDHOLM INTEGRO-DIFFERENTIAL EQUATIONS**

by

L. E. Garey+ and C. J. Gladwin

University of New Brunswick

Abstract: Numerical methods for second order ordinary differential equations are incorporated into a two-part method for the direct solution of Fredholm type integro-differential equations. Henrici's concept of conditional stability is considered for such equations and multi-derivative multi-step methods are used to construct unconditionally stable methods of order 2 for a certain family of test equations. The methods are illustrated with numerical examples.

47 **Tree-Codes in Lossless Image Compression**

S. S. Magliveras*, UN Nasir D. Memon, ASU Khalid Sayood, UN

Maximal rates of compression for images correspond to minimum entropy scans. Most image compression schemes, however, employ a particular fixed scan over all images. We can significantly improve the compression rate for a particular image by selecting a minimum entropy scan. In earlier work we showed that the problem of finding a minimum entropy scan is NP-hard. We also developed efficient heuristics for computing optimal or near-optimal scans. The total *cost* in transmitting a compressed image using an optimal scan is the cost of information content of the compressed image under this scan, plus the cost of transmitting information to recover the scan at the receiving end. Since the total number of scanning trees $|T| = t(n)$ for an $n \times n$ grid is extremely large, ($\limsup t(n) = 4^{n^2}$), any gain in the compression rate is usually lost, or even turned into an overall loss, by the amount of information needed to communicate the scan. In this paper we take a coding-theoretic approach. We consider a code C of spanning trees of covering radius ρ in the space of all spanning trees T. Since the entropy function is continuous in the metric of the grid-graph, replacing a minimum entropy tree-scan S by the codeword tree T corresponding to S results in a small increase in entropy. On the other hand the cost of transmission of the scan is now reduced to $log_2(|C|)$. We investigate possible codes, and attempt to minimize the overall cost. Many theoretical questions remain unanswered. Implementation results show significant improvements over using a fixed scan.

48 **CYCLES IN BIPARTITE GRAPHS:Some More results**

K. Jay Bagga, Department of Computer Science, Ball State University
Badri N. Varma*, Department of Mathematics, UWC - Fox Valley

Several sufficient conditions in erms of degrees or number of edges or both for a graph to have hamiltonian like properties have been improved in the case of balanced bipartite graphs. In this paper some such results are further extended to the case of general bipartite graphs.

Tuesday, February 23, 1993
8:40 a.m.

49 Integral Matrices with Given Row and Column Sum Vectors

Yunsun Nam
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Let m and n be positive integers, and let $R = (r_1, \cdots, r_m)$ and $S = (s_1, \cdots, s_n)$ be nonnegative integral vectors with $r_1 + \cdots + r_m = s_1 + \cdots + s_n$. Let $Q = (q_{ij})$ be an $m \times n$ nonnegative integral matrix. Denote by $\mathfrak{A}^Q(R,S)$ the class of all $m \times n$ nonnegative integral matrices $A = (a_{ij})$ with row sum vector R and column sum vector S such that $a_{ij} \leq q_{ij}$ for all i and j. We study a condition for the existence of a matrix in $\mathfrak{A}^Q(R,S)$. The well known existence theorem follows from the maxflow-mincut theorem. It contains an exponential number of inequalities. By generalizing Gale-Ryser theorem, we get some conditions under which this exponential number of inequalities can be reduced to a polynomial number of inequalities. We build a kind of hierarchy: under weaker and weaker conditions, more and more inequalities (still a polynomial in n) are involved in a necessary and sufficient condition for the existence of a matrix in $\mathfrak{A}^Q(R,S)$.

50 ON ORTHOGONAL DOUBLE COVERINGS OF THE Kn

Hans-Dietrich O.F. Gronau, University Rostock

An orthogonal double covering of the complete graph Kn is a collection of n spanning subgraphs G(1), G{2}, ..., G{n} of the K{n} such that
(i) every edge of the Kn belongs to exactly 2 of the G{i}'s
and
(ii) every pair of G{i}'s intersect in exactly one edge. We survey results on the existence of orthogonal double coverings, where the G{i}'s are members of a certain class of graphs.
It is proven that an orthogonal double covering exists for all n≥2 with at most very few small exeptional cases, if the G{i}'s onsist of short cycles (i.e. cycles of length 3,4,5), which also completes a conjecture of Chung and West.

It is joint work with R.C.Mullin.

51 CONDITIONAL CUTS IN HYPERCUBES

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The notion of connectivity generalization was introduced by F. Harary. For an n-dimensional hypercube Q_n and a given integer p $(1 < p \leq n)$, let F be a vertex cut of Q_n whose cardinality $|F|$ is minimum such that F contains at most p neighbors of each vertex in $Q_n - F$. In previous work [Oh and Choi], the minimum $|F|$ (i.e., p-conditional connectivity) was computed to provide a generalized measure of fault-tolerance in n-cube networks. In this paper, we compute for each p the number $N(n,\ p)$ of all such F's: $N(n,\ 2) = n(n-1)$; $N(n,\ 3) = 2^3 \cdot \binom{n}{3} \cdot (n-2)$; $N(n,\ p) = 2^p \cdot \binom{n}{p}$ for $p \geq 4$.

52 GRAPHS THAT ARE EDGE-DISJOINT UNIONS OF SPANNING TREES, AND K-POINT SEPARATING FAMILIES OF FUNCTIONS ON R^n

W. Edwin Clark*, Gregory L. McColm, Boris Shekhtman, Department of Mathematics, University of South Florida, email: eclark@math.usf.edu

We say that a graph is an *n-tree* if it is a union of n edge-disjoint spanning trees. We show that every graph with k ≥ 2 vertices and n(k-1) edges has a non-trivial subgraph that is an n-tree. We also establish a determinantal criterion for a graph with k vertices and n(k-1) edges to be an n-tree. We apply these results to prove the following: Say a family $\mathcal{F}$ of functions from R^n to R is *k-point separating* if for every k-subset S of R^n there is a function $f \in \mathcal{F}$ such that f is one-to-one on S. We show that if the functions in $\mathcal{F}$ are required to be linear (or smooth) then a minimum k-point separating family $\mathcal{F}$ has cardinality n(k-1). In the linear case we extend this result to a larger class of fields -- including all infinite fields as well as some finite fields (depending on k and n). We also obtain some partial results for continuous functions on R^n.

KEYWORDS: graph, spanning tree, separation, determinant, linear functional, smooth function, continuous function

Tuesday, February 23, 1993
9:00 a.m.

53

Self-Dual Embeddings of Cubes

Mark S. Anderson, Rollins College

Self-dual embeddings of even-dimensional cubes are described in terms of voltage graphs. In particular, the 6-cube is viewed as a covering of K_4. The triangles in K_4 lift to the hexagonal faces of the self-dual embedding of the 6-cube in an orientable surface of genus 33.

54

On Hamiltonian Cycles in CnXsCm

Joseph B. Klerlein*, Western Carolina University
Edward C. Carr, West Virginia University

In 1978 Trotter and Erdos gave necessary and sufficient conditions for the direct product, CnXCm, of two directed cycles to be hamiltonian. In this paper we give some sufficient and some necessary conditions for hamiltonian cycles in CnXsCm, the directed graph obtained from CnXCm by joining vertices which are s units apart in a single m-cycle.

55

Iterative Networks and their Properties

Rama K. Govindaraju*, M. S. Krishnamoorthy, Narsingh Deo

We introduce three new families of undirected graphs as a basis for interconnection networks. The three families are based on *Fibonacci graphs, relatively prime graphs,* and *natural number graphs.* Fibonacci graphs are based on Fibonacci numbers , relatively prime graphs are based on relatively prime numbers, and natural number graphs are based on primes. These graphs are iteratively constructed in the sense that a graph of $n+1$ nodes is a subgraph of a graph of n nodes in the same family. These graphs can be "succinctly represented", that is, we do not need the adjacency matrices explicitly to determine if two arbitrary nodes i and j are adjacent. We study several properties of these graphs like connectivity, diameter, fault-diameter, max-degree, etc. We also design algorithms for packet routing on these network graphs, and study their embedding capabilities. These graphs can also be used as inputs for testing and experimenting with graph algorithms.

56

REP-TILING EUCLIDEAN SPACE

Andrew Vince, University of Florida

A *rep-tiling* $\mathcal{T}$ is a self replicating, lattice tiling of R^n. *Lattice tiling* means a tiling by translates of a single compact tile by the points of a lattice, and *self-replicating* means that there is a non-singular linear map $\phi : R^n \to R^n$ such that, for each $T \in \mathcal{T}$, the image $\phi(T)$ is, in turn, tiled by $\mathcal{T}$. This topic has recently come under investigation, not only because of its recreational appeal, but because of its application to the theory of wavelets and to computer addressing. The talk concerns the construction of rep-tilings. Our construction is based on a close connection between rep-tilings and radix representation.

Keywords: tiling, tesselation, self-replicating, lattice, radix.

Tuesday, February 23, 1993
10:50 a.m.

57

Uniformly Resolvable Designs $\lambda > 1$

Peter Danziger, University of Toronto

A k-URD(v, g, λ, r) is a resolvable design on v points with block sizes g and k. Each parallel class contains only one blocksize, and there are r parallel classes with blocks of size g, this implies there are $\frac{\lambda(v-1)-r(g-1)}{k-1}$ parallel classes of size k. In this talk we consider the case of arbitrary λ and show that for $k = 2$ and 3, in most cases, these designs exist for sufficiently large v. Specifically if

$$\epsilon_{k,g} = \begin{cases} 1 & \text{if } g \equiv 0 \bmod k \\ k & \text{otherwise} \end{cases}$$

and $u = \frac{v}{g\epsilon_{k,g}}$ we show that if u is composite and p is the smallest factor of u then there exists a k-URD(v, g, r) for all $r \leq \frac{\lambda u}{p}$ except when $k = 3$, g is even and λ is odd.

58

Barred Permutations

*Paul Klingsberg, Saint Joseph's University
Cynthia Schmalzried, Swarthmore College

We investigate several enumeration questions that are associated with the insertion of indistinguishable bars into permutations of two sorts. The first sort we call *Stirling permutations* because they generalize Gessel and Stanley's Stirling permutations (into which they also insert bars); in our case just as in theirs, the barred permutations provide a combinatorial interpretation of the coefficients that occur in a certain family of generating functions. These coefficients also are the values of the order polynomials for a certain family of posets, so that there is a close connection between our generating functions and the order polynomial generating function. We exhibit the family and draw the connection. The second sort of permutation we use is this: for an arbitrary finite poset $(P, <_P)$, we insert bars into permutations that are linear extensions of $<_P$. Some of our Stirling permutation results generalize to this context; it also turns out that this sort of barred permutation affords a concrete representation of Stanley's proof of the closed form of the order polynomial generating function $\sum_{t \geq 0} \Omega(P, t)x^t$.

59

Gray Codes for Listing the Acyclic Orientations of Some Graphs

Matthew Squire* and Carla D. Savage
Department of Computer Science
North Carolina State University

Douglas B. West
Department of Mathematics
University of Illinois at Urbana-Champaign

Given a graph G, the *acyclic orientation graph of* G, denoted $AO(G)$, is defined such that the vertices of $AO(G)$ are the acyclic orientations of G, and two acyclic orientations are joined by an edge if they differ by the reversal of a single edge. A Hamilton cycle in $AO(G)$ gives a Gray code listing of the acyclic orientations of a graph. We prove that for certain graphs, G, $AO(G)$ is Hamiltonian, and give explicit constructions of Hamilton cycles in $AO(G)$. We also give examples of graphs whose acyclic orientation graph is *not* Hamiltonian.

60

Superpermutations and Complete Injective Superstrings

*Dan Ashlock, Jenett Tillotson, Iowa State

A superstring S of a set A of strings is a string that contains every element of A as a substring. A well known examples are De Bruijn sequences which are superstings of the set of all strings of a fixed length over a given alphabet.
A superpermutation is a superstring of the set of all permutations
A k-complete injective superstring is a superstring of the set of strings of length k<n over an alphabet of size n that have no repeated characters. We present a very short superpermutation that we conjecture to me minimal and, up to normailization, unique together with some results on complete injective superstrings.

Tuesday, February 23, 1993
11:10 a.m.

61

Large sets of quadruple systems from PSL(2,q)

Sidney W. Graham and Donald L. Kreher* Michigan Technological University

A quadruple system of order v and index λ is a collection $\mathcal{B}$ of 4-element subsets of $\mathcal{X}$ (called *quadruples*) such that every 3-element subset of $\mathcal{X}$ occurs in exactly λ quadruples. A partition of all the quadruples into disjoint quadruple systems of order v and index λ is said to be a large set of quadruple systems. We examine when the orbits of quadruples under the action of PSL(2,q) on the projective line can be used to form a large set of quadruple systems of order $q+1$ and index λ.

62

Degree Sums, k-Factors and Hamilton Cycles in Graphs

R. J. Faudree* — Memphis State University

J. van den Heuvel — University of Twente

We prove (a generalization of) the following conjecture of R. Häggkvist: Let G be a 2-connected graph on n vertices where every pair of nonadjacent vertices has degree sum at least $n-k$ and assume that G has a k-factor; then G is hamiltonian. This result is a common generalization of well-known theorems of Ore and Jackson, respectively.

63

Long Snakes In Powers Of Complete Graphs

Jerzy Wojciechowski, West Virginia University

Let K_n^d be the product of d copies of the complete graphK_n, $n \geq 2$, $d \geq 1$. In particular, for $n = 2$ we have $K_2^d = Q^d$, where Q^d is the d-dimensional hypercube. A snake in a graph G is an induced cycle in G, *i.e.* a cycle without chords. Let $S(K_n^d)$ be the length of the longest snake in K_n^d. Abbott and Katchalski proved that for any even integer $n \geq 2$, there exists a constant $c_n > 0$ such that $S(K_n^d) \geq c_n n^d$, for every $d \geq 2$, and asked if the same result is true for $n = 3$. We present a generalization of the above result to arbitrary integer$n \geq 2$.

During the XXIII Southeastern International Conference, Boca Raton 1992, Erdős posed the problem of deciding whether there is a number k such that for every $d \geq 2$ the vertices of Q^d can be covered using at most k snakes, and if it can be done in such a way that the snakes are pairwise vertex-disjoint. We present a result showing that the answer to both of the above questions is positive with $k = 16$.

Keywords: snakes-in-the-box, induced cycles, complete graphs,

64

Matchings, Cutsets, and Chain Partitions in Ranked Posets

Jerrold R. Griggs*, University of South Carolina

We say a ranked poset has the (strong) matching property M_1 (resp., M) if for every pair of consecutive ranks (resp., every pair of ranks) there is an order-preserving matching from the smaller to the larger rank. We say a graded poset has the cutset property C_1 if the minimum rank-set is a minimum cutset. These and related properties are compared to the Sperner property and to conditions on chain partitions, and the possible implications are determined. A series of poset examples is provided that realize every possible combination of properties.

Keywords: posets, Sperner theory, cutsets, chain partitions

Tuesday, February 23, 1993
11:30 a.m.

65

RESOLVABLE COVERINGS

E. R. Lamken
Princeton University, Princeton, NJ

W. H. Mills*
Center for Communications Research, Princeton, NJ
and DIMACS

A minimum covering of pairs by triples in a $6n$-element set contains $6n^2$ triples. Can such a covering be resolvable? A. Assaf, E. Mendelsohn, and D. R. Stinson showed that this is not possible for $n < 3$, and that for $n \geq 3$ such a resolvable covering exists if $n \notin \{6, 7, 8, 10, 11, 13, 14, 17, 22\}$. In the present paper we show that such resolvable coverings exist for these nine values of n.

66

Cycle Decomposition of the Line Graph of the Complete Graph

B.A. Cox*, C.A. Rodger Auburn University

We consider an m-cycle decomposition of L(Kn), the line graph of the complete graph on n points. Some general results are presented, one corollary of which is that the existence problem for eight-cycle systems of L(Kn) is completely settled.

67

A Gray Code for Necklaces of Fixed Density

Carla Savage and *Terry MinYih Wang**

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Raleigh, North Carolina 27695-8206

A necklace of n beads in two colors is an equivalence class of binary n-tuples under rotation. The density of a binary string is defined to be the number of 1's in the string. In this talk, we present a Gray code for n-bead necklaces of density d, so that (i) each necklace is listed once and only once by its representative in the equivalence class, and (ii) successive representatives, including the last and the first, differ only by the transposition of a 0 and a 1.

Key words: Gray code, necklace.

68

RECOGNITION OF DILWORTH POSETS

*Wiktor Piotrowski - North Dakota State University
Maciej M. Syslo - Wroclaw University

A finite partially ordered set P is a k-Dilworth poset if there exists a k-saturated linear extension of P. The problem of recognition of -Dilworth posets will be discussed. We survey known results for k=1 and generalize them for all k>0, proving that the problem is NP-complete and constructing polynomial algorithms in some classes of posets.

Key words: poset, linear extension, jump number

Tuesday, February 23, 1993
11:50 a.m.

69 Designs, Codes and Rigidity Theorems

J. D. Key, Clemson University

A p-ary code $C = C_p(\mathcal{D})$ is associated with a design $\mathcal{D}$ by taking the subspace spanned by the incidence vectors of the blocks of $\mathcal{D}$ in the vector space of functions from the point set of $\mathcal{D}$ to the prime field $\mathbf{F}_p$. If the prime p divides the order of the design then this code, along with $C^\perp$ (under the standard inner product), $C + C^\perp$, and $C \cap C^\perp$, can often be of assistance in characterizing the design amongst those of the same parameters. In particular, when the design is a natural one defined by a finite geometry, then the code of the design is usually the "best" in certain ways, and the code of a design with the same parameters might be classified according to its relation with this code, and in this way the "rigidity theorems" occur. Some cases where this has been fruitful in the past, for example for affine planes, Hadamard designs, biplanes, unitals, will be discussed and some more recent results concerning Steiner triple and quadruple systems (joint work of the author with F. E. Sullivan) will be outlined. In most of these examples the Reed-Muller and generalized Reed-Muller codes are those that arise in the rigidity theorems.

KEY WORDS: codes, designs

71 Gray-Coded Optimal Cube-Connected Cube Networks

Yinqiu Luo* and Jie Wu, Florida Atlantic University

The Cube-Connected Cubes Network (CCCubes) is a variant of the hypercube architecture, proposed for the divide-and-conquer paradigm and in particular for distributed environments with heavy localized communication. A CCCubes is a hypercube with each node represented as a cube, called an inner cube. This paper studies a specific connection of these inner cubes based on the Gray-code. It is shown that CCCubes under this connection can be used to solve a class of DESCEND/ASCEND parallel algorithms effectively. Results are compared with the compatible Cube-Connected Cycles.

Key Words: Cube-Connected Cubes, Divide-and-Conquer Algorithms, Gray Codes

72 A CHARACTERIZATION OF UPPER IDEALS OF DIGRAPHS AND THEIR PRESERVERS

LeRoy B. Beasley*, Utah State University, Logan, UT
Norman J. Pullman, Queen's University,

An upper ideal of digraphs has the property that the union of any element of the upper ideal and any other digraph is in the upper ideal. We classify upper ideals by the types of graphs with exactly 2 links which are subgraphs of the minimal elements of the upper ideal. We then use this classification to investigate the linear maps on the set of digraphs which map an upper ideal to itself and the complement to the complement.

Tuesday, February 23, 1993
12:10 p.m.

73 **Persistent Pairs in SCCDs**

*NC Phillips and WD Wallis, SIUC

A Single Change Covering Design (SCCD) is a design on v object with block size k in which the blocks are ordered in each block exactly one object is changed to form the next block. A persistent pair is a pair of objects persisting through a longest possible sequence of adjacent blocks. We discuss elementary properties of persistent pairs and show how overlapping persistent pairs can aid the construction of tight "large" designs.

74

A NOTE ON HAMILTON CYCLES IN CLAW-FREE GRAPHS

M.D. Plummer, Vanderbilt University,

Several interesting--and as yet unsettled--conjectures concerning Hamilton cycles in claw-free graphs have been formulated. One of the most well-known is due to C. Thomassen and states that every 4-connected line-graph is Hamiltonian. Several other conjectures equivalent to that of Thomassen have been set forth and in this paper we formulate yet another and show it to be equivalent to the rest. Our conjecture arose from the study of matching extension in claw-free graphs.

keywords: claw-free, Hamilton cycle, line graph

75 **Hypercube Embedding in Balanced Hypercubes**

Ke Huang* and Jie Wu, Florida Atlantic University

Balanced hypercube is a variant of standard hypercube structure. It has many desirable properties as that in a standard hypercube. In addition, the balanced hypercube is more fault-tolerant. In a balanced hypercube, every node has a matching node that share the common adjacent node set. This paper studies the embedding of standard hypercubes in compatible balanced hypercubes. The implementation of a basic parallel algorithm paradigm is also discussed.

