Edge Saturation for complete graphs

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Let $\mathcal{C}$ be a family of edge-colored graphs. A t-edge-colored graph G is $(\mathcal{C}, t)$-saturated if G does not contain any element of C, but for any edge $e \notin G$ and any color $i \in [t]$, the addition of e to G in color i creates some element of $\mathcal{C}$ in G. Let $\mathrm{sat}_t(n, \mathcal{C})$ denote the minimum number of edges in a $(\mathcal{C}, t)$-saturated graph on n vertices.
Let $\mathcal{C}_r(H)$ be the family consisting of every edge-colored copy of H in which exactly r colors are used on $E(H)$. We identify $\mathrm{sat}_t(n, \mathcal{C}_2(K_3))$ when $t \geq 2$. For $\mathcal{R}(K_k) = \mathcal{C}_{\binom{k}{2}}(K_k)$, we show $\mathrm{sat}_t(n, \mathcal{R}(K_k)) \geq c \log n$, which improves a lower bound from Barrus, Ferrara, Vandenbusshe, and Wenger, and matches their upper bound. We also identify the order of growth for $\mathrm{sat}_t(n, \mathcal{C}_r(K_k))$ for $t \geq r$ as r ranges from 1 to $\binom{k}{2}$.

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