Full $\mathcal{H}$-colourings

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Given two graphs G and H, a *full homomorphism* from G to H, also called a *full H-colouring* of G, is a function $\varphi: V_G \to V_H$ such that $uv \in E_G$ if and only if $\varphi(u)\varphi(v) \in E_H$. If a full H-colouring exists for a graph G we say that G is fully H-colourable. It is easy to verify that the family of fully H-colourable graphs is hereditary, and hence, it can be characterized by a set of forbidden induced subgraphs. It is known that for any fixed H, the number of forbidden induced subgraphs characterizing fully H-colourable graphs is finite. Thus, for a fixed H, the class of fully H-colourable graphs is polynomial-time recognizable.
For a family $\mathcal{H}$ of graphs we say that a graph G is *fully $\mathcal{H}$-colourable* if there is a fully H-colouring of G for some $H \in \mathcal{H}$. A cardinality argument easily shows that there are choices of $\mathcal{H}$ such that the recognition of fully $\mathcal{H}$-colourable graphs is not decidable. In this talk we will discuss some choices of $\mathcal{H}$ having nice forbidden induced subgraph characterizations, in some cases even yielding linear time recognition algorithms for fully $\mathcal{H}$-colourable graphs.
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