Finding the Distribution of the Length of the Longest Path of Acyclic Orientations on Complete Multi-Partite Graphs

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Motivated by a question by Peter J. Cameron, we investigate the distribution of the length of the longest path in a randomly sampled acyclic orientation on a graph. Focusing on the setting of complete multipartite graphs, we obtain a generating function by deriving and solving a recursive system of equations. The multivariate generating function we obtain can be viewed as a refinement of a generating function given by Donald E. Knuth for the total number of acyclic orientations on a multi-partite graph; namely, an additional variable u marks the longest path length, and the generating function reduces to Knuth's generating function when $u = 1$. Normalizing our enumerative generating function provides a probability generating function addressing the question of Cameron. We will conclude by discussing evidence toward the conjecture that the the distribution of the longest path length is asymptotically Gaussian.