Key Words: Embedding, Hypercubes, Load Balanced Graphs

76

Maximal Chains in Subposets of the Boolean Poset

Bela Bajnok* (University of Houston-Downtown) and
Shahriar Shahriari (Pomona College)

Let P(n) denote the poset of all subsets of an n-set ordered by inclusion. A maximal Chain in P(n) is a chain of length n. Suppose we remove certain nodes from P(n). We want to know if the remaining poset still contains a maximal chain. We will present the following two results:

1.) After removing n-3 (arbitrary) nodes from each level between level 1 and n-1, the remaining poset still contains a maximal chain.
2.) After removing n-2 (arbitrary) chains between level 1 and n-1, the remaining poset still contains a maximal chain.

Key Words: poset, maximal chain.

Tuesday, February 23, 1993
3:20 p.m.

77

On the Cycle Structure of Star Graphs

Ke Qiu* Henk Meijer Selim G. Akl
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The star graph is an attractive alternative to the popular hypercube for interconnecting processors on a parallel computer [1]. A scheme was recently proposed which decomposes an *n*-star into vertex disjoint cycles [2]. In this paper, we further study the cycle structure of the star graph. This includes characterizing the cycles obtained from the decomposition and investigating the relationships among these cycles. In particular, we consider the graph obtained by collapsing every cycle into a single node. We study this graph in detail and show several properties of this graph. For example, we show that this graph is Hamiltonian. The results obtained are useful in that we are able to use the cycle structures of the star graph to develop several embeddings that embed meshes and tori of certain dimensions into a star graph with various dilations. It remains to be seen how the decomposed cycles can be used in other applications, in particular, in designing parallel algorithms for the star graph.

Keywords: Graph theory, star graph, cycle decomposition, Hamiltonian cycle, mesh, torus, embedding, parallel computation.

1. S.B. Akers and B. Krishnamurthy, "A Group Theoretic Model for Symmetric Interconnection Networks," *IEEE Transaction on Computers*, Vol. c-38, No. 4, April 1989, pp. 555-566.

2. K. Qiu, H. Meijer, and S.G. Akl, "Decomposing a Star Graph into Disjoint Cycles", *Information Processing Letters*, Vol. 39, No. 3, 1991, pp. 125-129.

78

On Super Line Graphs of Hypercubes

K. Jay Bagga* and Maria R. Vasquez
Department of Computer Science
Ball State University

Recently Bagga et al defined a new generalization of the concept of line graph. The super line graph of index r of a graph G has the sets of r edges in G as its vertices, and two of these are joined if an edge in one is adjacent to an edge in the other. In this paper we study the super line graphs of the hypercubes, and in particular give several results on super line graphs of index 2.

79

Strong Asymmetric Digraphs With Prescribed Median and Periphery

Steven J. Winters, Western Michigan University

For verticies u and v in a strong digraph D, the *directed distance* d(u,v) from u to v is the length of a shortest u-v path in D. The *distance* d(v) of D is the sum of the directed distances from v to the vertices of D. The *median* M(D) of D is the subdigraph of D induced by those vertices having minimum distance. The *eccentricity* e(v) of v in D is the directed distance from v to a vertex furthest from v in D. The *periphery* P(D) of D is the subdigraph induced by those vertices of D having maximum eccentricity. The median is one way of defining the "middle" of a digraph, while the periphery is one way of defining the "exterior" of a digraph. For any two digraphs D_1 and D_2, we show that there exists a strong asymmetric digraph H such that $P(H) \cong D_1$ and $M(H) \cong D_2$. In addition, we give the expected result that the distance from the median to the periphery and from the periphery to the median of a digraph can be arbitrarily large, and the possibly unexpected result that the median and periphery of a digraph can overlap in any common induced subdigraph.

80

Domination Numbers of Some Grid-like Graphs

Eleanor O'M. Hare* (Dept of Computer Science)
William R. Hare (Dept of Mathematical Sciences)
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Several computer generated values for the domination number of Cm x Pn are presented, leading to corresponding conjectures These include values of 2 <= m <= 12 and all n. Similar empirical results for the domination number of Cm x Cn are given, and conjectures made for values 2 <= m <= 7 and all n. The conjectures for m = 2, 3, and 4 are proved for the graphs Cm x Pn and for m = 3, 4, and 5 in the case of graphs Cm x Cn.

Key Words: domination number, grid graphs

Tuesday, February 23, 1993
3:40 p.m.

81 **Load Balancing and Fault Tolerance in Hypercube Systems**

Hemant More* and Jie Wu, Florida Atlantic University

On multiprocessing systems like hypercubes, the utilization would be improved if all the processors are working at all times. In reality, some processors are idle while some are heavily loaded. This paper attempts to balance the load on hypercubes when a new load arrives. Load balancing is also required when a node fails or a link fails in the system. The system would be robust when fault tolerating features are incorporated in it. An approach to integrate the solutions to the three aspects which could give rise to the failure or underutilization of the hypercube systems is presented. The analysis of the link and node failures is done in terms of the {\em preferred nodes} which would take over the operation of the failed node. Every node will have a unique preferred node. The preferred node, if overloaded after it takes over the load from failed node, could initiate load balancing. There would be a preferred node assigned for each node if at least one link with its neighbors is active. If all the links of a node with its neighbors are broken, it is assumed to go out of circulation even if it is still active.

Key Words: Fault tolerance, Hypercubes, Load balancing.

82

MINIMUM SIZE OF AN AGREEMENT SUBTREE

Wayne D. Goddard, University of Pennsylvania
Grzegorz Kubicki*, University of Louisville

We consider binary trees whose all endvertices (leaves) are labeled without repetition. The operation of pruning of a tree T is removing some leaves from T and suppressing all inner vertices of degree 2 which are formed by this deletion. Having two or more labeled binary trees, their largest common pruned tree is called an agreement subtree. For two binary trees, considering both rooted and unrooted versions, we discuss how many leaves must be present in their agreement subtree.

83 **The Existence of Asymmetric Digraphs having Periphery Appendage Number 3**

Songlin Tian

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The (directed) distance d(u,v) from a vertex u to a vertex v in a digraph D is the length of a shortest (directed) u-v path in D if such a paths exist; otherwise, $d(u,v) = \infty$. The eccentricity of a vertex v in D is the distance from v to a vertex furthest from v. The periphery P(D) of D is the subdigraph induced by those vertices of maximum eccentricity. For an asymmetric digraph D, the periphery appendage number PA(D) of D is the minimum number of vertices that must be added to D to produced a strong asymmetric digraph H whose periphery is isomorphic to D. It was known that $PA(D) \leq 3$ for every asymmetric digraph D. In this paper, a family of asymmetric digraphs having periphery appendage number 3 is presented.

Key words: asymmetric digraph, directed distance, eccentricity, periphery, periphery appendage number

84 **Vizing's Conjecture on Domination**

B. Hartnell*, Saint Mary's University, Nova Scotia
D. Rall, Furman University, South Carolina

A conjecture (1963) of Vizing states that the cardinality of every dominating set of the Cartesian product G x H is at least as large as the product of the domination numbers of the graphs G and H. If the vertices of a graph G can be partitioned into n subsets, each of which induces a complete subgraph of G, and the domination number of G equals n, call G decomposable. Barcalkin and German (1979) showed that if G is decomposable or is a spanning subgraph of a decomposable graph with the same domination number, then G satisfies Vizing's conjecture (for any H).

In this talk, we indicate how the decomposable graphs can be extended to give a generalization of their result.

Tuesday, February 23, 1993
4:00 p.m.

85

A Construction that Connects Cubic Cages

Dan Ashlock and *David Schweizer
Iowa State University and College of the Holy Cross

We present a construction that extends some examples of Tutte. It directly obtains the (3,k)-cage from the (3,k-1)-cage for k less than or equal to eight and also connects the cages in larger jumps, obtaining the Tutte-Coxeter graph directly from the Petersen graph. The construction is not general (to date no putative (3,9)-cages have been obtained) but is automatable in a fashion that appears to restrict substantially the search space for cages.

key words: (3,k)-cage, graphical construction

86

THE TRANSITION DIGRAPH OF THE RANDOM f-GRAPH PROCESS

Krystyna T. Balińska, Technical University of Poznań, Poznań, POLAND
and
Louis V. Quintas*, Pace University, New York, U.S.A.

Starting with n labeled vertices and no edges, sequentially introduce edges so as to obtain a sequence of graphs each having no vertex of degree greater than f. The latter are called f-graphs. At each step the edge to be added is selected with equal probability from among those edges whose addition would not violate the f-degree restriction. A terminal graph of this procedure is called a sequentially generated random edge maximal f-graph and the procedure the Random f-Graph Process.

Let $D(n,f)$ be the digraph whose nodes correspond to the unlabeled f-graphs of order n and such that there is an arc from the node corresponding to graph H to the node corresponding to the graph K if and only if K is obtainable from H by the addition of a single edge. The digraph $D(n,f)$ is called the transition digraph of the R f-G P of order n.

A variety of properties and problems concerning the order, size and node degrees of the digraph $D(n,f)$ are presented.

87

ON THE PATH-PERFECTNESS OF THE COMPLETE BIPARTITE GRAPH

Kiran Chilakamarri, Central State University; Peter Hamburger, Raymond E. Pippert*, and W. Douglas Weakley, Indiana University-Purdue University Fort Wayne2

A graph G has been defined by Fink and Straight to be path-perfect if it can be partitioned into edge-disjoint paths of each length from 1 to n. Clearly, the size of such a graph must be n(n+1)/2. When G is the complete bipartite graph K(s,t), with s at least t, the fact that the longest path has length 2t imposes the additional constraint that 2t must be at least n. Whether or not these necessary conditions are are also sufficient to ensure that K(s,t) be path-perfect has been an open question for more than two decades. We discuss recent progress we have made on this problem.

Key words: decomposition, packing, path-perfect

88

Perfect Dominating Sets in Cube-Connected Cycles

Douglas Van Wieren*, Marilynn Livingston, Quentin F. Stout
University of Michigan, Ann Arbor

Cube-connected cycles are a family of cubic graphs with relatively small diameters and regular structure, making them attractive models for parallel architecture design. The existence of perfect dominating sets for any structural model of parallel computation is both useful for the construction of efficient algorithms for that structure and indicative of practical design constraints. This paper gives a simple algorithmic method for constructing perfect dominating sets on cube-connected cycles where they exist, and proves nonexistence for all other cases. Specifically, standard perfect dominating sets (distance equal to 1) are shown to exist for cube-connected cycles of order k, k not equal to 5. Moreover, the existence of perfect dominating sets for all distances greater than 1 is disproved (with the trivial exception --- the distance equaling or exceeding the diameter of the graph).

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4:20 p.m.

89

On some extremal properties of bipartite graphs of girth eight

Gene Fiorini University of Delaware

Let $G = G(n,n)$ be a simple, bipartite graph on $2n$ vertices with partitions of equal cardinality n and girth at least eight. Let $e(G)$ be the number of edges in G and $c_{2k}(G)$ the number of cycles of length $2k$ in G. We prove that for $n \geq 1$, $e(G) \leq (\lfloor n^{\frac{1}{3}} \rfloor + 1)n$. As a consequence of this we also show that for $n = (q+1)(q^2+1)$, q a positive integer, $c_8(G) \leq \binom{q+1}{2}^2 \binom{q^2+1}{2}$ with equality in both inequalities if and only if G is the point-line incidence graph of a generalized quadrangle of order q, if such exists.

90

BIG CYCLES IN RANDOM EDGE MAXIMAL 2-GRAPHS

Krystyna T. Balińska*, Technical University of Poznań, Poznań, POLAND
Louis V. Quintas, Pace University, New York, U.S.A.

A graph having no vertex of degree greater than 2 is called a 2-graph. A 2-graph to which no edge can be added without violating the 2-degree restriction is called an edge maximal 2-graph (EM 2-graph). Various cycle distribution probabilities for uniformly distributed labeled EM 2-graphs are obtained and compared with those for sequentially generated labeled EM 2-graphs, the latter being random graphs generated by adding one edge at a time to an initial set of labeled vertices with no edges.

The focus is on the probability that such a graph contains a cycle of length n, $n-1$, $n-2$ or that it contains a cycle of length greater than $n/2$. The comparisons are made for both small and large order graphs.

Key words: edge maximal 2-graphs; cycles

91

Independent Path Systems in Powers of Graphs

Ralph J. Faudree, Memphis State University
Ronald J. Gould, Emory University
*Terri E. Lindquester, Rhodes College
Richard H. Schelp, Memphis State University

A graph G of order at least $2k$ is said to be k-linked if for any ordered set $\{v_1, v_2, \cdots, v_k, w_1, w_2, \cdots, w_k\}$ of $2k$ vertices of $V(G)$, G has k disjoint paths $P_1, P_2, \cdots, P_k$ such that P_i connects v_i and w_i for $i = 1, \cdots, k$. We explore this property in powers of connected graphs and obtain several results. In particular, we show that if G is connected with $|V(G)| \geq 2k$, then G^{2k-1} is k-linked. For graphs G with $\kappa(G) = m \geq 3$, we show that if $tm \geq 4k$, then G^t is k-linked. Moreover, we obtain results of this nature for powers of cycles.

92

Twisted Perfect Dominating Subgraphs of Hypercubes

Italo J. Dejter*, Department of Mathematics and Computer Sciences, University of Puerto Rico, Rio Piedras PR 00931

Paul M. Weichsel*, Department of Mathematics, University of Illinois, Urbana 61801

A number of examples of perfect dominating sets of n-cubes are produced whose induced components are subcubes of the same dimension but occurring parallel to different hyperplanes, by means of the structure of n-cubes in relation to Steiner triples and Hamming codes. One of these examples, given in the 8-cube, has as components lines surprisingly in all coordinate directions. This suggest the conjecture that the total number of different hyperplanes to which the r-cube components of a perfect dominating set are parallel is either 1 or 4 or 8, and that, with the exclusion of the mentioned example in Q_8 and its extensions to higher dimensional cubes, a lower bound for the number of coordinate directions in which these components occur is (n/2)+r-1.

Tuesday, February 23, 1993
4:40 p.m.

93

High Girth and Extendability

Pavol Gvozdjak*, Simon Fraser University, Burnaby, BC
Jaroslav Nesetril, Charles University, Czech Republic

A graph G on at least 2k+2 vertices is said to be k-extendable if it contains a perfect matching and every matching of size k can be extended to a perfect matching.

Most examples of graphs having high extendability that are found in the literature have small girth (e.g. k-extendable graphs include K_{k+1,k+1} and K_{2(k+1)}). Indeed, it is a nontrivial problem to construct graphs having both large extendability and large girth.

Here we present a construction of graphs G_{k,n} which are k-extendable and have girth at least n. Graphs G_{k,n} may be viewed as certain generalizations of the hypercube graphs Q_m. The constructed graphs are, in addition, regular. They are interesting in a broader context as there are only a few known constructions of graphs that have a large girth and a large edge density.

94

Polynomial Algorithms for Hamiltonian Cycle In Cocomparability Graphs

J.S.Deogun University of Nebraska, Lincoln and
G.Steiner* McMaster University, Hamilton, Ont., Canada

Finding a Hamiltonian Cycle in a graph is one of the classical NP-complete problems. In this paper we present a polynomial time solution for this problem in cocomparability graphs. Our approach is based on exploiting the close relationship between this problem and the bump number problem on a partially ordered set.

95

Transformations on Channel Graphs for the k-path 2-terminal reliability Problem

Jacqueline Nadon and Ehab Elmallah* University of Alberta

A channel graph is a directed multistage acyclic graph with unique source and sink vertices, in which any two vertices in the same stage are nonadjacent. The k-path 2-terminal reliability problem is to compute the probability that there exist k-vertex disjoint paths between the source and the sink vertices, given that edges fail independently of each other.

The above problem generalizes the conventional 2-terminal reliability problem. Recently, M. Kraetzl and C. Colbourn have shown two useful reliability transformations on channel graphs that do not increase the 2-terminal reliability. In this talk, we show similar results for the k-path problem, and discuss methods for obtaining lower bounds on the reliability by applying the transformations.

Keywords: network reliability, k-disjoint paths, interconnection networks.

96.

CLOSED NEIGHBORHOOD DOMINATION AND PACKING

Peter J. Slater, University of Alabama in Huntsville

The fractional closed neighborhood domination and closed neighborhood packing parameters Wf and Rf, respectively, arise naturally as the duals of efficient domination and redundance, respectively. Among the results to be presented is the Automorphism Class Theorem for Wf and Pf.

best - pete

Tuesday, February 23, 1993
5:00 p.m.

97

Chordal Intersection Graphs of Semigroups.

Michael Ackerman*, F. R. McMorris, and Steve Seif
University of Louisville

The intersection graph of a semigroup S, denoted G(S), is defined as the graph whose vertex set is the set of all proper subsemigroups of S with an edge between vertices S_1 and S_2 of G(S) if and only if $S_1 \cap S_2 \neq \varnothing$. We characterize those commutative semigroups whose interesection graph is chordal and extend our result to the general problem of characterizing finite semigroups whose intersection graph is chordal.

Key words: graph of a semigroup, semilattice, chordal graph

98

HAMILTONIAN DEGREE CONDITIONS FOR TOUGH GRAPHS

Chính T. Hoàng , Lakehead University

Keywords: hamitonian graphs

We prove the following theorem: Let G be a graph with degree sequence $d_1, d_2, \ldots, d_n$ and let t be a positive integer at most three. If G is t-tough and $\forall i, t \leq i < \frac{n}{2}, d_i \leq i \Rightarrow d_{n-i+t} \geq n - i$ then G is hamiltonian. In the case $t = 1$, our theorem generalizes a well-known theorem of Chvátal. A similar result is established for pancyclic graphs.

99

Path Decompositions of Multigraphs

Leizhen Cai, University of Saskatchewan, Email: lcai@cs.usask.ca

Let G be a loopless multigraph. For each vertex x of G, denote its degree (the total number of edges incident with x) by $d_G(x)$, and its multiplicity (the maximum number of parallel edges incident with x) by $\mu_G(x)$.
Define

$$\phi_G(x) = \begin{cases} \text{the least even integer} \geq \mu(x) & \text{if } d_G(x) \text{ is even} \\ \text{the least odd integer} \geq \mu(x) & \text{if } d_G(x) \text{ is odd} \end{cases}$$

In this paper, it is shown that every multigraph G admits a ϕ-regular path decomposition — a partition $\mathcal{P}$ of the edges of G into simple paths such that every vertex x of G is an endpoint of exactly $\phi_G(x)$ paths in $\mathcal{P}$. This result implies Lovász's path decomposition theorem, and Li's perfect path double cover theorem (conjectured by Bondy) as special cases. It also gives an upper bound on the number of paths in a minimum path decomposition of a multigraph. Furthermore, the result can be transformed into a theorem on path covers of weighted graphs. A generalization of Gallai's path decomposition conjecture is also proposed.

Key words: path decomposition, ϕ-regular path decomposition

100

On the domination numbers of some hypercubes

K. Jay Bagga and Mingrui Li*, Ball State University

The domination number problem is known to be NP-complete in general. In this talk we present results on the domination numbers of a sublass of hypercubes. We use a construction for the hypercubes Q_n where $n = 2^k - 1$, to extend the result to some other values of n.

Wednesday, February 24, 1993
8:40 a.m.

101

**ADDITIVE PERMUTATIONS OF CADINALITY EIGHT
PART I: PERMUTATIONS CONTAINING AT MOST THREE NEGATIVE ELEMENTS.**

Jaromir Abrham
Department of Industrial Engineering, University of Toronto, Toronto, Canada M5S 3J7

Jean M. Turgeon*
Mathématiques et statistique, Université de Montréal, Case postale 6128, Montréal, QC, Canada H3C 3J7

An ordered set $X = (x_1, x_2, \ldots, x_n)$ of relatively prime integers is called a *basis of additive permutations* (an *A-basis*) if there exists a permutation $Y = (y_1, y_2, \ldots, y_n)$ of X such that the vector sum $X + Y = (x_1+y_1, x_2+y_2, \ldots, x_n+y_n)$ is again a permutation of X. Y is then called an *additive permutation* of X. All bases of additive permutations (A-bases) of cardinality 7 and less, and many other A-bases, have been known for some time. The A-bases of cardinality 8 with at most three negative elements, and their additive permutations, are enumerated in this paper. Because of the size of the problem, the A-bases with four negative and four positive elements will be studied in Part II of this paper.

