Embedding Factorizations for 4-uniform Hypergraphs

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Let $\binom{X}{h}$ be the collection of all h-subsets of an n-set $X \supseteq Y$. A coloring (partition) of S is *r-regular* if the number of times each element of X appears in each color class (all sets of the same color) is the same number r. Given an r-regular coloring of $S \subseteq \binom{X}{h}$, we are interested in finding conditions under which this coloring is extendible to an s-regular coloring of $\binom{X}{h}$. The case $h = 2, r = 1$ is extensively studied in the literature and is closely related to completing partial symmetric Latin squares, but very little is known for $h \geqslant 3$. The case $S = \varnothing, s = 1$ was studied by Sylvester in the 18th century, and remained open until the 1970s.

In this paper we completely solve the case $S = \binom{Y}{h}, h = 4$. This settles the first open case of recent problem of Bahmanian and Newman. These results can be seen as extensions of the famous Baranyai's theorem, and make progress toward settling a 40-year-old problem posed by Cameron.

Keywords: embedding, factorization, edge-coloring, decomposition, Baranyai's theorem, amalgamation, detachment