Edge rotations in triangulations on the sphere

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A *triangulation* is a finite simple plane graph such that each face is bounded by a cycle of length 3. A famous local transformation in triangulations, called a *diagonal flip*, is to replace an edge e with another diagonal in the quadrilateral region formed by two triangular faces sharing e. It was proved by Mori and Nakamoto that for any surface F^2, there exists $N(F^2)$ such that any two n-vertex triangulations on a surface can be transformed into each other by $O(n)$ diagonal flips if $n \geq N(F^2)$. In this paper, we consider another local transformation in graphs on surfaces, called an *edge rotation*, which fixes exactly one end of an edge e and turn e. It is known that a diagonal flip in an n-vertex triangulation G on a surface F^2 can be represented by a sequence of $O(n)$ edge rotations in a graph H on F^2, where H is obtained from G by removing at least one edge of G. So, it is easy to see that any two n-vertex simple graphs on a surface F^2 can be transformed into each other by $O(n^2)$ edge rotations, where both graphs are obtained from two (distinct) n-vertex triangulations on F^2 and $n \geq N(F^2)$. Then this time, we improve the above estimation to $O(n)$ for the spherical case.

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