102

Upper Bounds on Domination Number in Terms of Clique Size

Stephen G. Penrice, SUNY College at Cortland

We investigate the relationship between clique number and domination number. The main new result, which motivates our other questions, is the following: If G is a connected graph which does not contain a chordless path on 5 vertices, then G has a dominating set consisting of a clique and at most one other vertex. If Γ is a class of graphs and there is a constant k such that G has a dominating set which can be covered by k cliques whenever G ϵ Γ is connected, then we say that Γ is stongly γ-bound. If there is a function f such that $\gamma(G) \leq f(\omega(G))$ whenever G ϵ Γ is connected, we say that Γ is weakly γ-bound. Our main result implies that the class of graphs which do not contain a chordless path on 5 vertices is strongly γ-bound; we generalize this result and give an example that shows that several classes of graphs defined by forbidden subgraphs are not strongly γ-bound. We also characterize the hereditary classes of graphs which are weakly γ-bound.

103

GENETIC ALGORITHMS: A RANDOM GRAPH THEORY PERSPECTIVE

R.Marshall, CS Dept., Florida Tech, Melbourne, FL 32901

In this paper, we discuss representational, structural, and operational aspects of genetic algorithms in the framework of the theory of random graphs. Genetic algorithms are stochastic search techniques in which given a problem and a representation (typically binary), a set of plausible 'solutions' is assumed and then encoded using bit strings. Probability)based syntactic operations are performed on these bit strings to obtain new strings which are then evaluated for their fitness in contributing to a domain) defined goal. Since genetic algorithms are typically employed for solving problems involving large search spaces, the solutions obtained are very much dependent on the convergence properties of these algorithms. Consequently, we attempt to relate convergence properties of genetic algorithms to issues governing the evolution, properties and complexity of random graphs.

Keywords: Random Graphs, Genetic Algorithms, Stochastic Search

104

Investigating the Complexity of the Tenacity of Trees

David Mann, Northeastern University

For a graph G and a cutset $S \subset V(G)$, $\omega(G-S)$ is the number of components of G-S and $\tau(G-S)$ is the maximum order among all the components of G-S. The <u>tenacity</u> of G is defined as $T(G) = \min\left\{\frac{|S| + \tau(G-S)}{\omega(G-S)}\right\}$ where the minimum is taken over all cutsets S. A restriction of the tenacity is the <u>beta-tenacity</u>, $T_\beta(G)$, which is defined similarly except that the minimum is taken over all cutsets S with $\omega(G-S) = \beta(G)$, the independence number of G. The beta-tenacity of trees is shown to be polynomial and the complexity of the tenacity of trees is discussed.

KEYWORDS: vulnerability, tenacity, independence number

Wednesday, February 24, 1993
9:00 a.m.

105

Mathematical Properties of the Card Game SET

Robert Geist, Sandra Hedetniemi, Stephen Hedetniemi*
David Jacobs, Alice McRae and Robert Reynolds
Clemson University, Clemson, SC 29634-1906

SET is the name of a highly regarded card game, copyrighted by Mrsha J. Falco in 1988, and designed for players aged 6 to adult. The game consists of a deck of 81 cards, each of which can be represented by a distinct 4-tuple (a,b,c,d), where each of a,b,c and d is an element of {0,1,2}. Given a set of 12 randomly chosen cards, the object of the game is to find three cards, called a SET, say x = (a1,b1,c1,d1), y = (a2,b2,c2,d2) and z = (a3,b3,c3,d3) such that x + y + z = (0,0,0,0), where addition is Mod 3. In this paper we consider some of the algebraic, combinatorial, probabilistic and computational properties of SET, and we present several unsolved problems.

106

Guarding the Guards: a monitoring problem

G. Gunther*, Sir Wilfred Grenfell College, Newfoundland
B. Hartnell*, Saint Mary's University, Nova Scotia
D. Rall, Furman University, South Carolina

Let the vertices of a graph G represent the individual components of a system and the edges represent communication links. Since the system may fail when any of its components become unreliable, it is necessary for each component to be checked by one or more external components . This gives rise to a variation on domination where the vertices have restricted or bounded influence. For instance, if a graph has the property that there is some way for every component to monitor exactly k other components and in turn be monitored by k other components, we shall say that the graph admits a k-monitor solution.

Key phrases: bounded domination, reliability.

107 **A Genetic Algorithm for Set Covering Problems**

Roy P. Pargas Alice A. McRae*, Clemson University

We present a heuristic for the problem of set cover. The heuristic is called a genetic algorithm, a method of searching a large solution space based loosely on the theory of evolution of living organisms. We describe the data structures used and the manner in which populations of solutions are initialized and gradually evolve and improve. The genetic algorithm described is quite flexible and can be modified to solve many different problems, from graph domination to airline scheduling. To illustrate, we present results of our genetic algorithm for chessboard domination and total domination. The algorithm is implemented on an INTEL iPSC/2 hypercube.

Keywords: Genetic Algorithm, Set Cover

108

Polynomials for Directed Graphs

Gary Gordon* and Lorenzo Traldi, both of Lafayette College

Several polynomials are defined on directed graphs and rooted directed graphs which are all analogous to the Tutte polynomial of an undirected graph. Various interrelations between these polynomials are explored. Many of them have the number of maximal directed trees (arborescences) as an evaluation, and are related to the one-variable greedoid polynomial. Applications to reliability are also examined.

Keywords: Tutte polynomial, arborescence, greedoid.

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109

Some Combinatorial Problems suggested by the game of SETS

Phyllis Z. Chinn, Humboldt State University, Arcata, CA 95521

The game of SETS is played with a deck of "attribute cards." Such a deck is constructed from fixed values of various attributes (e.g. shape, color, etc.) by making one card for each possible selection of one value for each attribute. The SETS deck has three values for each of four attributes. The SETS game is equivalent to selecting collinear points (an affine line) in a four dimensional affine space over GF(3), or to identify blocks from a 2-(81, 3, 1) block design.

The presenter will teach the rules for this interesting combinatorial game and discuss some questions that are motivated by the game. It is a suitable game to teach in a variety of classes including finite math, discrete math or general education. Questions range from simple to very difficult, some answered and some open.

Key words: combinatorial games, finite affine geometries, using manipulatives to motivate mathematical discoveries.

110

MULTIPLY-SURE DISTANCES IN GRAPHS

Joseph E. McCanna*, Steven J. Winters, Western Michigan University

Given two vertices a and b in a graph G, we seek m internally (edge) disjoint a,b-paths P1,...,Pm which minimimize either the max or the sum of the values |P1|,...,|Pm|. This gives us several different new definitions of d*(a,b), the m-distance from a to b. After answering questions of metricity, we consider the relationship of these new distances to each other and to the standard distance. The diameter*, radius*, center* and perifery* of G can be defined in terms of d*. We present several results. One major one is: Every graph G is the center* Z*(H) of some other graph H for any choice of d*. In contrast, G=Perif*(H) for some H only if Rad*(G)>2m-1. This condition is sufficient only for some choices of d*, or if G is not isomorphic to Km. We also consider bonds on the diameter* of G.

Key words: distance, center, perifery, diameter, radius

111

Interblock Optimal Path Problems

James Geller, Ashish Mehta*, Yehoshua Perl N.J. Institute of Technology
Ronald Becker University of Cape Town

The problem of efficiently finding optimal paths (optimizing a given weighting function) between vertices of a directed graph is considered for weighted graphs which have a special structure. These graphs are those which are or which can be subdivided into blocks which have relatively few vertices adjacent to edges connecting different blocks . An example is the subdivision of the USA into states so that there are relatively many edges between cities within the same state, but relatively few between cities in different states. We assume that all locally optimal paths for all pairs of vertices in each block are precomputed by a version of Dijkstra's algorithm, and that their weights are stored. A relatively small auxiliary graph is constructed based on the original graph. This graph is preprocessed and then affords an efficient method of finding an optimal path between any two vertices of *different* blocks. A variation of this procedure also enables us to calculate *globally* optimal paths between two vertices in the same block. Since optimal paths between vertices in different blocks are assumed to be needed less frequently than those between vertices in the same block, this method provides a satisfactory compomise between storage needs and response time. We investigate general properties of weighting functions for which the procedures developed here are valid. For the special case of undirected graphs where the weight of a path is the weight of its smallest edge, the maximum weight spanning tree can be used to derive an algorithm for finding a path of maximum weight with better complexity than in the general case.
keywords: directed graph, shortest path, optimal path, path weighting function, undirected graph

112

Edge-Tenacity of Graphs

Barry Piazza*, U. of Southern Mississippi
Fred Roberts, Rutgers U.
Sam Stueckle, Northeastern U.

The tenacity of a graph has been introduced as a vulnerability measure which takes into account the number of deleted vertices, the number of remaining components, and the maximum order among the remaining components. It is related to the integrity and toughness of a graph. We examine the edge analog of the tenacity. The <u>edge-tenacity</u> of a graph G is defined as $T'(G) = \min\left\{\frac{|S|+\tau(G-S)}{\omega(G-S)}\right\}$, where the minimum is taken over all edge-cut-sets S of G, $\tau(G-S)$ is the maximum order among the components of G-S, and $\omega(G-S)$ is the number of components of G-S. For a graph with p vertices, q edges, and edge-connectivity λ, we show that $\frac{\lambda}{2}+\frac{1}{p} \leq T'(G) \leq \frac{q+1}{p}$. A graph shall be called <u>edge-tenacious</u> if $T'(G) = \frac{q+1}{p}$. We examine both non-edge-tenacious graphs and edge-tenacious graphs.
KEYWORDS: vulnerability, tenacity

113

On the minimum average length for k-intersecting families

Anthony J. Macula , Westfield State College, Westfield, MA

A separating system $\mathfrak{X} = \{N_j\}$ on $[n]$ is called **k-intersecting** if $|\cap\beta| \leq 1$ for all $\beta \subseteq \mathfrak{X}$ with $|\beta| \geq k$. Let $m(n,k)$ be the minimum m for which there exists a **k-intersecting** family on $[n]$. A **predetermined combinatorial search algorithm** that identifies a defect in a set of n elements can be thought of as a separating system on $[n]$. Let $A(\mathfrak{X})$ denote the **average search length of** $\mathfrak{X}$. Let $A(n,k) = \min\{A(\mathfrak{X}) : \mathfrak{X}$ is k-intersecting on $[n]\}$.

I. Let $\sigma(m,k) = \sum_{i=0}^{k}\binom{m}{i}$. Then $m = m(n,k)$ iff $\sigma(m-1,k) \leq n \leq \sigma(m,k)$

II. If $\sigma(m-1,k) \leq n \leq \sigma(m,k)$ and $n = \sigma(m-1,k) + r$, then

$$\frac{(m-1)\sigma(m-1,k) + rm + 1 - \sigma(m-1,k+1)}{\sigma(m-1,k) + r} \leq A(n,k) \leq \left[1 - \frac{1}{(k+1)\,\sigma(m,k)}\binom{m-1}{k}\right] m$$

Keywords: separating system, average search length, predetermined algorithm.

114

On a sequence of generalised distance domination numbers

J.H.Hattingh* and J.C. Schoeman, Rand Afrikaans University, Johannesburg, South Africa

Let $n \geq 1$ be an integer. The *closed n-neighbourhood* $N_n[u]$ of a vertex u in a graph $G = (V, E)$ is the set of vertices $\{v | d(u, v) \leq n\}$, i.e., all those vertices which are at distance at most n from u. The *closed n-neighbourhood of a set X of vertices*, denoted by $N_n[X]$, is the union of the closed n-neighbourhoods $N_n[u]$ of vertices u in X. A set $D \subseteq V(G)$ is an *n-dominating set* of G if $N_n[D] = V(G)$. A set $I \subseteq V(G)$ is *n-independent* if $d(u, v) > n$ for all distinct $u, v \in I$. A set $X \subseteq V(G)$ is *n-irredundant* if for all $x \in X$ $PN_n(x) = N_n[x] - N_n[X - \{x\}] \neq \emptyset$. For each of these types of subsets, we define upper and lower *distance* parameters as follows: The *upper (lower) n-domination number* $\Gamma_n(G)$ ($\gamma_n(G)$), *n-independence number* $\beta_n(G)$ ($i_n(G)$) and *n-irredundance number* $IR_n(G)$ ($ir_n(G)$) are respectively the largest (smallest) cardinality of a minimal n-dominating, a maximal n-independent and a maximal n-irredundant set of vertices of G. We establish necessary and sufficient conditions for a sequence m_1, m_3, m_4, m_5, m_6 of positive integers to satisfy, for some graph G, $ir_n(G) = \gamma_n(G) = m_1$, $i_n(G) = m_3$, $\beta_n(G) = m_4$, $\Gamma_n(G) = m_5$ and $IR_n(G) = m_6$.

Keywords: distance, domination, independence, irredundance

115

NEW CONSTRUCTIONS FOR DEBRUIJN TORI

Glenn Hurlbert, Arizona State University (*)
Garth Isaak, Lehigh University

An (M,N;m,n)_k de Bruijn torus is a k-ary MxN toroidal array with the property that every k-ary mxn matrix appears appears exactly once contiguously on the torus. Having a vast number of applications their existence has been studied for many years. Answering a question of Chung, Diaconis, and Graham, we show for n>9 that (N,N;n,n)_k tori (so-called 'square squares') exist if and only if n is even or k is a perfect square, furthering the work of Fan, Fan, Ma, and Siu on k=2. We conjecture that for large enough m and n (M,N;m,n)_k tori exist for all 'allowable' values of M and N, and we are able to construct the first examples of such tori in which M and N are not powers of k.

116

Contractible Edges In Hamilton Paths

R.L. Hemminger*, Vanderbilt U and
R.E.L. Aldred and Chen Jian, U of Otago, New Zealand.

We will discuss our latest work concerning contractible edges. In particular, we characterize the 3-connected graphs having a Hamilton path that contains exactly two contractible edges of the parent graph. As expected, they bear some similarity to the 3-connected graphs having a longest cycle containing exactly three contractible edges. Their similarities, as well as their differences, will be discussed.

Keywords: Contractible edges, Hamilton path.

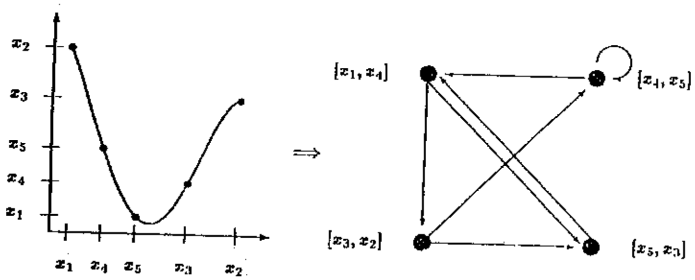
Wednesday, February 24, 1993
11:30 a.m.

117

Characteristic Polynomials of Straffin Digraphs

David C. Fisher – University of Colorado at Denver

Let f be a continuous scalar function where the recursion $x_{i+1} = f(x_i)$ has a periodic solution. Straffin introduced digraphs to study these systems (see the example). These digraphs have been used to give simpler proofs of Sharkovskii's Theorem and the log of their spectral radius gives a lower bound for the "topological entropy" of f. This talk will prove two properties of the characteristic polynomials of Straffin digraphs of periodic orbits: (a) The constant term is ± 1; (b) The coefficients are odd.



A Function and Its Straffin Digraph. Here f is continuous and $x_{i+1} = f(x_i)$ has a period 5 solution with $x_1 < x_4 < x_5 < x_3 < x_2$. Then $[x_5, x_3] \cup [x_3, x_2] \subseteq f([x_1, x_4])$, $[x_1, x_2] \cup [x_4, x_5] \subseteq f([x_4, x_5])$, $[x_1, x_4] \subseteq f([x_5, x_3])$, and $[x_4, x_5] \cup [x_5, x_3] \subseteq f([x_3, x_2])$. The digraph shows these relations.

118

Two Extremal Problems for Minus Domination Number

John Gimbel, University of Alaska

Given a graph G, a minus domination function on G is a map $\pi: V(G) \to \{-1,0,1\}$ where for each vertex u in G,

$$\sum_{v \in N[u]} \pi(v) \geq 1 .$$

The value of π is the sum

$$\sum_{v \in V(G)} \pi(v) .$$

The minus domination number of G, denoted $\gamma^-(G)$, is the minimum value of all minus domination functions defined on G. We consider the minimum minus domination number taken over all bipartite graphs of fixed order and establish a conjecture of Dunbar, et al. Further, we show that for any integer m and any integer k≥3, there is a graph G with $\gamma^-(G)=m$ and a girth of k.

119

The Spectrum of a Cayley Graph of Odd Order

Jon A. Sjogren, AFOSR

The spectrum of a graph is usually defined as the multi-set of eigenvalues of its vertex incidence matrix. Given a finite group G, together with a set of generators T, the Cayley graph essentially has group elements as vertices and generators as edges. A generalization with multiple edges is also allowed. Such a graph is regular, and one of the eigenvalues is just the degree of any vertex. In case G has odd order, our main result states that all of the other eigenvalues have even multiplicity. This makes use of a decomposition by direct sum of any symmetric matrix with circulant blocks, and the Feit-Thompson theorem. Dual results exist for factorization of the number of spanning trees in such graphs. Important open problems remain in this new field. We conjecture that any Cayley graph for the non-abelian group of order 21 has eigenvalues with generic multiplicity (1,2,6,6,6).

120

LINEAR OPERATORS WHICH PRESERVE COMBINATORIAL ORTHOGONALITY

LeRoy B. Beasley, Utah State U. and *Daniel J. Scully, St. Cloud State U.

A (1,-1,0)-matrix is said to be a potentially sign-orthogonal sign pattern if the inner product of any pair of distinct rows or columns equals zero or contains terms that are positive and terms that are negative. A sign-orthogonal matrix is a matrix such that some matrix with the same (1,-1,0) sign pattern is an orthogonal matrix. Evidently all sign-orthogonal (1,-1,0)-matrices are potentially sign-orthogonal sign patterns, but not conversely. In this article we characterize the linear operators on the $n \times n$ real matrices that map the set $\mathcal{X}$ of matrices with potentially sign-orthogonal sign patterns to itself and the complement of $\mathcal{X}$ to itself. We also show that the same operators are the only ones that map the set $\mathcal{Y}$ of sign-orthogonal matrices to itself and the complement of $\mathcal{Y}$ to itself.

Key Words: orthogonal matrix, linear operator, linear preserver, structured matrices.

Wednesday, February 24, 1993
11:50 a.m.

121

On Canonical Ramsey Numbers

Hanno Lefmann, University of Idaho

In Canonical Ramsey Theory one investigates the behaviour of objects with respect to arbitrary colorings of some structure. For Ramsey's theorem itself, a canonical version was given by Erdős and Rado in 1952. Here we will focus on quantitative aspects. In particular, we will give new lower and upper bounds for these Canonical Ramsey numbers.

122

A Minimum Factoring of Cycles

*Julie R. Carrington
(Rollins College)
Richard M. Caron
Robert C. Brigham
(University of Central Florida)

A subset, D_k, of the nodes of G is a k-dominating set if every node in G is either in D_k or is adjacent to at least k nodes of D_k. A factoring of G is a partitioning of the edges into spanning subgraphs and a global dominating set, D, is a subset of the nodes such that every node is either in D or is adjacent to some node of D in every factor. We consider the relationship between global domination and k-domination and a combinatorial problem which results from this relationship.

123

Why we will know R(4,5) soon: gluing graphs using regular sets

Brendan D. McKay, Australian National University, Canberra
Stanisław P. Radziszowski,* Rochester Institute of Technology, NY

A graph G is (k,l,n)-good if it has n vertices, no complete subgraph on k vertices and no independent set of order l. We present an algorithm, which given a family S of $(k-1,l,p)$-good graphs and a $(k,l-1,q)$-good graph H, produces all $(k,l,p+q+1)$-good graphs G, such that for some vertex x the neighborhood of x induces a graph isomorphic to a member of S, and the set of vertices not equal or adjacent to x induces a graph isomorphic to H. The key concept of the algorithm is the regular set of cones between two graphs, with elegant algebraical properties. This algorithm dramatically outperforms previous algorithms used in the search for Ramsey graphs.

