## Characterizing Critical Graphs for Oriented Book Embeddings

Stacey McAdams, Louisiana Tech University

Book embeddings play a crucial role in modeling problems in VLSI design and fault-tolerant computing. Traditionally, book embeddings have been studied in the context of undirected graphs and directed acyclic graphs, with strong connections to stack layouts. This work builds upon the research of McAdams and Kanno, which introduced an alternative definition of book embeddings that accommodates oriented graphs containing directed cycles, referred to as dicyclic graphs. Expanding on this framework, we formalize the concept of an oriented book embedding and the corresponding oriented book thickness, the minimum number of pages required for an oriented book embedding. While previous work focused on strictly unidicyclic graphs, we extend these results to strictly bi-dicyclic graphs, characterizing critical graphs for thickness 1 and providing structural insights into their embeddings.

Keywords: book embedding, book thickness, oriented book embedding, oriented book thickness, directed cycle, critical graph