New lower bounds for $R(3,33)$, $R(3,34)$ and four three-colour Ramsey numbers

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We give six new lower bounds for classical Ramsey numbers. A triangle-free Cayley graph on $\mathbb{Z}_{240}$ with independence number 32 shows $R(3,33) \geq 241$, improving the previous bound of 230, and the Burr–Erdős–Faudree–Schelp inequality then gives $R(3,34) \geq 244$ (previously 239). Circulant three-colourings of K_{182}, K_{198}, K_{286} and K_{319} give $R(3,3,11) \geq 183$, $R(3,3,12) \geq 199$, $R(3,3,15) \geq 287$ and $R(3,3,16) \geq 320$, improving bounds of Exoo and Tatarevic (2015) by 5 to 29. The constructions come from a counterexample-guided SAT search over connection sets invariant under a multiplier of small order, which reduces the 120 distance classes of $\mathbb{Z}_{240}$ to 37 orbits; a maximum-clique solver supplies blocking clauses. Each certificate is verified by three independent exact solvers using orbital branching over the stabilizer of a vertex. The two-colour bounds will appear in the next revision of Radziszowski's dynamic survey of small Ramsey numbers.