Using this algorithm, and other tools including large scale integer linear programming, several new results regarding some small classical Ramsey numbers have been obtained, namely: new upper bounds $R(4,5)\le 26$, $R(5,5)\le 50$, $R(4,6)\le 41$ were proved, and more than 350000 nonisomorphic (4,5,24)-good graphs were constructed. The authors are confident that the same method will lead shortly (within 1 or 2 months) to the evaluation of the exact value of the classical Ramsey number $R(4,5)$.

124

Gray Codes from Antimatroids

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FRANK RUSKEY(*), *Dept. of Computer Science, U. Victoria, Victoria, B.C., V8W 2Y2*

We show three main results concerning Hamiltonicity of graphs derived from antimatroids. These results provide Gray codes for the feasible sets and basic words of antimatroids. For antimatroid $(E,\mathcal{F})$, let $J(\mathcal{F})$ denote the graph whose vertices are the sets of $\mathcal{F}$, where two vertices are adjacent if the corresponding sets differ by one element. Define $J(\mathcal{F};k)$ to be the subgraph of $J(\mathcal{F})^2$ induced by the sets in $\mathcal{F}$ with exactly k elements. Both graphs $J(\mathcal{F})$ and $J(\mathcal{F};k)$ are connected, and the former is bipartite. We show that there is a Hamiltonian cycle in $J(\mathcal{F})\times K_2$. As a consequence, the ideals of any poset $\mathcal{P}$ may be listed in such a way that successive ideals differ by at most two elements. We also show that $J(\mathcal{F};k)$ has a Hamilton path if $(E,\mathcal{F})$ is the poset antimatroid of a series-parallel poset. Similarly, we show that $G(\mathcal{L})\times K_2$ is Hamiltonian, where $G(\mathcal{L})$ is the "basic word graph" of a language antimatroid $(E,\mathcal{L})$. This result was known previously for poset antimatroids.

Wednesday, February 24, 1993
3:20 p.m.

125

STRONGLY CONNECTED MIXED GRAPHS AND CONNECTED DETACHMENTS OF GRAPHS

C.St.J.A. Nash-Williams, West Virginia University and University of Reading, England

Let G be a finite graph and b be a function from V(G) into the set P of positive integers. A b-detachment of G is a graph with the same edges as G which is obtained from G by splitting each vertex v into b(v) vertices. We report a new proof of a necessary and sufficient condition for G to have a connected b-detachment. This proof, unlike those previously published, does not require matroid techniques. We also report what is in essence a generalisation of an auxiliary result used in this proof. This generalisation states a necessary and sufficient condition on a finite strongly connected mixed graph M and a function f: $V(M) \rightarrow P \cup \{0\}$ for it to be possible to convert some undirected edges of M into directed edges in such a way that each vertex v becomes the terminal vertex of f(v) newly directed edges and M remains strongly connected.

126

On p-Edge Clique Covers of Graphs

by Robert C. Brigham[1], Ronald D. Dutton[1*], F. R. McMorris[2], and Joe B. Mize[1]

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The existence of a p-edge cliqe cover for a graph is related to induced subgraphs of a new graph having a simply defined structure. Properties of this new graph are discussed, and it is employed in the consideration of two problems involving the p-edge clique cover number.

Key words: Clique covers, competition graphs, p-edge clique covers

127

Ring-valued invariants of graphs

John Pedersen, University of South Florida

Invariants of graphs such as the chromatic polynomial and the various link polynomials can be defined in a general way as "polynomially recursive" functions into rings. This is done in the context of replacement systems for graphs, which also encompasses the algebraic setting of equivalence of words in an algebra given by presentations. In particular, semigroup presentations correspond to replacement systems for labelled paths. Some properties of these polynomially recursive invariants will be examined, with an emphasis on their relation to solutions of decision problems for algebraic structures.

128

Correcting OCR-generated Page Images Using Permutations

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In processing a page image by an Optical Character Recognition (OCR) device, a certain text string is generated which may not be the same as the correct string. The difference may be due to the incorrect reading order selected by the employed zoning algorithm or poor recognition of characters. A difference algorithm is commonly used to find the best match between the generated string and the correct string (which may be obtained by using a sophisticated but accurate zoning algorithm). The output of such an algorithm will then be a sequence of matched substrings which are not in the correct order. A human operator is therefore required to take the OCR generated string and produce the correctly ordered version by moving matched substrings to their correct positions. It is of interest then to minimize the number of moves needed to obtain the correct string. The problem can be modeled as a sorting problem where a string of n integers ordered in a random manner, must be sorted according to a specific order.
In this paper, we derive bounds on the time complexity of sorting a given string and present a near-optimal algorithm for that.

Key Words: OCR, Permutation, Sorting, String.

Wednesday, February 24, 1993
3:40 p.m.

129

On 2-Heavy Graphs

Chi Wang, *Department of Mathematics, University of Louisville*

Given a graph, a threshold assignment assigns weights on subset of vertices of the graph of size at most 2 such that for any subset of vertices of the graph, it is non-stable if the sum of weights over vertices and pair of vertices of the set is larger than a fixed threshold value, and it is stable if the same sum is less than the threshold value minus one. A graph is heavy if the total weight required on pairs of vertices in any threshold assignment is at least the number of edges of the graph.

In this paper, we introduce and study a subclass of heavy graphs, 2-heavy graphs. We characterize 2-heavy graphs by a special perfect fractional b-matching and by solvability of linear inequality system. Some necessary conditions and sufficient conditions of 2-heavy graphs are given. Classes of 2-heavy graphs are characterized. Some known results on heavy graphs are strengthened.

130

THE SQUARE ROOT OF A TOURNAMENT

Raymond R. Fletcher III*, University of Texas of the Permain Basin, Odessa, TX;
David G. Robinson, Guilford College, NC

A digraph G with the property that each pair of vertices of G is joined by a unique directed path of length 2 we call a *tournament square root graph* (TSRG). The square of the adjacency matrix of such a graph yields the adjacency matrix of a tournament. Determining all TSRG's is a directed analog of the well known "friendship" problem which seeks a description of all simple graphs with the property that each pair of vertices is joined by a unique (undirected) path of length 2. We construct several infinite classes of TSRG's and give characterizations for TSRG's which contain vertices of maximum and minimum possible valence.

Key words: Tournaments, digraphs, unique path properties.

131

Structure of the Hoffman-Singleton Graph

Cong Fan* and Allen J. Schwenk
Western Michigan University

We will use an eigenvalue approach to simplify the proof of the uniqueness of the Hoffman-Singleton graph and identify its automorphism group. We shall then study the problem of packing H-S graphs into the complete graph K_{50}.
Key Words: Hoffman-Singleton graph, Moore graph, eigenvalue, automorphism group.

132

An O(log log n) Parallel-Time Family of Permutation Algorithms

J.Seguel, D.Bollman*, A.Gonzalez, University of Puerto Rico

We develop a kind of tensor notation which we use to express algorithms for the family of digit-index permutations. Using this notation we are able to show that the set of images of any digit-index permutation on {0,1,2,...,n-1} can be performed in O(log log n) parallel steps. This notation also has the advantage that it greatly facilitates the development of efficient implementations in a functional programming language. We illustrate this with an implementation of bit-reversal in the functional language Sisal, which outperforms its Fortran counterpart on a Cray 2.

133

Efficient Generation of Subsets with a Given Sum

Eugene Neufeld and Dominique Roelants van Baronaigien U of Victoria

A loopless generation algorithm is an algorithm that generates combinatorial objects such that the computation necessary to determine each successive object is O(1) worst case. We present the first loopless algorithm for listing all subsets of the set $\{1, 2, \ldots, n\}$ that have sum p.

134

The existence of regular and almost regular r-tournaments

Susan Marshall, Simon Fraser University

An r-tournament T on n vertices consists of an n-set $V(T)$ of vertices and and the $\binom{n}{r}$ r-subsets of $V(T)$, each of which is assigned one of the $r!$ possible linear orders. The ordered r-subsets of $V(T)$ are called *arcs* of T. An r-tournament is called *regular* if each vertex occurs in each of the r positions the same number of times (that is, in the same number of arcs). The following question was asked by E Barbut and A Bialostocki: Given integers r and n such that $n \geq r \geq 2$ and $\binom{n}{r} \equiv 0 \pmod{n}$, does there exist a regular r-tourament on n vertices? The answer is known to be affirmative in the case that $\gcd(n, r)$ is a power of a prime. In this talk we give an affirmative answer for all n and r satisfying the given conditions. In addition we define the notion of an almost-regular r-tournament, and show that and almost regular r-tournament on n vertices exists for all n and r with $n \geq r \geq 2$.

Keywords: r-tournament, regular

135

Linear Algorithms for Capacity Domination in Trees and Series Parallel Graphs

D. I. Carson and O. R. Oellermann

University of Natal

Durban, South Africa

136

Permutations made with two queues or one stack

*Edward T. Ordman and William Schmitt,
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If the numbers 1..n are permuted by passing them through a channel with two parallel queues, the result is the merger of two increasing sequences. Knuth (Vol. 3, Sorting, 5.1.4, p.64), observes that the number of such permutations is the n_th Catalan number, the same as the number of permutations produced by a channel with one stack, but points out that his proof is difficult and the one-to-one correspondence is not very natural (in fact, computing it involves work equal to sorting n numbers). We observe that, regarding the two channels as progammable devices, we can display one-to-one mappings {merger of two sequences} <--> {program for two-queue channel} <--> {program for one-stack channel} <--> {Catalan set, e.g. binary trees} where each correspondence is straightforward and an element of one set can be mapped to another set in time at worst O(n).

We are working on problems with more stacks or queues.

Key Words: permutation, stack, queue, Catalan number.

Wednesday, February 24, 1993
4:20 p.m.

137

TWO DIMENSIONAL LINAER CONGRUENTIAL GRAPHS

C. C. Koung* and J. Opatrny
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Montreal, Canada

A *Linear Congruential Graph* of degree $2d$ is a graph on the vertex set $\{0, 1, \ldots, n\}$ whose edge set is generated by a set of generators $\{f_1, f_2, \cdots, f_d\}$, where each generator is a linear function. We present a generalisation of these graph to the n-dimensional case in which the vertex set are vectors of integers and generators are linear functions on the vectors. In particular we show that there are 2-dimensional linear congruential graphs of degree k and diameter d which contain more vertices than one dimentional linear congruential graphs for many values of k, d. Some properties of these graphs will be mentioned.

138

TOURNAMENTS WITH A GIVEN NUMBER OF KINGS AND SERFS

J.W. Moon, University of Alberta

A *tournament* T_n consists of a set of n *nodes* $1, 2, \cdots, n$ such that each pair of distinct nodes i and j is joined by exactly one of the *arcs* $\overrightarrow{ij}$ or $\overrightarrow{ji}$. If the arc $\overrightarrow{ij}$ is in T_n we say that i *beats* j. Node i *covers* node j if node i beats every node that node j beats. This covering relation is transitive and defines a partial order on the nodes of T_n. The maximal and minimal elements of this partial order have been called *kings* and *serfs*, respectively. Reid [Cong. Num. 29(1980)809-826] characterized the sets of integers $n, B, K,$ and S for which there exists a tournament with $B+K$ kings, $B+S$ serfs, and B nodes that are both kings and serfs. Our object is to give a shorter proof of Reid's result that requires fewer ad hoc constructions.

Key Words: tournaments, kings, serfs.

139

A New Heuristic for Finding Steiner Trees in Graphs

D. I. Carson, H. P. Hlongwane, and O. R. Oellermann*

University of Natal

Durban, South Africa

140

An Exact Algorithm for Maximum Entropy Sampling

Chun-Wa Ko, DIMACS, Jon Lee*, Yale University
Maurice Queyranne, University of British Columbia

We study the computational complexity of finding extremal principal minors of a positive definite matrix. In particular, we focus on the NP-hard problem of maximizing the determinant over the set of principal submatrices of a given order. This problem arises in the area of statistical design, where one wishes to select a subset of some correlated Gaussian random variables having maximum entropy. In this case, the input matrix is the covariance matrix of the random variables and the entropy is the logarithm of the determinant. We establish an upper bound for the entropy, based on the eigenvalue interlacing property, and we incorporate this bound in a branch-and-bound algorithm for the exact solution for the problem. We present computational results for estimated covariance matrices corresponding to sets of environmental monitoring stations in the United States.

Key words: Combinatorial Optimization, Eigenvalues, Experimental Design, Acidic Precipitation.

141

THE MAXIMAL ULTRAMETRIC LESS THAN A COMPLETE WEIGHTED GRAPH

John C. Higgins / Department of Computer Science / Brigham Young University

Key Words: complete graph, weight function, ultrametric

Given the complete undirected graph G on n-vertices with a positive, real valued weight function W, it is well known that there is a unique maximal ultrametric weight function U less than W. This paper presents an algorithm that given W finds U in time polynomial in n. The algorithm operates by sequentially examining the set of all 3-subgraphs of G and imposing the local ultrametric condition on each of these subgraphs. It is demonstrated that this process achieves stasis after at most n^4 reassignments of edge weights and that the stable assignment is U, the maximal ultrametric on G less than W.

142

On Tournament Partitions of Graphs

Gary MacGillivray, University of Vicoria, B.C., Canada
Joseph Yu*, Memorial University of Newfoundland, Nfld., Canada

Let C be the set of all tournaments and H be a fixed digraph. An $(H, C)-$*partition* of a digraph G is a partition of $V(G)$ into $|V(H)|$ pairwise disjoint subsets $V_h, h \in V(H)$, such that for each h, V_h induces a tournament and if $x \neq y \in V(H)$, then there is an arc from V_x to V_y only if $(x,y) \in D(H)$. We show that the problem of deciding whether or not a given digraph has an (H,C)-partition is polynomial.

Key words: tournaments, digraphs

143

Size and Independence in K_4-free graphs

Kathryn L. Fraughnaugh *
University of Colorado at Denver, Denver, Colorado

Stephen C. Locke
Florida Atlantic University, Boca Raton, Florida

We investigate lower bounds on the size of K_4-free graphs in terms of their order and independence. For several ranges of independence relative to order and for graphs with maximum degree three and four, we find the best possible such lower bounds. We also evaluate some Ramsey-type numbers over the classes of graphs with maximum degree three and with maximum degree four.

144

A Boolean Expression for the GF(2) Determinant

Charles M. Fiduccia, Supercomputing Research Center, Bowie, MD

The formula size of the familiar expression, call it$\Delta_n(X)$, for the determinant $|X|$ of an arbitrary $n \times n$ matrix $X = (x_{ij})$ is $n \cdot n!$. We give a simple three-level*boolean* expression for $|X|$ that is valid over the finitefield GF(2), and whose size is $n^2 \cdot 2^{n-1}$. Moregenerally, if X is an $m \times n$ matrix, over GF(2), theassertion that its columns are linearly independent is captured bythe boolean expression (0=false, 1=true)

$$\Phi_{mn}(X) := \prod_{S \neq \emptyset} \bigvee_{i=1}^{m} \sum_{j \in S} x_{ij}$$

where S runs over the $2^n - 1$ non-empty subsets of $\{1, \ldots, n\}$. Since $|X| = 1$ iff the columns of X are independent, $\Phi_{nn}(X)$ is an expression for $|X|$. It has size$n^2 \cdot 2^{n-1}$ and leads to the *family of boolean identities* $\Delta_n(X) \equiv \Phi_{nn}(X)$. For instance, $n = 2$gives $ad + bc \equiv (a \vee c)(b \vee d)((a+b) \vee (c+d))$, which is a generalization of de Morgan's law, because$b = c = 1$ gives $ad + 1 \equiv (a+1) \vee (d+1)$. One might ask whichother boolean identities can be obtained in this way. Since$\Phi_{mn}(X)$ is defined for non-square matrices, we also havethat: For all $m < n$, $\Phi_{mn}(X) \equiv 0$, which shows that$1 + \Phi_{mn}(X)$ is a tautology. Moreover, since $\Phi_{mn}(X)$is *not* identically zero, when $m \leq n$, the identities$\Phi_{n,n+1}(X) \equiv 0$ appear to be particularly interesting,since they capture the transition point from linear indepenedenceto linear dependence.

145

Some nonexistence results for self-complementary circulant graphs

D.Froncek, A.Rosa, J.Siran*, McMaster University, Hamilton (Ont.)

We investigate the local structure of Cayley graphs. As an application, we show that there are no self-complementary circulant graphs on 12s + 9 vertices, s>=0.

Key words: Cayley graph, self-complementary graph, circulant graph.

146

TOURNAMENT GAMES AND CONDORCET VOTING

Jennifer Ryan* and David Fisher, University of Colorado at Denver

A Condorcet voting procedure asks each voter to rank candidates in order of preference. Then for each pair of candidates, a tally determines which candidate is preferred by a majority. The results can be modeled by a tournament where the nodes represent the candidates and the arcs point toward the winner of each two way race. If one candidate beats every other candidate, he/she is the winner. What if there is no such candidate? Felsenthal and Machover proposed that the "winner" be a probabilistic combination of candidates who win at least as often as they lose against every candidate. A winner is then picked at random using the probabilities. This is the unique optimal strategy to a generalized "Scissors, Paper and Stone" game played on the tournament. What is the maximum number of cards (the least common denominator of the probabilities) needed to pick a winner from n candidates? What are the minimum and maximum noninteger probabilities? What (if anything) can candidates do to increase their probability of inning?

147

CONSTANT TIME GENERATION OF TREES WITH BOUNDED DEGREE

*Kathleen A. McKeon and Bridget B. Baird, Connecticut College

In this paper we present an algorithm for generating trees of bounded degree of a given order. The trees under consideration are rooted d-trees, rooted trees in which the maximum degree is at most d, and k-ary trees, rooted trees in which every vertex has at most k children. We describe the algorithm for d-trees of order n and make note of the adjustments that must be made in the algorithm for k-ary trees. Using the level sequence representation defined by Beyer and Hedietniemi, all trees of a given order and type are generated, without repetition, by the algorithm in time proportional to the number of trees produced.

Key words: algorithm, rooted tree

148

"The Weighted Integer Matching Problem"

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The above problem is defined and examined for some initial cases. It tries to minimize a sum of products from selected lists of integers.
This problem comes up in the design of reliable networks, and specifically with Split graphs. It will be shown that a simple case suffices to characterize its computational complexity.

Keywords: combinatorial optimization, NP-Completeness.

Wednesday, February 24, 1993
5:20 p.m.

149

Tournament antichains of bounded length

Brenda J. Latka, Lafayette College

Let A be a k-set of finite tournaments which are pairwise incomparable under the embeddability relation. We say that A is a finite antichain of length k. We show that A can be extended to an arbitrarily large antichain of finite tournaments if and only if A fails to contain a member of one of five families of finite tournaments.

Keywords: Tournament, Antichain

150

The Existence of Interval-Balanced Tournament Designs

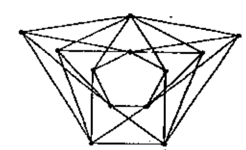
P. Rodney Dept. of Mathematics, University of Toronto

Let V be a set of n elements. A tournament design is a c-row array of the n(n-1)/2 pairs of elements from V such that every element appears at most once in each column. An interval-balanced tournament design, has the added condition that the appearances of each element are equitably distributed amongst the columns of the design. We settle the existence question for all IBTD's and discuss their application to scheduling round robin tournaments fairly with respect to the amount of rest allotted to each participant.

151 THE NUMBER OF EDGES IN A MAXIMAL TRIANGLE-FREE GRAPH

David Fisher, Kathryn Fraughnaugh, and Karen Casey, University of Colorado at Denver; Curtiss Barefoot, New Mexico Institute of Technology; Frank Harary, New Mexico State University*

A maximal triangle-free (abbreviated MTF) graph is a graph without triangles such that adding any edge to the graph creates at least one triangle. Since it is connected, an n node MTF graph has at least $n-1$ edges, and any n node triangle-free graph has at most $\lfloor n^2/4 \rfloor$ edges. Since a n node complete bipartite graph is MTF and can have either $n-1$ or $\lfloor n^2/4 \rfloor$ edges, these bounds are sharp. However for some values of n and e with $n-1 < e < \lfloor n^2/4 \rfloor$, there are no n node MTF graphs with e edges. For example, no 6 node MTF graph has 6 edges. We will answer the question: *For which values of n and e are there MTF graphs on n nodes with e edges?*, and relate it to the Murty/Simon conjecture about minimal diameter 2 graphs.



