On the Values of Independence Polynomials at -1

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The independence polynomial $I(G;x)$ of a graph G is $I(G;x) = \sum_{k=0}^{n} s_k x^k$, where s_k is the number of independent sets in G of size k. The decycling number of a graph G, denoted $\phi(G)$, is the minimum size of a set $S \subseteq V(G)$ such that $G - S$ is acyclic. Engström proved that the independence polynomial satisfies $|I(G;-1)| \leq 2^{\phi(G)}$ for any graph G, and that this bound is best possible. Levit and Mandrescu conjectured that for every positive integer k and integer q with $|q| \leq 2^k$, there is a connected graph G with $\phi(G) = k$ and $I(G;-1) = q$, and provided *ad hoc* constructions in support of this conjecture for all $k \leq 3$, and for $k = 4$ and $q \neq \pm 13$. In this talk we demonstrate that three graph operations—union, join, and a new operation called lateral joining—applied in various sequences to paths and cycles, suffice to prove the conjecture for all $k \leq 5$, and for $k = 6$ and $q \neq \pm 51$.