Almost all 5-regular graphs have a 3-flow

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Tutte conjectured in 1972 that every 4-edge connected graph has a nowhere-zero 3-flow. This has long been known to be equivalent to the conjecture that every 5-regular 4-edge-connected graph has an edge orientation in which every out-degree is either 1 or 4. Together with Nick Wormald we showed recently that the assertion of the conjecture holds asymptotically almost surely for random 5-regular graphs. It follows that the conjecture holds for almost all 4-edge connected 5-regular graphs.
Jaeger generalised the conjecture of Tutte by conjecturing that for any integer $p \geq 1$, the edges of every $4p$ edge-connected graph can be oriented so that the difference between the out-degree and the in-degree of every vertex is divisible by $2p + 1$. Similar to Tutte's conjecture, it is known that the general case can be reduced to the $(4p + 1)$-regular case. Together with Noga Alon we showed earlier that the assertion of Jaeger's conjecture holds for almost all $(4p + 1)$-regular graphs, provided that p is large enough.

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