A MTF graph. This graph has no triangles and yet the addition of any edge creates a triangle.

152 **A Strategy for Displaying State Diagrams for LR(0) Parsers**

Benjamin R. Seyfarth, University of Southern Mississippi
Manuel E. Bermudez*, University of Florida

We consider the problem of displaying the LR(0) automaton of a given context-free grammar. The construction algorithm of LR(0) automata yields a machine that exhibits some unique characteristics; particular groups of states have essentially the same (or subsets of the same) transitions. We present an algorithm for establishing a compact, uncluttered layout of the states in the corresponding state diagram. The algorithm consists of three parts. First, a linear arrangement is established, and the initial pattern of transitions fixed accordingly. Second a "shuffle" procedure is applied, in which states are grouped (and re-arranged) according to their need for physical proximity. Third, the actual distances among the various states is minimized to save space.

The resulting algorithm yields a credible (though not optimal) layout of the states of an LR(0) automaton. We believe that our algorithm can be generalized to so that arbitrary graphs and finite-state automata can be displayed in a near-optimal fashion.

Thursday, February 25, 1993
8:40 a.m.

153

A Note on Parker's Paper on Mutually Orthogonal Latin Squares

Lawrence Somer, The Catholic University of America, Washington, DC 20064

We will prove the following theorem:

Theorem 1: Let t and r be given positive integers such that $t < r$. Suppose that a complete set of $n-1$ mutually orthogonal latin squares of order n contains t mutually orthogonal latin squares of order r with $r < n$. Let $c = [r^2/n]$, where $[x]$ denotes the greatest integer less than or equal to x. Then n must satisfy the following inequality

$$(n-t-1)c(-cn-n+2r^2) \leq r^2(r-1)(r-t-1).$$

The proof extends results given in the paper [1] by E.T. Parker and gives improved bounds in many cases.

In Theorem 1, if $t = r-1$, then the theorem can be interpreted as giving lower bounds on the size n of a finite projective plane which has a subplane of order r. In this case, however, the bounds for n are not improved over known results.

Reference

1. E. T. Parker, "Nonextendability Conditions on Mutually Orthogonal Latin Squares," Proc. Amer. Math. Soc. 13 (1962), 219-221.

Key words. orthogonal latin squares

154

Bandwidth Sum of the Sum of k Sum-deterministic Graphs

Kenneth Williams, Western Michigan University

For graph $G = (V, E)$, 1-1 mapping $f : V \longrightarrow \{1, 2, \ldots, |V|\}$ is called a *proper numbering* of G. The *bandwidth sum of G*, denoted $s(G)$, is the number

$$min\{\sum_{uv \in E} |f(u) - f(v)| : \text{ f is a proper numbering of G}\}$$

and a proper numbering that achieves $s(G)$ is said to be *optimal*. For g an optimal numbering we consider any numbering h, (h is not necessarily proper) of G with $h : V \longrightarrow H = \{i_1, i_2, \ldots, i_n\}$ for integers $i_1 < i_2 < \ldots < i_n$. Then if $h(u) = i_{g(u)}$ for all $u \in V$ (i.e. h assigns values to vertices in the same order as g) and $\sum_{uv \in E} |h(u) - h(v)| = \min \sum_{uv \in E} |f(u) - f(v)|$ such that f is any numbering of G from H we say G is *sum deterministic*. The sum-deterministic graphs include K_n, $\overline{K_n}$, C_n, P_n and $K_{1,n}$.

It is well known that determination of the bandwidth sum for arbitrary graphs is NP-complete. The sum of k graphs, $\Sigma_{i=1}^k G_i$, is defined to be the graph with vertex set $\cup_{i=1}^k V(G_i)$ and edge set $\cup_{i=1}^k E(G_i) \cup \{uw : u \in V(G_i), w \in V(G_j) \text{ for all } i \neq j\}$. For $G = \Sigma_{i=1}^k G_i$, with each G_i sum-deterministic, a polynomial time algorithm to establish $s(G)$ is established.

Keywords: bandwidth sum, sum-deterministic, polynomial time algorithm.

155

On the Numerical Stability of Necked States of Circular Plates in Tension

Pablo V. Negron-Marrero*, Univ of Puerto Rico, Rio Piedras, PR
Barbara L. Santiago-Figueroa, InterAmerican University Bayamon, PR

We study numerically the stability in the energy sense of the necked states of circular plates in tension. We use a finite difference approximation of the boundary value problem giving the necked states. This yields a nonlinear system of equations that depends parametrically on the tension. To study the bifurcation diagram of this system we use continuation methods and a numerical technique for computing bifurcating branches developed by Rheinboldt. We give numerical results for a large class of physically reasonable constitutive functions that exhibit the standard Poisson ratio effects and get conditions for bifurcation for this class of functions.

156

Isogonal Imbeddings of Some Product Graphs

Ghidewon Abay Asmerom Virginia Commonwealth University

Among the many 2-cell imbeddings of a connected graph, most of the interest has been focused on four types of imbeddings: the minimum genus, the maximum genus, the self-dual, and the symmetrical imbeddings. In this paper we discuss another type of imbedding; the isogonal imbedding. Isogonal imbeddings are imbeddings where $r = r_n$. Duals of imbeddings of regular graphs are isogonal; in particular, self-dual imbeddings of Cayley graphs are isogonal. All symmetrical imbeddings are also isogonal. Here we will present isogonal imbeddings of the tensor, augmented tensor, and the strong tensor products. The first factors will include: trees, cycles and $6k-3$loop pseudographs imbedded in S_k, and the second factors will be Cayley graphs.

Thursday, February 25, 1993
9:00 a.m.

157 EXISTENCE OF DBIBDs WITH BLOCK SIZE SIX

F.E. Bennett *
Mount Saint Vincent University

R. Wei and J. Yin
Suzhou University

A. Mahmoodi
University of Toronto

Let v, k and λ be positive integers. A transitively ordered k-tuple $(a_1, a_2, \ldots, a_k)$ is defined to be the set $\{(a_i, a_j) : 1 \le i < j \le k\}$ consisting of $k(k-1)/2$ ordered pairs. A *directed balanced incomplete block design* (DBIBD) with parameters v, k and λ, denoted by $DB(k, \lambda; v)$, is a pair (X, B) where X is a v-set (of points) and B is a collection of transitively ordered k-tuples of X (called blocks) such that every ordered pair of points of X appears in exactly λ blocks of B. In this paper, the necessary conditions for the existence of a $DB(6, \lambda; v)$ are shown to be sufficient except for the case $(v, \lambda) = (21, 1)$.

158

CUBIC GRAPHS AND CAPILLARY RECRUITMENT IN LUNGS

John J. Watkins, Colorado College

Biologists have observed that, as the demand for oxygen transfer increases, individual capillaries in the lungs will be recruited — that is, perfused with blood. In order to decide statistically whether or not this recruitment is random it is necessary to know, in a given network of capillaries, exactly which subnetworks are allowable —that is, which patterns of capillaries would actually permit the flow of blood. Since these networks are almost always cubic graphs, a relatively simple 'pruning' technique allows us to find the allowable subgraphs efficiently enough to satisfy the biologists. In particular, enumerating these subgraphs in the special case of a treelike network of capillaries produces alternate Fibonacci numbers.

159

The Numerical Solution of the von Karman Equations using Multigrid Methods

Mariano Marcano* and Pablo V. Negron-Marrero
University of Puerto Rico, Rio Piedras,P.R.00931.

In this paper we study the deformations of a plate subjected to a compressive force. We use the nonlinear-plate theory of von Karman from which we get a semilinear system of partial differential equations. We will study the numerical solution of the system using technics that involve the use of vectorization and parallelism.

160 Adding and Deleting Crossings from Drawings of Graphs

J. E. Green-Cottingham* and R. D. Ringeisen, Clemson University

In this talk, we examine so called 'good drawings' of graphs on surfaces, where here we also mean that the drawings are 2-cell. We have developed methods for adding and deleting one crossing from an existing good drawing of a graph and creating another good drawing of the same graph with one more or one less crossing, respectively. We will discuss these methods, as well as, present examples of drawings including thrackles of graphs.

161

Some Recent Progress in BIBD and RBIBD Constructions

Malcolm Greig, Dept. of Statistics, Univ. of British Columbia.

This talk will give an update of the existence status of BIBD's for k less than 10. For RBIBD's, some recent progress has been made in joint work with Furino, Yin and Miao. For RB(8,7) there are 36 open cases, with the largest open case being v=2928; for RB(16,15) the largest open case is v=90192. The RB(8,1) status will be updated, and this recent work will be outlined.

Keywords: Frames, balanced incomplete block designs, resolvable balanced incomplete block designs.

162

Some new sub-classes of well-covered graphs

Ramesh S. Sankaranarayana, University of Alberta

A well-covered graph is one in which every maximal independent set has the same size. In this paper, we introduce a hierarchy of three new classes of well-covered graphs. We study the complexity of some of the standard problems like recognition, hamiltonian cycle and path, etc., for these classes. We show that the clique partition problem, which is NP-complete for well-covered grahs, is in P for these classes. We show that for the first class, recognition is co-NP-complete, and the dominating set problem is NP-complete. The second class, which is a sub-class of the first on e, has some unsolved problems like recognition and dominating set. The third class is a sub-class of the second one and has recognition and dominating set in P, with the hamiltonian set and path problems being NP-complete. It has also the interesting property that it properly contains the family of very well covered graphs.

163

An Efficient Distributed Muutual Exclusion Algorithm

Kia Makki, University of Nevada, Las Vegas

A distributed system consists of a geographically dissed collection of computer systems which are uniquely identified. The problem of providing for mutual exclusion in a distributed system is a non-trivial one which has inspired a variety of solutions. In this paper we present an efficient algorithm for distributed mutual exclusion in a distributed systems with no shared memory, whose nodes communicate only by messages. Our algorithm is token based and requires constant messages per critical section execution in the best case nd O(n) messages in the worst case.

164

Planar Graphs with Few Vertices of Small Degree

Jerrold R. Griggs and Yan-Chyuan Lin*, U of South Carolina

Erdös and Griggs asked for $a_k(n) = \min|\{v : v \in V(G), deg(v) < k\}|$, where G runs over all planar graphs on n vertices. Constructions of Grünbaum and Motzkin (1963) determine that $a_k(n) = 0$ for $k \leq 5$ and $n \geq 14$ and that $a_6(n)$ is 4 (resp; 5) when $n \geq 7$ is even (resp; odd). Last year, West and Will showed that for $k \geq 12$ and $n \geq n_o(k)$, $a_k(n) = \lceil \frac{(k-8)n+16}{k-6} \rceil$. We can solve nearly all of the remaining cases, which seem to be more complicated. For $k = 7,8,9,10$ and $n > n_o(k)$, $a_k(n) = \lceil \frac{(k-6)n+12}{k-3} \rceil$. We conjecture that for $k = 11$, $a_{11}(n) = \lceil \frac{5n+18}{8} \rceil$ for $n > n_o(11)$; we can prove this for all but one equivalence class of n modulo 48.

Keywords: Planar graph, plane triangulation, Euler relation.

Thursday, February 25, 1993
11:10 a.m.

165

Row-Column Directed Block Designs II

R. C. Hamm* and D. G. Sarvate, College of Charleston

A balanced incomplete block design (BIBD) is called a row-column directed BIBD (RCBIBD) if

(i) it is directed in the usual sense, i.e., each ordered pair of points occurs an equal number of times in the blocks (directed column wise), and

(ii) the blocks are arranged in such a way that

(a) each point occurs an equal number of times in each row, and

(b) each ordered pair of points occurs an almost equal number of times in the rows.

The present note supplements the results given in Row-Column Directed Block Designs, Discrete Mathematics 92 (1991) 321-328. In particular we give construction techniques and a non existence result for RCDBIBDs.

166

Locating Sources and Sinks in Comparability Graphs

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In this work, we consider the study of transitive orientations of a graph. Our approach is different from traditional ones. We focus on maximal cliques of a graph and we find conditions that elements of a clique partition should satisfy for containing sources or sinks of a transitive orientation. We consider also the case when the clique graph is a clique. We show that there exists an edge between sources and sinks of any two maximal cliques. This implies that the cases with at most three maximal cliques are comparability graphs.

key words: transitive orientation, clique graph.

167

A Distributed Algorithm for Deadlock Handling

M. Freeman and A. Amin*, Univ. of Alabama in Huntsville

A distributed algorithm for deadlock handling in distributed database systems is presented. It is based on the Sinha-Natarajan algorithm, and incorporates an optimal strategy for deadlock resolution. A simple proof of correctness of the algorithm is given using the notion of a well formed transaction tree and ensuring that allowed operations on the tree preserve the well formedness of the tree. Some results on a more complicated model of deadlock are also described.

Keywords: distributed algorithm, deadlock.

168

The Complexity of Near-Optimal Graph Imbeddings

Jianer Chen, Saroja P. Kanchi,* Arkady Kanevesky, Texas A&M University
Sanjay Joshi, University of Southern California

It is known that imbedding a graph G into a surface of minimum genus $\gamma_{\min}(G)$ is NP-hard, whereas imbedding a graph G into a surface of maximum genus $\gamma_M(G)$ can be done in polynomial time. In this paper, it is proved that for any function $f(n) = O(n^\epsilon)$, $0 \leq \epsilon < 1$, the problem of imbedding a graph G of n vertices into a surface of genus at most $\gamma_{\min}(G) + f(n)$ remains NP-hard. Moreover, a polynomial time algorithm is presented for imbedding a graph Ginto a surface of genus $\gamma_M(G) - 1$.

Thursday, February 25, 1993
11:30 a.m.

169

Z-Cyclic Wh(v), $v = qp_1^{m_1} \ldots p_n^{m_n} + 1$

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The main result of this study is the following theorem: If q, p_i, $i = 1,\ldots,n$ are primes such that $q \equiv 3 \pmod 4$, $p_i \equiv 1 \pmod 4$, $i = 1,\ldots,n$ and if there exists a Z-cyclic Wh(q+1) then there exists a Z-cyclic Wh(v), $v = qp_1^{m_1} \ldots p_n^{m_n} + 1$

KEY WORDS : block designs, cyclic block designs, whist tournaments, cyclic whist tournaments

170

Results of Cyclic Gossiping in Some Graphs

James A. Knisely, Bob Jones University
Renu Laskar*, Clemson University

Let G be a finite, connected, undirected graph. Gossiping is the total exchange of information between vertices in the graph. Gossiping in G occurs when the information of each vertex has been sent to all other vertices of G. The minimum time required to complete gossiping in a graph will be called the gossiping time of the graph. The color classes of a proper coloring of edges define matchings that can be used to gossip in G. Cyclic gossiping is gossiping that occurs by using those matchings in a cyclic manner.

Cyclic gossiping bounds or times will be given for several classes of graphs including complete bipartite graphs, caterpillars, Fibonnacci and Binomial Trees. A review of previous results in the area of cyclic gossiping will also be given.

Keywords: gossiping, trees

171

STACK-BASED, PROBABILISTIC MODELS FOR GENERATION OF NESTED PARENTHESES

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Considerable progress has been made in modelling the process of generating syntactically correct block-structured programs by a first-order Markov source augmented by a push down stack. If the nesting structure of programs are represented as strings of nested parentheses, the steady state behavior of the source is represented by a set of recurrence relations which is readily solvable. A better model, needed for a variety of control constructs and expressions, is strings of nested parentheses of several "colors". The recurrence relations in this case may not have a simple analytic solution, but the behavior can still be approximated by simulation and applied to actual program data.
This approach yields both a technique for analyzing existing programs to extract information theoretic measures of their syntactic complexity, and for generating sets of random psuedo-programs with similar charateristics to the existing programs.

KEY WORDS: Markov processes, information theory, software measures

172

PROBLEMS RELATED TO CHROMATIC NUMBER OF THE PLANE

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We will survey a good number of new problems and results related to the cromatic number of the plane. Most of them have appeared during just the past year on the pages of *Geombinatorics*, a quarterly dedicated to open problems of combinatorial and discreet geometry.

Thursday, February 25, 1993
11:50 a.m.

173

A survey of Menon difference sets — the abelian case

James A. Davis, U of Richmond   Jonathan Jedwab, H-P Labs

A (v,k,λ) difference set is a k-element subset D of a group G of order v for which the multiset $\{d_1 d_2^{-1} : d_1, d_2 \in D, d_1 \neq d_2\}$ contains each nonidentity element of G exactly λ times. A difference set is called abelian or nonabelian if the underlying group is, and reversible if $\{d^{-1} : d \in D\} = D$. Difference sets are important in design theory because they are equivalent to symmetric (v,k,λ) designs with a regular automorphism group. Abelian difference sets arise naturally in the solution of many problems of signal design in digital communications, including synchronization, radar, coded aperture imaging and optical image alignment. The restriction of reversibility frequently leads to constructive or nonexistence results for difference sets.

A Menon difference set (MDS) has parameters of the form $(v,k,\lambda) = (4N^2, 2N^2 - N, N^2 - N)$; alternative names used by some authors are Hadamard difference set or H-set. The Menon parameters provide the richest source of known examples of difference sets, and account for all known parameter sets (v,k,λ) of reversible difference sets with one exception. The central research question is: for each integer N, which groups of order $4N^2$ support a MDS? This question remains open, for abelian and nonabelian groups, despite a large literature spanning thirty years. The techniques so far used include algebraic number theory, character theory, representation theory, finite geometry and graph theory as well as elementary methods and computer search. Considerable progress has been made recently, both in terms of constructive and nonexistence results. Indeed some of the most surprising advances currently exist only in preprint form, so one intention of this survey is to clarify the status of the subject and to identify future research directions. Another intention is to show the interplay between the study of MDSs and several diverse branches of discrete mathematics.

In this talk we examine the abelian case.

174

On Lower Cost Parallel Algorithms for Tree-Decomposable Graphs

Richard Borie, University of Alabama Arobinda Gupta, University of Iowa

This paper considers parallel algorithms for recognizing a member of a tree-decomposable graph class, finding its balanced tree decomposition, and solving optimization problems on tree-decomposable graph classes. It contributes the following new results:

1. A proof that every n-node k-terminal tree-decomposable graph has a balanced $O(\lg n)$-height tree decomposition of width $2k$.
2. An NC algorithm that finds an $O(\lg n)$-height width-4 tree decomposition for any 2-terminal series-parallel graph. This algorithm requires $O(\lg^2 n)$ time on $O(n^5)$ processors.
3. More practical parallel algorithms for solving optimization problems on tree-decomposable graphs, due to the small width of the tree decomposition while maintaining $O(\lg n)$ height.

175

Counting Left-over: A paradigm for locating local optimal choice

P. S. Nair, Department of Computer Science, Creighton University,

A greedy algorithm always makes the choice that looks best at the moment. It makes a locally optimal choice in the hope that this choice will lead to a globally optimal solution. A paradigm for locating local optimal choice is presented. The power and the simplicity of this scheme is illustrated through a sequence of examples.

Index terms: greedy algorithms, heuristics.

176

A Planar Geometric Graph of Chromatic Number Four

Klaus G. Fischer, George Mason University

Let p and q be square free relatively prime integers so that $p = 3 \bmod 16$, $q = 11 \bmod 16$ and $p \cdot q = 1 \bmod 32$. Let G be the graph whose vertices are points in $Q(\sqrt{p}, \sqrt{q})^2$ with an edge between two of them if their Euclidean distance is 1. We show that this graph has an additive 4–coloring. Specifically, we prove the existence of a group homomorphism $v: \Sigma = \{\sum_1^n \xi_j : \xi_j \in Q(\sqrt{p}, \sqrt{q}, i), \xi_j \cdot \bar{\xi}_j = 1\}$ to $\frac{Z}{(4)}$ so that $v(\xi) \neq 0$ whenever ξ is a complex number in $Q(\sqrt{p}, \sqrt{q})^2$ of complex norm 1. Since the graph $Q(\sqrt{3}, \sqrt{11})^2$ contains finite graphs which require four colors, it follows that the chromatic number of this graph is 4.

Key words: Additive coloring, chromatic number of a graph

Thursday, February 25, 1993
12:10 p.m.

177

A survey of Menon difference sets — the nonabelian case

James A. Davis, U of Richmond Jonathan Jedwab, H-P Labs

The central research question for nonabelian Menon difference sets is the same as for the abelian case: for each integer N, which groups of order $4N^2$ support a MDS? This question has not been settled even for 2-groups, unlike the abelian case, although many constructions are known. The main nonexistence results for nonabelian groups rely on the presence of a large cyclic or dihedral quotient group. On current evidence it appears that the criteria for existence in nonabelian groups are far more complex than for abelian groups. A remarkable recent result is the existence of a particular nonabelian MDS despite the nonexistence of any abelian MDS with the same parameter N.

