Zonal Labelings of Plane Graphs Using Arbitrary Abelian Groups

Andrew Bowling*, Wabash College

Let G be a plane graph. A *zonal labeling* is a labeling $\ell : V(G) \to \mathbb{Z}_3 \setminus \{0\}$ of the vertices of G with the nonzero elements of $\mathbb{Z}_3$ such that the vertices on the boundary of each region sum to 0 in $\mathbb{Z}_3$. Zonal labelings are of interest in part due to their connection to proper 3-edge colorings of cubic maps and, by extension, the Four Color Theorem. Here we generalize this by allowing our group to be an arbitrary abelian group Γ. We call such a labeling a Γ-*zonal labeling*, and we explore several families of graphs which admit a Γ-zonal labeling for a variety of abelian groups. We also explore an interesting restriction on Γ-zonal labeling which has a strong connection to a certain type of edge coloring in 2-connected plane graphs.

Keywords: plane graphs, graph labelings, edge colorings, abelian groups