

New bounds for some 3-color Ramsey numbers

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Given graphs G_1, \dots, G_r , the *Ramsey number* $R(G_1, \dots, G_r)$ is the smallest n such that if the edges of K_n are r -colored, there is a copy of G_i in color i for some i . The most famous cases are the “classical” Ramsey numbers $R(K_s, K_t)$, which are notoriously difficult to compute exactly, but other classes of graphs such as books, cycles, and wheels have also received significant interest. Ramsey numbers for cycle graphs C_k have been studied extensively, but in the three color case there remained two gaps in the table of exact small values, namely the upper bounds $R(C_3, C_6, C_6)$ and $R(C_5, C_6, C_6)$ were not tight. We tighten these upper bounds and show $R(C_3, C_6, C_6) = R(C_5, C_6, C_6) = 15$ using Boolean satisfiability (SAT) solvers. Moreover, we improve the lower bound for $R(K_4, K_4 - e, K_4 - e)$ using a generalization of block-circulant and Cayley graphs in nonabelian groups.

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