In this talk we survey the nonabelian case. We also deal with reversibility and open research problems, both in the abelian and nonabelian cases.

178

On sum zero sum problems and greedy colorings

A. Bialostocki and D. Schaal* University of Idaho

In this talk we shall report on some of the recent activity on zero sum problems. We shall present results from computer experiments that support several conjectures and relate zero sum problems to greedy colorings. In particular, we consider the following problem, motivated by a result of Beutelspacher and Brestovansky.

Let Z_m denote the additive cyclic group of order m. Denote by $n = f(m,k)$ ($n = f_{zs}(m, Z_m)$) the minimum integer n such that in every k-coloring (Z_m-coloring), say Δ, of the integer set $S = \{1, 2, \ldots, n\}$ there are m integers in S, $x_1 < x_2 < \ldots < x_m$ which satisfy

(i) $x_1 + x_2 + \ldots + x_{m-1} < x_m$ and

(ii) $\Delta(x_i) = \Delta(x_j)$ for every $i, j \in \{1, 2, \ldots, n\}$ $(\sum_{i=1}^{m} \Delta(x_i) = 0 \text{ in } Z_m)$.

It is not difficult to show that $f(m,2) \leq f(m, Z_m) \leq f(m,m)$.

We will address the following problem: for which m's does $f(m,2) = f(m, Z_m)$?

Key words: Ramsey theory, additive number theory, zero sums, greedy approach.

179

Computing Watchman Route in Parallel

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A watchman route in the presence of polygonal obstacles is a closed path such that each point in the free-space is visible to some point along the route. Finding a shortest watchman route is known to be NP-hard for polygons with holes. For simple polygonal obstacles, shortest watchman route can be computed in polynomial time. We present a parallel algorithm for computing a shortest watchman route in the exterior of a convex polygon. The algorithm runs in O(log n) time in the CREW-PRAM computational model. We also discuss potential approaches for computing shortest watchman route in the presence of a non-convex polygonal obstacle.

Keywords: Visibility, Parallel Algorithms, Computational Geometry

180

INCIDENCE PROPERTIES OF LINES IN THE N-DIM TORUS

Raúl Figueroa and Pablo M. Salzberg*, Department of Mathematics and Computer Sciences, University of Puerto rico, P.O. Box 23355, Río Piedras, Puerto Rico 00931.

Given a finite set L of rational lines in the n-dimensional Euclidean space, we determine the incidence pattern when these lines are immersed in the n-dimensional Torus. Furthermore, we examine how some incidence patterns given initially among the lines in L are reflected after their immersion in the Torus. This problem aroused when studying a new model for CT scanning based on combinatorial properties of finite geometries.

181

Contributions to the Existence of Some Arrays

D.V. Chopra; Wichita State University

An array T with m constraints (rows), N runs (treatment - combinations or columns), and with S levels is merely a matrix of size (m x N) with S symbols (say, 0,1,2,...,S-1). For S=2 the array T is called a binary array. Imposing some combinatorial constraints on T leads us to some interesting and useful structures which are valuable in applications to statistics and combinatorics. In this paper we consider and study binary arrays T with some specified combinatorial structure(s). We also discuss the use of such arrays to statistical design of experiments (in particular to fractional factorial designs).

Key Words: array, constraints, runs, fractional factorial designs, levels of an array.

182

On Direct Computation of Chromatic Numbers of Competition Graphs

J. Richard Lundgren and Sarah K. Merz, U. of Colorado at Denver
Craig W. Rasmussen*, Naval Postgraduate School

The competition graph $C(G)$ of a symmetric digraph D has beenvariously defined to be either the square G^2 or the two-step graph$S_2(G)$, where $G = G(D)$ is the undirected graph underlying D. If G has loops at each vertex, then $C(G) = G^2$; if G is loopless, then $C(G) = S_2(G)$. Previous work on competition graphs has emphasized characterization, not only of the competition graphs themselves but also of those graphs whose competition graphs are interval. The latter characterization is of interest when it is desired to have a competition graph that is easily colorable, e.g. in a scheduling or assignment problem. We now consider the following question: Given a graph G, does the structure of G tell us anything about the chromatic number of the competition graph $C(G)$? Preliminary results show that in some cases we can calculate this chromatic number exactly. We anticipate extension of these results to the more general setting of competition graphs of directed graphs.

183

Partitioning Vertices of Chordal Graphs

Nesrine Abbas* and Lorna Stewart University of Alberta

In this paper we consider the problem of partitioning the vertices of a graph into the minimum number of sets so that a class of monotone functions of the diameters is minimized. We prove that the problem is NP-hard when the input graph is a split graph and the given function is $\max_{X_i \in \chi}$ *diameter*(X_i), where X_i is a set in the partition χ. We also present an efficient parallel algorithm for interval graphs that computes the minimum number of sets for a wide variety of such functions.

Keywords: graph clustering, graph partitioning, chordal graphs, interval graphs, split graphs.

184

Undirected graphs and visibility graphs of simple polygons

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The conditions under which an undirected graph G with n vertices is the visibility graph of a simple polygon of n vertices are studied. A necessary (but not sufficient condition) is that the graph contain a Hamiltonian cycle. Other necessary conditions include the following: 1) G must not have minimal simple cycles of length >= 4, 2) G must not contain certain types of subgraphs of 4 vertices.

Keywords: Graph Algorithms; Computational Geometry

Thursday, February 25, 1993
3:40 p.m.

185

SOME "FUZZY" IDEAS

Mary Deutsch-McLeish, University of Guelph, Canada

This talk will introduce fuzzy sets through a brief mention of a couple of applications in medical diagnosis. One involves fuzzy probabilities and the other fuzzy neural networks. The main focus of the talk, however, will be to introduce the concept of a "fuzzy" combinatorial design. Many algebraic structures have been fuzzified and considerable literature exists on the topic. In particular, properties of fuzzy graphs, groups and relations are well known. It is from these concepts that a 'reasonable' definition of corresponding combinatorial structures is attempted. Usefulness is also an issue, for example, to construct approximate designs when designs of certain orders do not exist.

Concerning experimental designs, there are situations where the topography for an agricultural application may break down and the design parameters cannot be fully met. There can also arise imprecisions in the measurements in carrying out experiments. This talk is intended as a preliminary exploration of some ideas for handling the general question of uncertainty in designs.

186

Embedding edge-colorings into m-edge-connected k-factorizations

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We find necessary and sufficient conditions for the embedding of an edge-colored K_ν into an edge-colored K_n, the edge-coloring being a k-factorization of K_n. The proof uses the Hilton method of amalgamations, which is of interest in itself. Amalgamations are then used to give a different proof of a theorem of Nash-Williams which requires each k-factor in the k-factorization to be 2-edge-connected.

187

UNIT INTERVAL GRAPHS, NICHE GRAPHS, AND CATALAN NUMBERS

Charles A. Anderson* and J. Richard Lundgren
University of Colorado at Denver

A niche graph is a graph constructed from an acyclic digraph by using the same vertex set and inserting edges between each pair of vertices that have arcs to a common vertex and between each pair of vertices that have arcs from a common vertex. We investigate which unit interval graphs are niche graphs, and we show that all unit interval graphs are niche graphs if loops are allowed in the digraph. We use a well-known ordering property for unit interval graphs to define a "properly labeled" unit interval graph, and derive an interesting relationship between the number of properly labeled unit interval graphs and the Catalan numbers.

188

Vertex Prime Graphs and the Jacobsthal Function II

Itshak Borosh, Douglas Hensley and Arthur M. Hobbs* Texas A& M

A graph G with n edges has a vertex prime labeling (v.p.l.) if its edges can be labeled with the integers $1, 2, \ldots, n$ and the g.c.d. of the labels of all edges meeting at any vertex is 1. In a paper by T. Deretsky *et al*, it was conjectured that a union of disjoint cycles has a v.p.l. iff at most one of them is odd. To approach this interesting problem, we introduce the function $k(n) = \min\{m :$ among m consecutive integers, at least one is relatively prime to $4n^2 - 1\}$. This is related through the (odd) prime divisors of $4n^2 - 1$ to the Jacobsthal function.
Using methods developed for bounding the Jacobsthal function, we obtain bounds on $k(n)$ which enable us to show the conjecture is true for the cases

(1) $G = \bigcup_{i=1}^{r} t_i C_{2n_i}$ where $2 \leq n_1 < n_2 < \cdots < n_r$ and $0 < t_i \leq n_i$ for each i
or
(2) $G = tC_{2n}$ for any positive integers t and n.

Thursday, February 25, 1993
4:00 p.m.

189 **NEW EXAMPLES OF r-MULTISETS**

E. Lipkin, School of Math Sciences, Tel Aviv University

We call a multiset A_q of residues modulo q an r-multiset if A_q contains at least r elements $\not\equiv O(modq')$ for any divisor q' of q. Let ≥ 3 and be different from a power of two. For $r \geq 1$ define the function $q(r)$ as the smallest q for which there exists an r-multiset A_q of residues modulo q with no sum of its elements $\Sigma \varepsilon_i a_i$ equal to $2^s(modq)$, where $s \geq s_0$ for some s_0, and $\varepsilon_i = 0$ or 1. We compute values of the step function $q(r)$ for $1 \leq r \leq 20,000$. To make the amount of computations possible for a computer, we reduce computations to r-multisets which are arithmetic progressions $\{-dk_1, \ldots, -d, 0, d, 2d, \ldots, dk_2\}$ modulo q, such that d and q are relatively prime.

KEY WORDS: multiset of residues modulo q; subset sums

190 **Vertex-distinguishing proper edge-colorings**

A.C. Burris* and R.H. Schelp
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An edge-coloring is called *vertex-distinguishing* if any two distinct vertices are incident to different sets of colored edges. The minimum number of colors required for a vertex-distinguishing proper edge-coloring of a simple graph G is denoted by $\chi'_s(G)$. A simple count shows that $\chi'_s(G) \geq \max\{(i!n_i)^{1/i} : 1 \leq i \leq \Delta\}$ where n_i denotes the number of vertices of degree i in G. We prove that $\chi'_s(G) \leq C \max\{n_i^{1/i} : 1 \leq i \leq \Delta\}$ where C is a constant depending only on Δ: Other results concerning regular graphs, trees, and other special classes of graphs will also be presented.

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Frequency Assignment Problems
Barry Tesman, Dickenson College
In certain graph coloring applications there is a restriction as
to the choice of color one may assign to a vertex.  For example,
in the television frequency assignment problem, depending on the
nature of the programming, only certain frequencies from the
UHF and VHF bands may be assigned to a television network. In other
frequency assignment problems, users may get to specify a list
of acceptable assignments.  In this talk, a variant of T-colorings
(which were introduced to address the general frequency assignment)
called  list  T-colorings  is  introduced  which  addresses  this
situation. In particular, frequency assignment where users specify
a consecutive list of acceptable assignments is addressed.
Date: Mon, 8 Feb 93 15:38:13 est
From: "Tesman, Barry" <TESMAN@DICKINSON.EDU>
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192 **A $\log n$ Set Representation of T_n**

Nancy Eaton, The University of Rhode Island

We study set representations of graphs. The following is a generalization of the more familiar intersection representations. We say a family $\mathcal{F} = \{A_x : x \in V\}$ of (not necessarily distinct) sets is called a *general representation* of G if there exists a set L such that $|A_x \cap A_y| \in L \iff \{x, y\} \in E$ for every pair x, y of vertices of G. Let gdim(G) be the minimum size of $|\cup \mathcal{F}|$ taken over all general representations of G. Let T_n be a tree on n vertices. The probabilistic method is used to show that $gdim(T_n) \leq c \log n$ where c is an absolute constant.

Keywords: graph, set representation, tree, probabilistic method.

Thursday, February 25, 1993
4:20 p.m.

193

On a Problem of F.Galvin

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In this paper, a proof of the following theorem conjectured by F.Galvin is given: *Let $n, k \geq 0$ be integers. Given any sets A_j, $j \in J$ with $|A_j| \leq n$ for all $j \in J$ and $\sum_{j \in J} |A_j| < n(k+1)$, there exists a set $X \subseteq \bigcup_{j \in J} A_j$ such that (1) $|X| \leq k$ (2) for any set $I \subseteq J$ with $0 < |I| \leq n$ one has $|\bigcap_{i \in I} A_i \setminus X| \leq n - |I|$.* Some interesting consequences are derived including an application in Transversal Theory. Finally, a polynomial algorithm (in n and k) is presented for the problem of finding the set X described in the theorem.

194

Polychrome Spanning Trees in Small Complete Graphs

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Given a properly edge-colored complete graph K_n, we are interested in determining all spanning subtrees in K_n in which all edges have different colors. Such a subtree is said to be a *polychrome spanning subtree* (PST). We will be interested in small *critical (*sometimes called *minimal)* colorings: the number of colors is n-1. (These are also called 1-factorizations or parallelisms.) For n= 4 there is only one such coloring and it is easy to see that only one of the two trees on 4 nodes occurs as a PST. For n=6, there is again a unique critical coloring, and based on observations of Lars Anderson, it follows easily that only two of the six trees on 6 nodes occur as a PST. For n=8, there are several possible critical colorings and 23 trees on 8 nodes. We determine all PST for two colorings of special interest: the Boolean coloring arising from $(Z_2)^3$ and the coloring $(Z_7)^*$ arising from Z_7 by adding a point at infinity. For the Boolean coloring there are only 5 types of PSTs. For $(Z_7)^*$ there are 15 types of PSTs.

KEY WORDS: edge-coloring, 1-factorizations, parallelisms, spanning trees

195

Hamiltonian Decompositions of Cayley Graphs on Abelian Groups of Odd Order

Jiuqiang Liu
Eastern Michigan University

Alspach has conjectured that any $2k$-regular connected Cayley graph $cay(A, S)$ on a finite abelian group A can be decomposed into k hamiltonian cycles. In this paper, the conjecture is shown to be true if $S = \{s_1, s_2, \cdots, s_k\}$ is a minimal generating set of an abelian group A of odd order.
Key Words: pseudo-cartesian product, color switching. Cayley graph.

196

A Closure Theorem for k-walks

*Gara Pruesse and Derek Corneil, University of Toronto

A j-walk through a graph is a spanning closed walk which visits each vertex at most j times. Among the conditions known to guarantee that a graph has a j-walk is the following generalization of Dirac's theorem [Jackson & Wormald, Pruesse]: If each vertex has degree at least n/(j+1) then the graph has a j-walk. Setting j=1 yields Dirac's theorem. The well-known proof of Dirac's theorem was observed by Bondy and Chvatal to in fact imply the following stronger result: Let G be a graph, and suppose that d(u) + d(v) <= n/2, where u and v are two non-adjacent vertices. Then G is hamiltonian if and only if G + (u,v) is. We show that the closure theorem cannot generalize to j-walks in the manner suggested by the generalized Dirac's theorem. Rather, we prove the following theorem, and show that it is tight.

THEOREM: Let G be a graph, and suppose that d(u) + d(v) <= n-j+1. Then G has a j-walk if and only if G + (u,v) does.

This result is surprising because, while the Bondy-Chvatal closure theorem implies Dirac's theorem, the generalized closure theorem does not (and cannot) imply the generalized Dirac theorem.

Key words: Hamiltonian, j-walk, Dirac's theorem, closure.

Thursday, February 25, 1993
4:40 p.m.

197

A Computer Generation of Polytope Sections of the 5-Cube

C. Caiseda, Inter American University of Puerto Rico

An interesting application of cut-complexes is to generate convex polytopes which are sections of the hypercube. Through the implementation of three graph transformations in *Mathematica*, we are able to generate all distinct convex polytopes which are sections ot the 5-cube. Some computational results are introduced.

198

Nearly Perfect Colorings and Strong Colorings of Graphs

Gayla Domke*, Georgia State University
Jean Dunbar, Converse College
Stephen Hedetniemi, Renu Laskar, Clemson University

Let G=(V,E) be a graph. A set S of vertices is nearly perfect if every vertex v not in S is adjacent to at most one vertex in S. S is a strongly stable set if every vertex v in V is adjacent to at most one vertex in S.

A nearly perfect coloring is a partition of the vertex set into nearly perfect sets. Similarly, a strong coloring is a partition into strongly stable sets. In this talk these concepts will be discussed.

Keywords: coloring, nearly perfect, strongly stable

199

Immersion Order Obstruction Sets

Nancy Kinnersley, University of Kansas

An obstruction set is a finite collection of ''forbidden graphs'' that are minimal with respect to a well-partial-order. As shown by Robertson and Seymour, any family of graphs that is closed under the immersion order has a polynomial-time membership test based on obstruction sets. The proof, however, is nonconstructive and thus no general method exists to find all the graphs in such a set, their number, or even the order of the largest one. In the Min Cut Linear Arrangement Problem, an instance is represented by a graph, and we wish to determine whether there is a linear arrangement (layout) of the vertices of the graph in which no vertical line (cut) drawn between consecutive vertices passes through more than k edges of the graph. When k is fixed, the family of graphs with cutwidth at most k is closed under the immersion order. In this paper we describe families of obstructions for Min Cut and discuss several methodologies that can be used to identify obstructions for this and similar layout problems.

Key Words: Nonconstructive complexity, obstruction sets, well-partial-order

200

Graphs with Stable Inclusive Connectivity Parameters

D. W. Cribb*, R. D. Ringeisen, Clemson University
J. Boland, East Tennessee State University

The inclusive edge (vertex, mixed) connectivity of a vertex v is the minimum number of edges (vertices, graph elements) whose removal yields a subgraph in which v is a cutvertex. All possible combinations of these three parameters with regards to edge addition stability, in which the value of the parameter will remain unchanged after the addition of any edge, is investigated along with other various properties.

Key words: Inclusive Connectivity, Stability

Thursday, February 25, 1993
5:00 p.m.

201

ADHERENCE IN FINITELY GENERATED FREE MONOIDS

M. Ito(*), Faculty of Science, Kyoto Sangyo University, Kyoto 603, Japan
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Key words: Left unitary submonoids Left unitary closures Adherences Regular languages Finitely generated submonoids

Let X be a finite alphabet and let X^* be the free monoid generated by X. Every subset of X^* is called a *language* over X. A language $L \subseteq X^*$ is said to be a *submonoid* of X^* if it is closed under the concatenation operation. If $Lx \cap L \neq \emptyset$ implies $x \in L$, then L is called a *left unitary submonoid*. These submonoids are characterized by the property of having prefix codes as bases. Every language L is contained in a unique smallest left unitary submonoid $\mathcal{Lu}(L)$ called the *left unitary closure* of L. The language $\sigma(L) = L \cup \{x \in X^* \mid Lx \cap L \neq \emptyset\}$ is called the *adherence* of L and $\sigma(L) \subseteq \mathcal{Lu}(L)$. In our talk, relations between the left unitary closure and the adherence of a language are discussed. In particular, we focus our attention on regular languages and we provide an algorithm for determining the prefix code that is the base of $\mathcal{Lu}(L)$ for the case of a regular language L. Furthermore, for finitely generated submonoids S of X^*, we give necessary and sufficient conditions for $\sigma(S)$ to be also a submonoid. Finally, we determine all languages T over X with $|T| \leq 4$ such that $\sigma(T^*)$ is a submonoid.

202

Edge Choosability in Some Regular Graphs

Luis Goddyn, Simon Fraser University, Burnaby, BC

In a graph coloring problem each vertex is allowed to take any one of the k colors in a fixed set {1,2,....,k}. We generalize by assigning to each vertex v its own set A(v) of "allowed" colors. The choosablility problem is to decide whether one can find a proper vertex coloring where each vertex v gets a color beloning to A(v). We say that G is k-choosable if for any function A mapping V(G) to the k-subsets of {1,2,...}, there exists a proper coloring c : V(G) -> {1,2,...} such that c(v) is in A(v) for all v in V(G). Thus, "k-choosable" implies "k-colorable", but not conversely. For example, K_{3,3} is 2-colorable but not 2-choosable (it is 3-choosable). Edge Choosablility is defined similarly.

The Edge Choosablility Conjecture asserts that a graph is k-edge choosable if and only if it is k-edge colorable. Up to now, this conjecture has only been verified for a very small list of graphs. Here we verify this conjecture for the class of regular planar multigraphs and more! The basic idea relies on a surprising theorem of Alon and Tarsi which gives an algebraic sufficient condition for a graph to be k-choosable. This is joint work with Mark Ellingham.

