Sums of cubes over odd-index q-Fibonacci numbers

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We study sums of the form $\sum_{k=1}^{n} F_{mk}^3$ and $\sum_{k=1}^{n} (-1)^k F_{mk}^3$ where m is an odd integer for the generalised Fibonacci numbers defined as $qF_{n-1} + F_{n-2}$.

Here, q is a non-zero integer. We study a generalization of the question to q-Fibonacci numbers which is a slightly modified recursive relation instead of regular Fibonacci numbers. We find summation formula for $\sum_{k=1}^{n} F_{mk}^3$ while q remains fixed.

Keywords: Fibonacci numbers, Lucas numbers, recursion.