Every tree on n edges decomposes $K_{n,n}$

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We prove that every tree on n edges decomposes the complete bipartite graph $K_{n,n}$. We do so by translating the decomposition problem into a labeling problem, namely $\vec{\beta}$–labeling (oriented beta–labeling). Our proof employs the polynomial method by identifying trees with functions in the transformation monoid $\mathbb{Z}_n^{\mathbb{Z}_n}$. A proof of the graceful tree conjecture (1967) follows as an immediate consequence of the current result.

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