On disjunction convex hulls by lifting

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We study the natural extended-variable formulation for the disjunction of $n + 1$ polytopes in R^d. We demonstrate that the convex hull $\mathcal{D}$ in the natural extended-variable space R^{d+n} is given by full optimal big-M lifting (i) when $d \leq 2$ (and that it is not generally true for $d \geq 3$), and also (ii) under some technical conditions, when the polytopes have a common facet-describing constraint matrix, for arbitrary $d \geq 1$ and $n \geq 1$. We give a broad family of examples with $d \geq 3$ and $n = 1$, where the convex hull is not described after employing all full optimal big-M lifting inequalities, but it is described after one round of MIR inequalities. Additionally, we give some general results on the polyhedral structure of $\mathcal{D}$, and we demonstrate that all facets of $\mathcal{D}$ can be enumerated in polynomial time when d is fixed.

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