203

Geography --- the game.

Aviezri Fraenkel, Weizmann Institute; Edward Scheinerman, Johns Hopkins University; Daniel Ullman *, George Washington University

The game of (undirected, edge) geography is played by two players who alternately move a token on a graph from one vertex to an adjacent vertex, erasing the edge in between. The player who first has no legal move is the loser. We show that the decision problem of determining whether a position in this game is a win for the first player is PSPACE-complete. Further, the problem remains PSPACE-complete when restricted to planar graphs with maximum degree 3. However, if the underlying graph is bipartite we provide a polynomial time algorithm for deciding whether the position is a win for the first player. In particular, in the version of the game where the first move consists of choosing a vertex, the second player wins if and only if the bipartite adjacency matrix of the graph is square and has odd determinant.

Keywords: Games on graphs, geography, complexity, PSPACE-complete.

204

Graphs that look like trees

Guoli Ding and Bogdan Oporowski*
Louisiana State University

Keywords: tree width, congestion number, bandwidth, cutwidth, tree partition width, tied.

We investigate several graph parameters each of which indicates how closely a graph resembles a tree. These parameters are the congestion number, tree analogs of the bandwidth and cutwidth, the tree partition width, and the tree width. The first main result is that the first three of these parameters are tied, that is, for every class of graphs, if one of them is bounded, then all of them are bounded. Moreover, for classes of graphs of bounded maximal degree, the first three parameters are also tied to the last two. A part of this result provides a proof of a conjecture proposed by Bienstock. The second main result presents a characterization of the classes of graphs of bounded tree partition width in terms of excluded topological minors. This result may be viewed as a finite version of a result of Halin.

205

Ports of Oriented Matroids

Jenny McNulty, University of North Carolina

The vertices of an affine arrangement do not in general form a matroid. Consequently, a special hyperplane must be added to destroy all parallelisms in order to obtain a matroid structure. Using the result that a port of a connected matroid uniquely determines the matroid, it follows that the vertices of an affine "connected" arrangement uniquely determine the vertices of the induced projective arrangement. Orienting each hyperplane of the affine arrangement gives an orientation on the vertices of the affine arrangement and on the vertices of the projective arrangement whenever the resulting matroid is orientable. It will be shown that an oriented port of an oriented matroid, uniquely determines the oriented matroid. In addition, necessary conditions for a port with a given orientation to be an oriented port will be given.

206

Nordhaus-Gaddum Type Theorem For Largest k-Chromatic Subgraphs of Simple Graphs

Joseph R. Barr, California Lutheran University

E.A.Nordhaus and J.W.Gaddum (1956) showed that every graph G of order n satisfies $2\sqrt{n} \leq \chi(G)+\chi(\overline{G}) \leq n+1$, where $\overline{G}$ is the complement of G.

Subsequently an inequality involving some graphical parameters $\mathfrak{P}$, that gives a lower and upper bound to the sum $\mathfrak{P}(G)+\mathfrak{P}(\overline{G})$ is called a Nordhaus-Gaddum (N-G) type inequality.

The graphical parameter $p_k(G)$, the k-partite number of G, is defined to be the least integer r for which there is a subset $E_1 \subset E$ of r edges such that $G-E_1$ is k-colorable. This paper establishes a Nordhaus-Gaddum (N-G) type inequality for the parameter $p_k(G)$. Moreover, a complete list of extremal graphs is given. As a preliminary step to prove the N-G type inequality for p_k, a lemma (lemma 5) that establishes a lower bound for $p_k(G)$ in terms of the number of edges of G is proved. Lemma 5 is effective for dense graphs, and seems seminal for further implementations.

207

HAMILTONIAN PROPERTIES OF DOUBLE VERTEX GRAPHS

Yousef Alavi*, Western Michigan University
Don R. Lick, Eastern Michigan University
Jiuqiang Liu, Western Michigan University

Let G be a (V,E) graph of order $p \geq 2$. The ***double vertex graph*** $U_2(G)$ of G is the graph whose vertex set consists of all 2-elements subsets of V such that two distinct vertices $\{x,y\}$ and $\{u,v\}$ are adjacent if and only if $|\{x,y\} \cap \{u,v\}| = 1$ and if $x = u$, then y and v are adjacent in G. In this paper we present necessary conditions for the double vertex graph $U_2(G)$ of a graph G to be hamiltonian. The complete bipartite graphs $K(m,n)$ for which the double vertex graph $U_2(K(m,n))$ are hamiltonian are characterized.

208

Combinatorics and the Conflict-Free Access Problem

Doreen L. Erickson* and Charles J. Colbourn
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When performing matrix operations in a synchronized parallel environment, one frequently wants access to subarrays of particular shapes known as templates (rows for example). The problem of efficiently storing these templates is commonly referred to as the conflict-free access problem. We discuss the relationship of the conflict-free problem to latin squares. In addition, we relate this problem to that of the chromatic number of a graph and show how this relationship can be employed to find bounds and even solve the conflict-free problem for certain sets of templates. Furthermore, we describe the types of coloring results which would be most relevant. Finally, we describe a problem on permutations whose solutions give solutions to the conflict-free access problem.

keywords: Conflict-free access, latin squares, chromatic number

Friday, February 26, 1993
8:40 a.m.

209

Automorphisms and the eigenvalues of Circulant graphs

Lei Wang, Northeastern University

In this paper the relationship between automorphisms and the eigenvalues of the undirected circulant graphs is studied. A equivalence relation of two vertices is defined by the equality of the two eigenvalues of the adjacency matrix. It is proved that if the connection set of the circulant graph is the union of equivalence classes of the vertices,then the orbit of vertex i under the action of the group of all the automorphisms that fix vertex 0 is included in $[i]$, the equivalence class that contains i. It is also pointed out that if for any vertex i the equivalence class containing i is equal to Ri, where R is a set of multiplication coefficients, then the orbit of i is just $[i]$. The general structure of the automorphism groups of circulant graphs are also studied.

210

Statistics for the Hitting Time of an Asymmetric Random Walk on a Loop

Thomas J. O'Reilly, Saint Joseph's University, Philadelphia, PA 19131

An asymmetric random walk is one in which the probability of a move in one direction is different from the probability of a move in the other direction. For an asymmetric random walk on a loop, difference equations for the expected number of moves needed to visit each node of the loop at least once are developed. Difference equations to solve for the variance are also derived. The result is that these equations are two dimensional vector difference equations whose coefficients are 2x2 matrices. Initial values and recurrence relations are found to solve for these coefficient matrices, and these are used to solve the difference equations. The result is an algorithm of order n, in both storage space and in time, which finds the mean (hitting time) and variance.

Program checks are also described including comparing the theoretical means and variances to sample means and variances. The sample means and variances are obtained by simulating the random walks using random numbers to choose the direction for each step.

211

The Largest Face of a 4-critical Planar Graph with Minimum Degree 4

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The first 4-color-critical planar graph with $\delta \geq 4$ was constructed by Koester in 1985. The graph has 40 vertices and 80 edges, and provides a counterexample to conjectures of Dirac, Gallai and Grötzch. More recently, Koester showed that this type of graph on n vertices exists for all sufficiently large n. These graphs have sparked new interest in this area.

We prove that the size of the largest face of a 4-critical planar graph with $\delta \geq 4$ is at most one half the number of its vertices. Let $f(n)$ denote the maximum of the sizes of largest faces of all such graphs with n vertices (n sufficiently large). We present an infinite family of graphs that shows $\lim_{n\to\infty} \frac{f(n)}{n} = \frac{1}{2}$.

Keywords: color-critical, planar, extremal graph theory

212

Graph theoretical obstacles to perfect hashing

George Havas and B.S. Majewski(*), Department of Computer Science, University of Queensland, Queensland 4072, Australia

A number of algorithms based on quasi-random graphs for generating perfect hash functions have been published. These include Sager's mincycle algorithm [3], a modification by Fox et al. of it [2] and finally probabilistic methods due to Czech, Havas, Majewski [1]. Each of these algorithms exploits different properties of graphs, like bipartitedness [3, 2] or acyclicity [1]. In this paper we formally justify the significance of these properties. Also we indicate causes of failure for some methods. In particular we show that acyclicity of a graph plays a crucial role in finding order preserving perfect hash functions. It is a sufficient but not necessary condition for algorithms to actually find a perfect hash function. We provide some examples for which various published methods methods fail, possibly taking exponential time to do so. Finally, based on our considerations of graph properties we propose yet another method for generating perfect hash functions.

213

Geometric-Algebraic Codes on a class Covering Plane Curves

H. Tapia (U. Autónoma Metropolitana-I, México)*
C. Rentería (I. Politécnico Nacional, México)

(Keywords: Covering curves, Goppa codes, Ideals)

According to V.D.Goppa, a *covering curve* over a finite field $GF(q)$, q being a power of a prime p, is an irreducible algebraic curve passing through all the points of the affine space $A(GF((q))$. The purpose of this talk is to describe a series of algebraic-geometric codes defined over a finite field $GF(q)$ on a class of smooth plane covering curves over $GF(q)$. This class of curves is parametrized by the field $GF(q)$. It is also shown that these codes can be visualized as ideals in a certain group algebra over $GF(q)$. J.P. Hansen and H. Stichtenoth have provided geometric codes with similar properties on different covering plane curves over the field $GF(2^{2n+1})$.

214

Expected performance of a greedy heuristic for the assignment problem

BORIS PITTEL, Department of Mathematics, The Ohio State University, Columbus, OH

A simple heuristic algorithm for the assigment problem is described. For the costs distributed independently and uniformly on the interval [0, 1], we prove that the expected running time is of order $O(n^2)$. The total cost of the assignment delivered by the algorithm is asymptotic, in probability and in the mean, to $(2e-1)/(e-1) \approx 2.58$.

215

CROSSINGS IN PLANAR AND RECTILINEAR DRAWINGS OF GRAPHS

M.L. GARGANO(*), J.W. KENNEDY, L.V. QUINTAS - PACE UNIVERSITY

Let G be a graph and consider the set of all possible drawings of G in the plane. If d is such a drawing, we denote the number of crossings of that drawing by x(d;G). The cross set of G is the set of all attainable values of x(d;G).

Let G be a graph and consider the set of all possible rectilinear drawings of G in the plane. We similarly define the number of crossings of a rectilinear drawing and the rectilinear cross set of G.

This paper considers the elements of the cross set and the rectilinear cross set for paths, cycles, circulants, complete graphs, and complete bipartite graphs.

KEYWORDS : cross set, planar drawing, rectilinear drawing

216

RELAXATION IN A QUADRATIC INTEGER PROGRAM

Gord Sinnamon, University of Western Ontario

Just-In-Time production goals translate into quadratic objective functions but scheduling problems are normally much too large to be accessible by general QIP methods. We present an exact solution to a Just-In-Time scheduling problem that features very fast solution of a relaxed problem (of interest in itself) and generally rapid patching of infeasibilities to produce an optimal solution to the original problem. Testing shows that although an optimal solution is usually found very quickly, long solution times do occur. In this case the algorithm may be interrupted and a simple heuristic applied to produce good, sub-optimal solutions.

Friday, February 26, 1993
10:50 a.m.

217!

Bipartite Graphs and Inverse Sign Patterns of Strong Sign–nonsingular Matrices

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A sign-nonsingular matrix is a matrix B such that each matrix X with the same sign pattern as B is nonsingular. If, in addition, the sign pattern of the inverse of X is the same for all X, then B is a strong sign-nonsingular matrix. A fully indecomposable matrix is a matrix whose associated bipartite graph is connected and (perfect) matching covered. The bipartite graphs of fully indecomposable, strong sign–nonsingular matrices are characterized and a recursive construction is given. This characterization is used to determine the sign patterns of the inverses of fully indecomposable, strong sign–nonsingular matrices, and to develop a recognition algorithm for such sign patterns. Those maximal strong sign–nonsingular matrices B whose sign patterns are uniquely determined by the sign patterns of their inverses are also characterized in terms of bipartite graphs.

218

A GENERALIZATION OF LOCAL CONNECTIVITY

Nell K. Rayburn, Austin Peay State University

A great deal has appeared in the literature about locally connected graphs, much of it concerning sufficient conditions for Hamiltonicity. The purpose of this paper is to establish the existence of graphs with properties involving the connectivity of neighborhoods at distance greater than one. We also present a sufficient condition for a line graph of a graph with maximum degree not exceeding three to be Hamiltonian.

219

k-basis and Independent k-basis in Interval Graphs

Gur Saran Adhar
Univ. of North Carolina at Wilmington

We present *NC* algorithms to compute Connected domination, Independent domination, and Total domination sets in interval graphs. We then use the method developed for domination problem to solve the more general problem of computing K-basis, and Independent K-basis.
The model of computation used is the CRCW P-RAM (Concurrent Read Concurrent Write Parallel RAM), where more than one processor can concurrently read from or write into the same memory location. Writing conflicts are resolved in a non-deterministic fashion.

Keywords: Domination, Total Domination, Connected Domination, Independent Domination, K-basis, Independent K-basis, Interval Graphs NC algorithm

220

An Efficient Algorithm for Bandwidth of an Interval Graph

Alan P. Sprague, U. of Alabama at Birmingham

We present an O(n log n) algorithm which, given an interval graph G and integer k, will determine if the bandwidth of G is at most k. We assume that an interval model of G is given. Two algorithms for this problem have already appeared in the literature; one is flawed, and the other, by Kleitman and Vohra, has time complexity O(kn). Our algorithm is essentially the Kleitman-Vohra algorithm, supported by data structures that enable the O(n log n) running time. The data structures are related to Bentley's segment tree, a well known data structure from computational geometry.

221

Completely regular designs

William J. Martin, University of Vermont

Completely regular subsets in distance-regular graphs were introduced by Delsarte in 1973. The Johnson graphs are distance-regular graphs built from the k-element subsets of a fixed v-set. Thus completely regular subsets in these graphs are combinatorial t-designs. In this talk, I survey results about these "completely regular designs", which seem to be very rare. This class includes some quite interesting designs and some not so interesting. At the end of the talk, I will describe the relationship between these questions and the famous conjecture of Delsarte that Johnson graphs contain no non-trivial perfect codes.

Keywords: t-design, Johnson graph, completely regular subset, perfect code, association scheme.

222

P L A N E S O F O R D E R 11 AND RELATED RESULTS

R.Killgrove,R.Sternfeld,G.Graham,S.Edwards,ISU;R.Tamez,CSCLA

The first two unresolved orders for projective planes are 11 and 12. How many are there of order 11? Are there any of order 12? We have started working on the former question. We have shown no Coxeter-free Fano-free plane with cyclic group for additive loop except the known one. Lueneburg suggested a possible plane with an incidence matrix having 19 copies of the incidence matrix for a Fano configuration down the main diagonal. For now we seek other planes with this property. We relate a local motion of a regular pentagon to the Fibonacci sequence. We use another but not local motion completion from triangles in Steiner systems.

223

On Construction of Subspectral Graphs

*M. Randić, A. F. Kleiner, and X. Guo, Department of Mathematics & Computer Science, Drake University, Des Moines, Iowa 50311, USA

Graph G_s is said to be subspectral to graph G if all eigenvalues of G_s are found among the eigenvalues of G, but G has additional eigenvalues not belonging to G_s. Subspectral graphs apparently are not as common as isospectral or cospectral graphs (when two graphs have the same spectrum). We start by illustrating three pairs of subspectral graphs, each owing its subspectrality to a distinct structural source. The first case will illustrate the role of nodal surfaces when nodes are centered on subset of vertices in G. The second case illustrates graphs which have for vertices a same environment of nonequivalent neighbors. The third pair of subspectral graphs will illustrate the most intriguing case when the relation for neighbors in G_s can be generalized to G. Each of the three distinct cases allow construction of novel subspectral graphs. Moreover classification of vertex neighborhoods allow classification of families of graphs. Such characterization for cycles, complete graphs, vertex transitive graphs, paths, stars and wheels will be outlined.

224

The Joint Distribution of Elastic Buckets in a Data Structure

William Lew† Hosam M. Mahmoud
The IBM Corporation The George Washington University

In a general memory management scheme for random search trees, the space is released in buckets of certain pre-designated sizes. For a search tree with branch factor m, the nodes may hold up to $m-1$ keys. Suppose the buckets of the memory management scheme that can hold less than m keys have key capacities $c_1, \ldots, c_p$. The search tree must then be implemented with multitype nodes of these capacities. After n insertions, let $X_n^{(i)}$ be the number of buckets of type i (i.e. of capacity c_i, $1 \le i \le p$). For the vector $\mathbf{X}_n = (X_n^{(1)}, \ldots, X_n^{(p)})^T$, we determine the asymptotic mean and covariance matrix. Under practical memory management schemes, all variances and covariances experience a phase transition: For $3 \le m \le 26$, all variances and covariances are asymptotically linear in n, for higher branch factors the variances and covariances become a superlinear (but subquadratic) function of n. The joint distribution of $\mathbf{X}_n$ is shown to be multivariate normal in a range of m.

Friday, February 26, 1993
11:30 a.m.

225

The graphs of thin type

Paul Terwilliger, University of Wisconsin, Madison

A finite, undirected, connected graph $G = (X, E)$, with path length distance function ∂, is said to be of *thin type* whenever for all vertices $x, y, z \in X$ with $\partial(x,y) = \partial(x,z)$, and all distances i, j, the number of vertices $w \in X$ such that $\partial(x,w) = i$, $\partial(y,w) = 1$, and $\partial(z,w) = j$ equal s the number of vertices $w' \in X$ such that $\partial(x,w') = i$, $\partial(y,w') = j$, and $\partial(z,w') = 1$. A graph of thin type is either distance regular or bi-distance regular. We will discuss recent progress in the classification of the graphs of thin type.

226

Spreads Of Generalized Quadrangles $GQ(q-1, q+1)$

Aleksandar Jurišić ~~University of Waterloo~~

A connection between spreads of $GQ(q,q)$ and spreads of $GQ(q-1,q+1)$ is established by reversing Payne's general construction of $GQ(q-1,q+1)$ from a $GQ(q,q)$ with a regular point. $AGQ(q-1,q+1)$ with a covering spread can be viewed as a cover of $AG(2,q)$.
Switching between the two spreads of $GQ(2,4)$ is generalized to arbitrary $GQ(s,t)$ with a covering spread. This gives new spreads of $GQ(s,t)$ and implies new infinite families of distance regular covers of complete graphs. determined.

Keywords: spreads, generalized quadrangles, distance regular graphs.

227

EDGE RECONSTRUCTION OF GRAPHS WITH SUFFICIENTLY LARGE CENTER

Andrew Vince and Yongzhi Yang*, University of Florida

It is proved that a graph with pruned center P is edge reconstructible if $p \geq 3$ and $\varepsilon(P) > \frac{1}{4}p(p+3)$, where $\varepsilon(P)$ and p are the number of edges and the number of vertices in P, respectively. In proving the theorem a result of Greenwell and Hemminger on the reconstructibility of the set of branches of a separable graph is improved.

Key Words: graph, reconstruction

228

On multicommodity flows and applications

Farhad Shahrokhi Univ. of North Texas

We consider those instances of concurrent multicommodity flow problem in which the demand graph is a subgraph of K_4 or a subgraph of two star graphs whose roots are adjacent. We provide for efficient approximation algorithms and a constructive proof of max-flow min-cut theorem. As a simple application we show that a variation of edge or vertex isoperimetric problem is solvable in polynomial time.

Friday, February 26, 1993
11:50 a.m.

229

Coefficients of Bitableau Straightening

Rosa Huang, Virginia Tech; Neil L. White*, University of Florida

The straightening algorithm expresses a bitableau as a linear combination of standard bitableaux. The method of interpolants, found by Rutherford for tableaux and generalized to bitableaux with negative letters and places by Clausen, provides the coefficients one at a time, by a combinatorial algorithm which avoids processing a stack of nonstandard tableaux as in the classical straightening algorithm. We generalize this method to the case of bitableaux of mixed negative and positive letters and places, in the sense of the Grosshans-Rota-Stein superalgebra. We describe the relationship of this generalization to other versions of the straightening algorithm, such as the dotted straightening algorithm.

Keywords: Straightening algorithm, bitableaux, interpolants, superalgebra.

