New results and old and new open problems and conjectures related to the chromatic number of the plane problem

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In 1994, I proved that for any a from the segment [0.41421356237, 0.44721359550], there is a 6-coloring of the plane, in which five colors forbid distance 1 and the 6th color forbids distance a. In 2024, a group of four German mathematicians expanded mysegment. They agreed with my conjecture that for $a = 1$, such 6-coloring does not exist.

This is an example of new results that will be conveyed. I will also present open problems and conjectures that I believe will pave the way for future advances related to the chromatic number of the plane and the chromatic number of Euclidean n-dimensional space.

Much of this material – but not all – in contained in the 2024, Springer New York book "The New Mathematical Coloring Book: Mathematics of Coloring and the Colorful Life of Its Creators." It is about double the volume of the original 2009 edition.