

Degree of Unsolvability of Arising from Infinitary Choice

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This result assesses the degree of unsolvability arising from augmenting the class of partial computable functions with a nondeterministic procedure Choice() that axiomatically returns an arbitrary nonnegative integer. That this cannot be realized by finitary nondeterminism is a consequence of the main result. A suggestion of a mathematical model of computation using a technique from quantum information theory is briefly mentioned. It is shown that the halting problem for this augmented class of nondeterministic computable “functions” has the same extremely high degree of unsolvability as a complete set of notations for all constructive ordinals, denoted by \mathbf{O} in the literature. Specifically, a computational procedure that includes calls to Choice() is given for which the set of inputs that produce guaranteed termination is precisely \mathbf{O} . It is shown for this procedure that \mathbf{O} is the basin of attraction for the subset of \mathbf{O} corresponding to the r.e. indices of the empty set. A sketch of the proof of this result is explained by a single simple figure presented together with very short pseudocode.

Keywords: Infinitary choice, attractor basin, recursive ordinals