From a Chessboard Problem to a Graph Coloring Problem

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A problem dealing with a chessboard has given rise to the concept of proper total domination in graph theory. A set S of vertices in a graph G is a proper total dominating set or a pt-dominating set if every two adjacent vertices of G have a different number of neighbors in S. The number $\sigma_S(v)$ of neighbors of a vertex v in S becomes the color of v. This leads to a proper coloring of the graph G with respect to S or a σ_S-coloring of G. The pt-chromatic number $\chi_{pt}(G)$ of G is the minimum number of distinct colors used by σ_S-colorings among all pt-dominating sets S of G. We present results and open problems in this area of research.

Keywords: proper total dominating set, pt-coloring, pt-chromatic number.