Dulmage-Mendelsohn Decomposition for the Minimum Odd Join Problem in Bipartite Graphs

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In matching theory, a series of structural theorems known as canonical decompositions, such as the Gallai-Edmonds decomposition, form the foundation of the theory. These decompositions are uniquely determined for each given graph and describe the structure of all perfect matchings within the graph. As such, they serve as powerful and versatile tools for studying matchings.

The minimum T-join problem is a classical combinatorial problem that encompasses several routing problems, such as the postman tour problem, and has close connections to matching theory. Given a graph G and a set T of vertices, a set F of edges is called a T-join if each vertex in G is incident to an even number of edges in F when it is not in T, and an odd number of edges in F when it is in T.

Although T-join theory exhibits many connections with matching theory, its canonical decomposition theory remains less developed. As such, we investigate canonical decomposition theory for T-joins and obtain a generalization of the Dulmage-Mendelsohn decomposition, a classical canonical decomposition in matching theory applicable to bipartite graphs, for the minimum T-join problem.

Keywords: T-join, postman tour, canonical decomposition, Dulmage-Mendelsohn decomposition