# A Classification of Trees Without Unique Subgraphs

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We study trees without unique subgraphs, in both the induced and non-induced settings. Graphs without unique induced subgraphs, called *anchor-free graphs*, form a natural candidate class for counterexamples to the Graph Reconstruction Conjecture, but their structure remains poorly understood. We develop a framework for studying anchor-free graphs based on a covering lemma that characterizes anchor-freeness via induced subgraph coverings of vertices. Using this framework, we prove that a tree is anchor free if and only it is a path of length one less than a prime. We also introduce the related notion of *unique-subgraph-freeness*, which acts as an edge-based analogue of anchor-freeness. We show that the line graph of every unique-subgraph-free graph is anchor-free and obtain a complete classification of unique-subgraph-free trees as either prime paths, stars, or extended stars. Together, these results provide a classification of trees without unique subgraphs and suggest a general approach for extending the theory beyond trees.
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