Ramsey numbers of signed graphs

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A signed graph is a pair (G, σ) where $G = (V, E)$ is a graph and $\sigma : E(G) \to \{+, -\}$ is a signature which assigns a sign to each edge of G. One well-studied operation on signed graphs is that of switching at a vertex $v \in V(G)$, by which we mean that every edge incident to v has its sign changed. Two signed graphs are called equivalent if one can be obtained from the other by a sequence of vertex switches.

We call a complete signed graph positive (negative) if every edge is positive (negative). We study the following Ramsey problem on signed graphs – for positive integers s and t, what is the smallest n such that every signed complete graph on n vertices has an equivalent signed complete graph containing either a negative K_s or positive K_t? This "signed Ramsey number" is denoted $r_{\pm}(s, t)$. We show how a variety of approaches lead to upper and lower bounds on $r_{\pm}(s, t)$, settle an open problem by establishing the exact value of $r_{\pm}(4, t)$ for every t, and determine the asymptotics of $r_{\pm}(5, t)$ and $r_{\pm}(6, t)$.

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