Ted's Polytope

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A perturbation of each extreme point of a subset of extrema of the Birkhoff polytope B_n leads to the creation of a new polytope, called Ted's polytope $\mathcal{T}_n(\epsilon)$. The magnitude of the perturbation is controlled by a parameter $\epsilon = \epsilon(n) > 0$. Observe that B_n can also be defined as the convex hull of both the TSP polytope whose extrema are tours (connected 2-factor 'permutation matrices when viewed as adjacency matrices'), and the set of non-tour permutation extrema (disconnected 2-factor 'permutation matrices'). This is important to note for understanding since it's the set of tours (extrema of the TSP polytope) that are perturbed by stretching them away from the arithmetic centre of B_n by a factor of $1+\epsilon$. This leads to creation of the feasible region $\mathcal{T}_n(\epsilon)$ of a non-compact LP model, that can decide existence of a tour in a graph. But by designing for polynomial-time distinguishable tour extrema embedded in a subspace disjoint from non-tour extrema, **NP-completeness** strongholds come into play, necessarily expressed in a non-compact extended formulation of $\mathcal{T}_n(\epsilon)$. For illustration purposes, the external representation of $\mathcal{T}_4(1)$ is computed yielding 508 facet-inducing inequalities, in contrast to 16 facet-inducing inequalities of B_4.

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