Digraphs: Homomorphism, Continuity, Differentiation

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The edges of a digraph (V,E) (with no multiple edges, but self-loops for convenience, which we will leave implicit throughout) are often jointly and informally referred to as the topology of the digraph with the vertices sometimes described as its space. This informal topology-like notion can be sharpened to be rigorous by associating to each vertex v, without any changes to the digraph, a structured family of principal filters F_v based on v's neighborhood. The filters in F_v are said to converge to v, and the overall structure ((V,E), {F_v : v in V}) is a *convergence space*. Digraphs are then a special case of convergence spaces, where the intersection over each of the associated filter families is a principle filter. This construction allows continuous mappings from topological spaces into the vertex sets of digraphs, and continuous functions among digraphs are homomorphisms. Differentiation is introduced and follows from function spaces uniformly inheriting convergence structure. Three particular digraphs have a special role that each plays as a convergence space: (1) Sierpinski space as ({p,q} : {(p,q)}); (2) 3 vertices with 5 edges; (3) the rootless complete binary tree.

Keywords: digraph, filter, convergence space