230

Constructions of Chiral Polytopes

Barbara Nostrand*, Egon Schulte (Northeastern) and Asia Ivic Weiss (York)

A type of partially ordered structure called an abstract-polytope generalizes the notion of polyhedra in a combinatorial sense. This concept includes all of the classical regular polytopes as well as many well-known configurations. Chiral polytopes are repetitive structures with rotational symmetry which lack reflexive symmetry. The simplest of which can be thought of as a twisted torus or, more correctly, a twisted doughnut. While much is known about regular polytopes, there is still little known about chiral polytopes. Number-theoretic properties are found which predict the existence of such polytopes. We then use hyperbolic honeycombs to construct abstract polytopes with toroidal facets and the rotation group of the resulting polytope to enumerate its vertices and other structures.

KEYWORDS: Regular Polytopes, Chirality, Rotational Symmetry, Projective Linear Groups, Hyperbolic Honeycombs

231

DECOMPOSITION OF COMPLETE TRIPARTITE GRAPHS

R. Balakrishnan and R. Sampath Kumar
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For t dividing $|E(G)|$, the number of edges of a graph G, let $G|t = \{H : G$ is an edge-disjoint union of t copies of $H\}$. Harary and Robinson proposed the following problem: For which odd $t \geq 3$ and parameters m,n and s, is $K(m,n,s)|t \neq \emptyset$? In this paper, we establish the following results.

(i) If $2m+1 \equiv 0 \pmod t$, $t \geq 3$, then $2m+1 \neq 3t$ implies that $K(1,1,m)|t = \emptyset$ and $2m+1 = 3t$ implies that $K(1,1,m)|t = \{P_4\}$.

(ii) If at least two of m, n and s are divisible by t, $t \geq 3$, then $K(m,n,s)|t \neq \emptyset$.

Other results of similar type are also established for certain special values of m,n,s and t. Further, a new decomposition parameter is introduced.

232

A pseudo-Boolean approach to the max-flow problem

Jean-Marie Bourjolly, Concordia University *
Bruno Simeone, University of Rome

It is well-known that finding a minimum source-sink separating cut in a network is equivalent to minimizing a specific quadratic pseudo-Boolean function. In this paper, we show how to solve the latter by using an algorithm that provides a lower bound on the minimum of quadratic pseudo-Boolean functions in general. This algorithm also finds a max-flow and can be implemented efficiently.

Friday, February 26, 1993
12:10 p.m.

233 Authentication Codes and Designs

Rolf S. Rees,* Memorial U. and Douglas R. Stinson, U. Nebraska

We consider the problem of characterizing authentication codes in which the number b of encoding rules is minimum possible as a function of the number k of source states and the number v of messages, given the restriction that the deception probabilities be $P_{d_0} = \frac{k}{v}$ and $P_{d_1} = \frac{k-1}{v-1}$. Some interesting combinatorial designs are obtained.

234

Embedding Symmetric Configurations in Finite Planes

Jane W. Di Paola, FTICA

Except for the Desargues and Pappus configurations, the embedding of symmetric configurations in finite planes has been largely neglected in the literature. In this paper, we find new embeddings for the Mobius 8_3 and the Moebius 12_3 Moreover, a "folk" theorem tells us that every finite projective plane contains as a sub-configuration a cfz.$(n^2-1)_n$. A theorem by Bose gives this symmetric configuration directly in the case of planes derived from finite fields.

235 Computing k-independent Sets on Regular Bipartite Graphs

M.C. Kong* Yijun Zhao*

Given an undirected finite simple graph G=(V,E), the k-independent set problem is to find a maximum cardinality vertex set $S \subseteq V$ of G such that the distance between any two distinct vertices in S is greater than k, where k is a positive integer. For any fixed positive integer k, the k-independent set problem is NP-complete for general graphs. Indeed, it has been shown that the k-independent set problem remains NP-complete even for bipartite graphs when $k \geq 2$. In this paper, we prove the stronger result that the k-independent set problem is NP-complete for regular bipartite graphs. In particular, we prove that, for any fixed positive even integer k, the k-independent set problem remains NP-complete for regular bipartite graphs.

236 Nowhere-zero 4-flows in some regular graphs

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West Virginia University, Morgantown, WV 26506

In [Congressus Numerantium, 44 (1984) 33 - 40], Ellingham proved that if $r \geq 3$ and if G is a r-rugular graph with a 2-factor consisting of two chordless cycles A and B, then either G is hamiltonian (therefore G has a nowhere-zero 4-flow) or G has a subdivision of the Petersen graph. In this note, we shall show that all such graphs have nowhere-zero 5-flows and when $r \geq 4$, all such graphs have nowhere-zero 4-flows, regardless the existence of the subdivision of the Petersen graph.

Friday, February 26, 1993
2:00 p.m.

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Extreme Self-Dual Codes from Combinatorial Designs

Vladimir D. Tonchev Michigan Technological University

Methods for the construction of binary extremal self-dual codes from symmetric designs are discussed. Two new extremal singly-even codes with parameters (42,21,8) and (64,32,12) are constructed from certain symmetric 2-(41,16,6) and 2-(31,10,3) designs.

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On Bounding the Chromatic Number of L-Graphs.

Sean McGuinness, Matematisk Institut Ume} Universitet Ume}, Sweden.

We show that the intersection graph of L-shapes where each L-shape has an infinite vertical stem has its chromatic number bounded by an exponential function of its clique number. The proves a special case of a conjecture of Gyarfas and Lehel.

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A Neural Adaptive Tree Classifier

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A new decision tree classifier is proposed in this paper. The classification tree can be trained using adaptive nonparametric neural net. The tree classifier can combine the traditional decision tree technique, artificial neural network technique and hierarchical process in aspect of structure, training and classification. It is proved that the binary tree trained by this method is optimal. Such decision tree can also be used as cluster. For pattern classification with large samples, the the recognition speed is fast and the recognition rate is high using such binary tree classifier. The simulations have shown very encouraging results. Such classifier have new properties and good performances compared to neural network classifiers and conventional tree classifiers.

Key words: neural network, classifier, decision tree, pattern recognition

240

COUNTING PURE K-CYCLES IN UNITARY CAYLEY GRAPHS

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For each positive integer n we consider the Cayley graph $Cay(Z_n,U_n)$, where Z_n is the ring of integers modulo n and U_n is the multiplicative group of units modulo n. Let $p_k(n)$ denote the number of pure k-cycles of $Cay(Z_n,U_n)$. We show that p_k is a linear combination (with rational coefficients) of multiplicative arithmetic functions. In particular we give explicit formulas for $p_3(n)$ and $p_4(n)$ in terms of the primes dividing n.

Friday, February 26, 1993
2:20 p.m.

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Automorphism Groups of 1-factorizations

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It can be shown that any finite group is the automorphism group of some 1-factorization of the complete graph. In contrast to this, it has been shown that perfect 1-factorizations (1-factorizations for which the union of any two distinct 1-factors is connected) of the complete graph have automorphism groups with a very rigid structure. One can define a 1-factorization to be j-perfect if any union of j distinct 1-factors is connected. In this talk we will outline a number of general results showing how j-perfection (together with certain graph invariants, such as chromatic number) effects the nature of the automorphism group of the 1-factorization. We will also indicate some sharp results for the more specific case of 3-perfect 1-factorizations on the complete graph which parallel some of the results of the perfect (i.e., 2-perfect) case.

242

Vertex Coloring for Weighted Graphs With Application to Timetabling

Lynn Kiaer*, Rose-Hulman Institute of Technology,
Jay Yellen, Florida Institute of Technology.

Many scheduling problems can be modelled as vertex coloring problems, but the simple graph model is inappropriate for a significant number of instances in which the objective is not a conflict-free schedule but a conflict-minimizing one. Such problems frequently arise in university course timetabling, in the scheduling of common final exams, and in scheduling concurrent sessions at a conference or convention. The authors approached these problems by developing a weighted graph model that distinguished between varying levels of conflict and then designed algorithms for coloring its vertices. In this paper the authors briefly describe their weighted graph model for scheduling problems, along with a collection of fast heuristic coloring algorithms and an exact, branch-and-bound algorithm whose success hinges both on the speed of the heuristic algorithms in obtaining feasible solutions and on the existence of an embedded partition matroid that provides quick lower bounds.

243

3-stresses on Complete Tripartite Hypergraphs

Kevin Peterson, University of Florida

A 3-stress on a simplicial complex is an assignment of scalars to the triangles of the simplicial complex so that the corresponding linear combination of the altitude vectors of the triangles around a given edge is zero for every edge.

Under certain geometric conditions we prove that there is a 3-stress on a complete tripartite hypergraph if and only if there is a dependency of the tensor square of the edges through each vertex and a related dependency on the edges through that vertex.

Keywords: 3-stress, rigidity, tripartite hypergraphs, simplicial complexes

244

On the enumeration of M-trees and some related problems

Taojun Lu University of Waterloo (C & O)

An M-tree is a tree in which no two vertices have the same set of neighbors. The oriented and mixed M-trees are defined analogously. Later on when we say a tree we mean a tree which can be undirected, oriented or mixed. In a joint work with R.C.Read (Waterloo) and E.M.Palmer (Michigan State University), the M-trees are enumerated. The corresponding results in terms of cycle-index sums are obtained, and by making labeling substitutions in the cycle-index sums we get enumeration results for labeled M-trees. Some asymptotic results are also discussed.

In contrast to M-trees, the author enumerated what we shall call $\mathcal{M}$-trees (both labeled and unlabeled), trees in which all branches at each vertex are non-isomorphic. Finally, we present a simple proof of A.Schwenk's enumeration result for trees with a forbidden limb, and extend this to the enumeration of trees with more than one forbidden limb. Keywords: Enumeration, M-trees, $\mathcal{M}$-trees and limbs.

Friday, February 26, 1993
2:40 p.m.

245 <u>Symmetric Sequencings of Non-Solvable Groups</u>

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A sequencing of a group G (with order n) is an ordering of its n elements $1 = g_1, g_2, \ldots, g_n$ so that $g_1, g_1g_2, \ldots, g_1g_2\cdots g_n$ is also a listing of all the elements of G. Such a sequencing is called symmetric if $g_i^{-1} = g_{n+2-i}$ for $2 \le i \le n$. It is easy to see that if G admits a symmetric sequencing then G must have a unique element of order two. Recently we were able to show that if G is solvable with a unique element of order two, then G has a symmetric sequencing if and only if G is not Q_8, the quaternian group of order 8. In this talk we will show how the existence of of symmetric sequencings in the non-solvable case can be reduced to the question of the existence of 2-sequencings on certain specific simple groups. In particular, since it is already known that A_5 has a 2-sequencing, we can give an infinite class of non-solvable groups which have symmetric sequencings.

246

An edge coloring model for open shop scheduling problems

Daniel Costa, Swiss Federal Institute of Technology

We present in this talk an edge coloring problem with constraints in a bipartite multigraph. Such a model can be used for solving preemptive open shop scheduling problems with a renewable or a non-renewable resource. The goal is to find under which circumstances there exists a feasible schedule with a given completion time k (k>=the chromatic index of the underlying bipartite multigraph). The problem is known to be NP-Complete in the general case. We discuss some cases which are solvable in polynomial time. Also we will discuss the problem of finding an optimal schedule when there is a cost c(i) for processing a task at a period i.

247

Generalized Steinhaus Graphs

Neal Brand*, U. of North Texas and Margaret Morton U. of Aukland

A generalized Steinhaus graph of order n and type s is a graph with n vertices whose adjacency matrix $(a_{i,j})$ satisfies $a_{i,j} = \sum_{l=0}^{s-1} c_l a_{i-1,j-l} \pmod 2$ where $2 \le i \le n-1$, $i+s-1 \le j \le n$, $c_l \in \{0,1\}$, and $c_{s-1} = 1$. Generalized Steinhaus graphs in which each edge has probability $\frac{1}{2}$ are considered. Harary and Blass have shown that a certain special set of properties are satisfied by almost all graphs and that any first order property of graphs can either be deduced or its negation can be deduced from a finite number of these special properties. We show that almost all generalized Steinhaus graphs satisfy these properties. Therefore, the first order theory of "almost all" generalized Steinhaus graphs of any type is identical with the first order theory of random graphs. We also show that generalized Steinhaus graphs are pseudo-random.

248

Construction and Enumeration of Avoiding Sequences

James T. Lewis, University of Rhode Island

Let x, n, S be positive integers. The sequence $a_1, \ldots, a_n$ of positive integers with sum S is an <u>x-avoiding sequence of length n</u> if $a_i + a_{i+1} + \ldots + a_j \ne x$ whenever $1 \le i \le j \le n$; i.e., no set of consecutive terms sums to x. Such sequences are constructed and enumerated by considering independent sets in the associated "avoidance graph." Conditions for uniqueness are given. Results are also obtained when there is more than one x to be avoided.

Kew words: avoiding sequence, independent set

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Geometric Representations of the Maximal Subgroups of the Hesse Group

Edward Pervin, Software Compositions

The Hesse simple group of order 1451520, which is the rotation group of the 7-dimensional uniform polytope 3_{21}, is known to have eight classes of maximal subgroups. All but two of these classes can easily be described as stabilizers of certain geometric features of the 3_{21}. The two difficult subgroups, $U_3(3){:}2$ and $L_2(8){:}3$, are stabilizers of two distinct Steiner S(2,4,28) systems whose 28 objects are the axes of the 3_{21}, and whose tetrads are equatorial cubes of the 3_{21}.

Key words: polytope, Steiner system

250

HOMOMORPHICALLY FULL GRAPHS

Richard Brewster, Simon Fraser University/Capilano College,
Gary MacGillivray*, University of Victoria

A graph G is a homomorphic image of a graph H if there is a homomorphism of H to G which is onto on both vertices and edges. A graph H is called homomorphically full if it contains each of its homomorphic images as a subgraph. We give characterizations of the homomorphically full graphs in terms of the structure of their neighbourhoods, in terms of forbidden subgraphs, and in terms of other properties of their homomorphic images. We also exhibit a 1-1 correspondence between the set of homomorphically full graphs and a certain collection of posets, and discuss other properties of these (perfect) graphs.

Key words: colouring, homomorphism, perfect graph, poset

251

Least-time Paths in Clustered Temporal Networks

Thomas F. Hain*, University of South Alabama
Christopher Ward, Auburn University

Clustered temporal networks place constraints on time-dependent networks permitting the computation of least-time and minimum hop-count delayed-paths in feasible (polynomial) running times. In this paper a formalism for clustered temporal networks is discussed; a greedy algorithm for computing the least time delay for any path, given a starting time and a pair of nodes, is presented; and the development of this approach using spanning multigraphs to yield a system of algorithms for the determination of shortest paths is introduced. An area of application of these algorithms is in the routing of packets in low altitude multiple satellite (LAMS) networks.

252

A Variation on a Problem of Liu: Enumerating Strips of Numbers

Joel Brawley* and Robert Jamison,
Clemson University, Clemson SC 29634-1907

As a simple illustration of the use of Burnside's Lemma, C.L. Liu in his textbook "Introduction to Combinatorial Mathematics" calculates the number of slips of paper required to represent all n-digit sequences of numbers if a slip can also be read upside down when it makes sense to do so. In this paper we consider a simple variation of Liu's problem whose solution requires a blend of Burnside's Lemma and the Principle of Inclusion-Exclusion as well as a solid understanding of what it means for a group to act as a group of permutations.

Friday, February 26, 1993
3:20 p.m.

253 PSL(3,q) as Totally Irregular Collineation Group

Sonja Radas, University of Florida

Let $PSL(3,q), q$ odd, be a collineation group of a plane Π, such that it containes a perspectivity and is totally irregular (i.e. every point is fixed by a nontrivial collineation). Non-abelian simple strongly irreducible collineation groups containing a perspectivity have been characterised by Reifart and Stroth (1982). They are $PSL(2,q)$, q odd, $PSL(3,q)$, $PSU(3,q)$, A_7 or J_2. Hering and Walker proved that the projective plane which admits $PSL(3,q)$, q odd, as a strongly irreducible collineation group with a perspectivity, is $PG(2,q)$ (1979).

Let $PSL(3,q)$ be totally irregular, not strongly irreducible collineation group with a perspectivity , such that every subgroup of a Singer cycle S fixes the same set of points as S itself. Then Π is either of order q (Desarguesian plane), order q^2 (Desarguesian or generalised Hughes plane) or of order q^3 (Desarguesian or Figueroa plane).

254 Homomorphisms to Edge-Coloured Paths

Richard Brewster, Simon Fraser University/Capilano College

An *edge-coloured graph*, G is a (k+1)-tuple $(V(G), E_1(G), \ldots, E_k(G))$ where $V(G)$ is a set of vertices and each $E_i(G)$ is a set of unordered pairs of vertices; called the edges of colour i. A homomorphism $G \to H$, is a function $f : V(G) \to V(H)$ such that $uv \in E_i(G)$ implies $f(u)f(v) \in E_i(H)$.

We show that for any edge-coloured graph G and any edge-coloured path P, $G \to P$ if and only if the following two conditions hold:

(1) G contains no odd cycles,

(2) for all paths W, $W \to G$ implies $W \to P$.

For classical graphs the first condition alone suffices; where as, Hell and Zhu have shown for oriented graphs the second condition alone suffices.

Key words: homomorphism, colouring.

255 **Algorithms for the Analysis of AND-OR Graph Models**

Renee A. McCauley*, Winthrop University
W.R. Edwards, Jr., University of Southwestern Louisiana

AND-OR graphs are commonly used within the artificial intelligence community to represent soluion spaces for problems that can be naturally decomposed into independent subproblems. AND-OR graphs are also useful as abstractions of the logical structure of programs developed in languages based on the Horn-clause subset of first-order predicate logic, such as Prolog.

This paper presents techniques for "path-like" analysis of AND-OR graph models, a necessary first step for complexity analysis of AND-OR graphs. We define a logical analogue of a path for an AND-OR graph and present algorithms for computing counts of these "paths" for acyclic and cyclic graphs. These algorithms are essentially adaptations of analysis techniques for directed graphs. Also defined is a logical analogue of a path expression for AND-OR graph models.

256 **On the Choice Number of $K_{m,n}$**

D. G. Hoffman and *P. D. Johnson Jr.
Auburn University

For short, let $c(m,n) = c(K_{m,n})$, the choice number of $K_{m,n}$. Suppose $2 \le m \le n$. We assemble some generalities, some estimates due to Erdös, Rubin, Taylor, and Vizing, and show that $c(m,n) = m+1$ if $n \ge m^m$, $c(m,n) = m$ if $(m-1)^{m-1} - (m-2)^{m-1} \le n < m^m$, and $c(m,n) \le m-1$ otherwise.

Key words: choice number.

Friday, February 26, 1993
3:40 p.m.

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PSL(2,q) acting on a projective plane

Kristine Griffin University of Florida

Th action of PSL(2,q) on a projective plane of order n is considered. It is assumed that the action is totally irregular and that there is an involutorial homology. With these assumptions, structure of the plane is established with respect to the group and restrictions are found for q and n. This work follows on the ideas of Hering, Ho, and Goncalez in considering strongly irreducible and totally irregular action of a group on a plane.

Keywords:finite geometry, groups, projective planes.

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A Graph Theoretic Approach to the Classification Problem for Finite State Machines

Robert Goldberg*, Jerry Waxman - Queens College

Let M be a finite state automata and L(m) be its associated language. The classification problem for finite state machine seeks to decide for a machine M whether L(m) is empty, finite of infinite. Standard approaches utilize an exhaustive test of a number of strings exponential in the number of states of M.
To each machine M, there is naturally associated a labeled directed graph G(M) corresponding to the transitions of M. We show that by using the reachability information implicit in G(m), the classification problem may be solved in O(nxnxn) time where n is the number of states of M. We also demonstrate an O(nxn) parallel algorithm using n processors as well as a combinatorial circuit that can decide the classification problem in O(n) time.

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Construction of Isomers

R.Grund, A.Kerber, R.Laue* Math. Dpt. Uni Bayreuth, Germany

General approaches for the construction of discrete structures up to isomorphism explain a strategy for the generation of all chemical isomers of a given brutto formula. Homomorphisms compatible with group actions reduce the objects step by step to connected simple graphs of valency at least 3. Each reduction corresponds to a construction step. The generation strategy combines several steps and allows to check intermediate results against constraints.
The single construction steps are based on a gluing lemma, reducing to double cosets and generalizing some older theorems, and on orderly generation.
A program system MOLGEN is reported which currently for example produces all 4,264,429 isomers for C12H16 in 11m:10s cpu-time on a HP 705 workstation.

keywords: Polya-Theory, DENDRAL, constructions, isomorphisms problems, algorithms, double cosets