

Steiner k -Skolem labeled graphs

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A *Skolem labeled graph* is a graph $G = (V, E)$ admitting a vertex labeling onto the set $L = \{1, 2, \dots, n\}$ satisfying:

1. for each $\ell \in L$, exactly two vertices in V which receive label ℓ , and
2. if u and v receive label $\ell \in L$, then $d(u, v) = \ell$.

As an extension, we define a *Steiner k -Skolem labeled graph* to be a graph $G = (V, E)$ admitting a vertex labeling onto the set $L = \{k - 1, k, \dots, k + n - 2\}$ satisfying:

1. for each $\ell \in L$, exactly k vertices in V which receive label ℓ , and
2. if S_ℓ is the set of vertices which receive label $\ell \in L$, then $d(S_\ell) = \ell$, where d is the graph Steiner distance.

Both definitions can be reformulated into strong and weak versions. For $k \geq 3$, we show that paths and cycles of a certain minimum length admit a strong Steiner k -Skolem labeling and that $n \times k$ grid graphs admit weak Steiner k -Skolem labelings. For $k \geq 4$, we prove that every connected graph $G = (V, E)$ is an induced subgraph of a Steiner k -Skolem labeled graph of order at most $k|E|$.

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