Near-factorization of finite groups

Donald L. Kreher, Michigan Technological University

Let $(G, \cdot)$ be a finite multiplicative group with identity e. For $A, B \subseteq G$, define

$$AB = \{gh : g \in A, h \in B\}$$

and note that AB is a multi-set.

We say that (A, B) is a *near-factorization of G with index* λ if $|A| \times |B| = \lambda(|G| - 1)$ and each element of $G \setminus \{e\}$ occurs λ times in the product AB. We abbreviate this by writing $AB = \lambda(G \setminus \{1\})$. If (A, B) is a near-factorization with index λ, then we say that B is a λ-mate of B. A λ-mate with $\lambda = 1$ is simply called a mate.

Some new structural properties of near-factorizations in certain classes of groups are established. In particular if there is a near-factorization (A, B), then there is an explicate formula for B in terms of A. This leads to an efficient method for computing the λ-mate B of a subset $A \subseteq G$, if it exists.

All noncyclic abelian groups of order less than 200 were examined in a search for a possible nontrivial near-factorization with index 1 and all of these possibilities ere ruled out, either by theoretical criteria or by exhaustive computer searches. (In contrast, index 1 near-factorizations in cyclic or dihedral groups are known to exist by previous results.)

Collaborators: Bill Martin, Maura Paterson, and Doug Stinson.

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