Proper Total Domination in Trees

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A question involving a chess piece called a prince on the 8×8 chessboard leads to a concept in graph theory involving total domination. A vertex u in a graph G totally dominates a vertex v if u is adjacent to v. A subset S of the vertex set of a graph G is a total dominating set for G if every vertex in G is totally dominated by at least one vertex of S. For a total dominating set S of a graph G and a vertex v of G, the number of vertices in S which totally dominate v is denoted by $\sigma_S(v)$. A total dominating set S for a graph G is called a proper total dominating set if $\sigma_S(u) \neq \sigma_S(v)$ for every adjacent pair u and v of vertices of G. While every graph without isolated vertices has a total dominating set, not every graph without isolated vertices has a proper total dominating set. We present results classifying which paths, doubles stars, and star-like trees possess a proper total dominating set. Additionally, we present a sufficient condition for when caterpillars possess a proper total dominating set.

Keywords: trees, total domination, proper total domination.