Color 2-switches and neighborhood (k, λ)-balanced graphs

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For two graphs with the same degree sequence, a classic result in graph theory provides a way to transform one to the other using a sequence of 2-switches. For k-colored graphs we define a generalization of degree sequences and prove an analog of this theorem. For an n-vertex graph G in which each vertex is assigned one of k colors, we define the *color degree matrix* $D(G)$ to be the $n \times (k + 1)$ matrix in which $D_{i,j}$ is the number of color j vertices in the neighborhood of vertex i for $1 \leq j \leq k$, and $D_{i,(k+1)}$ is the color of vertex i.

We define color 2-switches and prove that two k-colored graphs have the same color degree matrix if and only if there is a sequence of color 2-switches that transforms one to the other. Our result is helpful in studying neighborhood (k, λ)-balanced graphs, that is graphs with a k-coloring so that in every vertex neighborhood and for any two colors, the number of vertices of one color differs from that of the other color by at most λ.

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