On van der Waerden numbers

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Van der Waerden's theorem (1927) states that for any given positive integers r and k, there is some number N such that if the integers $\{1, 2, \ldots, N\}$ are colored, each with one of r different colors, then there is a k-term arithmetic progression whose elements are of the same color. The least such N is the *van der Waerden number* $W(r, k)$. In general, $w(r; k_1, k_2, \ldots, k_t)$ is the smallest number N such that any coloring of the integers $\{1, 2, \ldots, N\}$ with r colors contains a k_i-term arithmetic progression of color i, for some i. In this paper we prove some bounds on van der Waerden numbers $w(r; k_1, k_2, \ldots, k_t)$ and some properties of monochromatic arithmetic progressions.